Financial Risk Management for Designing Multi-echelon Supply Chain Networks Under Demand Uncertainty

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Abstract: This paper presents a methodology to include financial risk management for the design of multiproduct, multi-echelon supply chain networks under uncertainty. The method is in the framework of two-stage stochastic programming. Definitions of financial risk and downside risk are adapted. Using these definitions, financial risk management constraints are introduced and a new two-stage stochastic programming model is established. Case studies illustrate the applicability of such financial risk management. Trade-offs between expected cost and risk are also analyzed.

Keywords: Supply Chain, Financial Risk Management, Downside Risk, Uncertainty

1. Introduction

In today’s highly competitive environment, it is important to maximize the total profit by managing supply chains. A variety of academic disciplines have been focused on supply chain for many years. [1] A supply chain begins with procurement of raw materials and ends with finished products shipped to customers. [2] It is an integrative approach used to manage the inter-related flows of products and information among suppliers, manufacturing plants, warehouses, and customers. [3] This paper considers the design of multiproduct, multi-echelon supply chain networks.

Early research on supply chain network design can be accredited to Geoffrion et al. [4] who firstly formulated a mixed integer linear program (MILP) model, the objective is to decide the optimal location of intermediate distribution facilities between plants and customers. Alikens et al. [5] reviewed significant contributions of best warehouses site planning, and extended the model considering site inventory. Vidal et al. [6] presented an extensive review of strategic production-distribution models. Special emphasis was placed on models for global logistics systems and further research in this area. Tsiakis et al. [2] designed a multiproduct, multi-echelon supply chain network in a MILP formulation. The objective was to minimize of the total cost of the network. Assavapokee et al. [7] proposed a MILP model to solve the problem of designing the infrastructure of the reverse production network to reduce the amount of waste stream. Cafaro et al. [8] presented a mixed-integer nonlinear programming (MINLP) model to optimally determine the structure of shale gas supply chain. Kalaitzidou et al. [9] proposed a model in a MILP framework for the design of supply chain networks by providing flexibility on facilities’ location and operation.

But the above researches were all deterministic. In reality, there are many uncertain factors including supplier changes, output fluctuating, and demand uncertainties. [10] These factors will have impactions on the total profit of supply chain, therefore it’s necessary to develop methods to address the problem of supply chain network design in the presence of uncertainty. In general, there are two approaches which can be used for dealing the problem of supply chain network design under uncertainty: one is probability based method, and the other is scenario based. Talaei et al. [11] proposed an optimization model to design a multi-product closed-loop green supply chain network, in which demands and costs were supposed to be fuzzy parameters. But in real world, it’s hard to obtain the probability distribution of uncertainties, so scenario based method is widely used. [12] Financial risk management is a scenario based method which provides new insights into the trade-offs between risk and profitability of aimed objectives, and has been used in the tactic level of
supply chain. [13] You et al. [14] considered the risk management for mid-term planning under demand and freight rate uncertainty. Cardoso et al. [15] proposed a MILP formulation to integrate different risk measures into planning of closed-loop supply chains. This paper will discuss the financial risk management in the strategic level such that designing of multi-echelon supply chain network.

This article is organized as follows: In section 2, we review the definitions of financial risk and downside risk. In section 3, detailed mathematical formulations of the multi-echelon supply chain network design problem based on the model proposed by Tsiakis et al. [2] are presented. Then the problem is formulated as a two-stage stochastic programming model to take into account risk management constraints. Finally, we demonstrate the effectiveness of this novel approach using several examples. Concluding remarks are given in the last section.

2. Financial Risk Management

2.1. Two-Stage Stochastic Programming

Financial risk can be defined as the probability of not meeting a certain target profit (maximization) or cost (minimization) level. For the two-stage stochastic problem (SP), the financial risk can be expressed by the following equations [13]

\[
\text{Max } E[\text{Profit}] = \sum_{s \in S} p_s q_s^T y_s - c^T x
\]  

s.t. 

\[ A x = b \]  

\[ T_s x + W y_s = h_s \quad \forall s \in S \]  

\[ x \geq 0 \quad x \in X \]  

\[ y_s \geq 0 \quad s \in S \]  

where \( x \) is the first-stage decision variables, \( y_s \) is the optimal second-stage solution for scenario \( s \) with probability \( p_s \). The objective is the second-stage profit minus the first-stage costs. \( q_s, T_s, \) and \( h_s \) are uncertain parameters.

2.2. Definition of Financial Risk

In the two-stage stochastic programming, if the second-stage profit is \( \text{Profit}_s(x) \), then financial risk associated with a target profit \( \Omega \) can be expressed by the following probability

\[
\text{Risk}(x, \Omega) = \sum_{s \in S} p_s z_s(x, \Omega)
\]

where \( p_s \) is the probability of scenario \( s \), and \( z_s \) is a binary variable

\[
z_s(x, \Omega) = \begin{cases} 
1 & \text{if } \text{Profit}_s(x) < \Omega \\
0 & \text{otherwise}
\end{cases} \quad \forall s \in S
\]

The expected value of the profit can be expressed as

\[
E[\text{Profit}(x)] = \sum_{s \in S} p_s \xi_s
\]

The above scenarios \( \xi_s \) are sorted in ascending sequence, such that \( \xi_{s+1} \geq \xi_s \). If the expected profit is known as \( \xi < \text{Profit}(x) < \xi \), then the relationship between expected profit and risk is

\[
E[\text{Profit}] = \bar{\xi} - \sum_{s \in S} \text{Risk}(x, \xi)(\xi_{s+1} - \xi_s)
\]

2.3. Downside Risk

If the definition of \( \delta(x, \Omega) \) is the positive deviation from a profit target \( \Omega \) for design variable \( x \), such that

\[
\delta(x, \Omega) = \begin{cases} 
\Omega - \text{Profit}_s(x) & \text{if } \text{Profit}_s(x) < \Omega \\
0 & \text{otherwise}
\end{cases}
\]

Then for profit with discrete distribution, downside risk can be written as

\[
DRisk(x, \Omega) = \sum_{s \in S} p_s \delta(x, \Omega)
\]

The relationship between expected profit and downside risk can be expressed as

\[
E[\text{Profit}(x)] = \bar{\xi} - DRisk(x, \bar{\xi})
\]

2.4. Financial Risk Constraints

When the objective of a model is to minimize the total costs, and at the same time minimize the financial risk at every profit level, a weight \( p_s, (\rho_s \geq 0) \) is introduced. By using \( \rho_n \), a multi-objective optimization can be reduced to single objective by imposing a penalty for risk at different target profits \( \Omega_n \)

\[
\text{Min } \sum_{s \in S} p_s q_s^T y_s + c^T x + \sum_{s \in S} \sum_{n \in N} p_s \rho_n z_{sn}
\]

s.t. \((2) - (5)\)

\[
q_s^T y_s + c^T x \geq \Omega_n - U_j (1 - z_{sn}) \quad \forall s \in S, n \in N
\]

\[
q_s^T y_s + c^T x \leq \Omega_n + U_j z_{sn} \quad \forall s \in S, n \in N
\]

\(U_j\) is a positive number big enough to ensure only one of (14) and (15) is true, \( z_{sn} \) is a binary variable to calculate the risk of the objective function.

3. Mathematical Models

3.1. Multi-echelon Supply Chain Networks Design

The problem consists of determining the number, capacity, and location of warehouses and distribution centers to be set as well as the transportation links that need to be established, and the flows and production rates of materials in the network.
3.2. Constraints

This paper is based on the model of Tsiakis et al. [2], and financial risk management constraints are applied and analyzed. There are six sorts of constraints. Binary variables and continuous nonnegative variables are included refer to location and logistics.

3.2.1. Network Structure Constraints

\[ X_{mk} \leq Y_m \quad \forall m, k \] (16)

\[ X_{kl} \leq Y_k \quad \forall k, l \] (17)

When warehouse \( m \) exists, there are transportations between warehouse \( m \) and distribution center \( k \) as shown in (16). The relation between distribution center \( k \) and customer \( l \) is expressed as (17).

3.2.2. Logical Constraints

\[ Q_{ijm} \leq Q_{ijm}^m \quad \forall i, j, m \] (18)

\[ Q_{ink} \leq Q_{ink}^m \quad \forall i, m, k \] (19)

\[ Q_{idl} \leq Q_{idl}^m \quad \forall i, k, l \] (20)

\[ \sum_{i} Q_{ink} \geq Q_{ink}^{m} \quad \forall m, k \] (21)

\[ \sum_{i} Q_{idl} \geq Q_{idl}^{m} \quad \forall k, l \] (22)

When distribution center \( k \) exists, there are transportations limitations as shown in (18) to (22).

3.2.3. Material Balances Constraints

\[ P_i = \sum_{m} Q_{ijm} \quad \forall i, j \] (23)

\[ \sum_{i} Q_{ijm} = \sum_{k} Q_{ink} \quad \forall i, m \] (24)

\[ \sum_{m} Q_{ink} = \sum_{l} Q_{idl} \quad \forall i, k \] (25)

\[ \sum_{l} Q_{idl} = Dem_{id} \quad \forall i, l \] (26)

There is no inventory for each site as shown from (23) to (26).

3.2.4. Resource Constraints

\[ p_{ij}^{\min} \leq P_i \leq p_{ij}^{\max} \quad \forall i, j \] (27)

\[ \sum_{j} p_{ij} P_{ij} \leq R_{ie} \quad \forall j, e \] (28)

Resource limitation for each product is written by (27) and (28).

3.2.5. Capacity Constraints of Warehouses and Distribution Centers

\[ W_{m}^{\min} \leq W_{m} \leq W_{m}^{\max} \quad \forall m \] (29)

\[ D_{k}^{\min} \leq D_{k} \leq D_{k}^{\max} \quad \forall k \] (30)

\[ W_{m} \leq \sum_{l} a_{mk} Q_{mlk} \quad \forall m \] (31)

\[ D_{k} \geq \sum_{l} p_{kl} Q_{dl} \quad \forall k \] (32)

Each warehouse and distribution center has capacity limitation for certain product as shown from (29) to (32).

3.2.6. Transportation Costs Constraints

Because the transportation cost is a piecewise linear function of the material flow, the transportation cost between plants and warehouses can be written as

\[ \sum_{r=1}^{NR_{im}} Z_{fjr} = 1 \quad \forall f, j, m \] (33)

\[ \sum_{r=1}^{NR_{jm}} Z_{fjr} \leq Q_{fjr} \leq \sum_{r=1}^{NR_{jm}} Z_{fjr} \] (34)

\[ \sum_{r=1}^{NR_{im}} Z_{fjr} \leq Q_{fjr} \leq \sum_{r=1}^{NR_{im}} Z_{fjr} \] (35)

\[ C_{fjm} = \sum_{r=1}^{NR_{jm}} (Q_{fjr} - \bar{Q}_{fjm, r} Z_{fjr}) \frac{C_{fjm, r} - C_{fjm, r-1}}{\bar{Q}_{fjm} - \bar{Q}_{fjm, r-1}} \quad \forall f, j, m \] (36)

The transportation cost between warehouses and distribution centers can be written as

\[ \sum_{r=1}^{NR_{mk}} Z_{fkr} = 1 \quad \forall f, m, k \] (37)

\[ \sum_{r=1}^{NR_{mk}} Z_{fkr} \leq Q_{fkr} \leq \sum_{r=1}^{NR_{mk}} Z_{fkr} \] (38)

\[ \sum_{r=1}^{NR_{mk}} Z_{fkr} \leq Q_{fkr} \leq \sum_{r=1}^{NR_{mk}} Z_{fkr} \] (39)

\[ C_{fkm} = \sum_{r=1}^{NR_{km}} (Q_{fkr} - \bar{Q}_{fkm, r} Z_{fkr}) \frac{C_{fkm, r} - C_{fkm, r-1}}{\bar{Q}_{fkm} - \bar{Q}_{fkm, r-1}} \quad \forall f, m, k \] (40)

The transportation cost between distribution centers and final customers can be written as

\[ \sum_{r=1}^{NR_{kl}} Z_{fkl} = 1 \quad \forall f, k, l \] (41)

\[ \sum_{r=1}^{NR_{kl}} Z_{fkl} \leq Q_{fkl} \leq \sum_{r=1}^{NR_{kl}} Z_{fkl} \] (42)
3.3. Objective Function

Because the demand \( Dem \) is uncertain, this paper takes scenario-based method, we assume that the probability of scenario \( s \) occurring is \( \psi_s = \frac{1}{S} \), then all variables corresponding to a certain scenario will have one additional index. The objective function can be expressed as

\[
\min \sum_m c_{m}^{WH} w_m + \sum_k c_k^{DH} D_k + \sum_{i,j} c_{ij}^{DI} \psi_{ij} \\
+ \sum_i c_i^{RI} (\sum_j Q_{jm}) + \sum_k c_k^{RI} (\sum_s Q_{ks}) \\
+ \sum_{f,j,m} c_{j,m} f_m + \sum_{f,m,k} c_{f,m,k} + \sum_{f,k,l} c_{f,k,l} \geq \Omega_u - U_s (1 - z_m)
\]

(45)

The first two terms are fixed costs of setting up warehouses and distribution centers which are design variables. The others are control variables related to different scenarios. The third term is manufacturing cost, the fourth and fifth terms are logistic costs of warehouses and distribution centers, the sixth term to the eighth term are the transportation costs of the whole network.

3.4. Financial Risk Management Model

As the objective of the supply chain network design is to minimize the expected cost, once the calculated objective is larger than the expected one, the risk exists. Financial risk management is aim to control the bias between real cost and the expected cost \( \Omega_u \). Two additional constraints of risk management should be added to the former model

\[
\min \sum_m c_{m}^{WH} w_m + \sum_k c_k^{DH} D_k + \sum_{i,j} c_{ij}^{DI} \psi_{ij} \\
+ \sum_i c_i^{RI} (\sum_j Q_{jm}) + \sum_k c_k^{RI} (\sum_s Q_{ks}) \\
+ \sum_{f,j,m} c_{j,m} f_m + \sum_{f,m,k} c_{f,m,k} + \sum_{f,k,l} c_{f,k,l} \geq \Omega_u - U_s (1 - z_m)
\]

(46)

\[
+ \sum_m c_{m}^{RH} w_m + \sum_k c_k^{RH} D_k + \sum_{i,j} c_{ij}^{RI} \psi_{ij} \\
+ \sum_i c_i^{RH} (\sum_j Q_{jm}) + \sum_k c_k^{RH} (\sum_s Q_{ks}) \\
+ \sum_{f,j,m} c_{j,m} f_m + \sum_{f,m,k} c_{f,m,k} + \sum_{f,k,l} c_{f,k,l} \leq \Omega_u + U_s z_m
\]

(47)

The objected function is

\[
\min \sum_m c_{m}^{WH} w_m + \sum_k c_k^{DH} D_k + \sum_{i,j} c_{ij}^{DI} \psi_{ij} \\
+ \sum_i c_i^{RI} (\sum_j Q_{jm}) + \sum_k c_k^{RI} (\sum_s Q_{ks}) \\
+ \sum_{f,j,m} c_{j,m} f_m + \sum_{f,m,k} c_{f,m,k} + \sum_{f,k,l} c_{f,k,l}
\]

(48)

\[
\forall f, k, l
\]

\[
+ \frac{\sum_{s \in S} c_{m}^{WH} w_{m} + \sum_k c_k^{DH} D_k + \sum_{i,j} c_{ij}^{DI} \psi_{ij} \\
+ \sum_i c_i^{RH} (\sum_j Q_{jm}) + \sum_k c_k^{RH} (\sum_s Q_{ks}) \\
+ \sum_{f,j,m} c_{j,m} f_m + \sum_{f,m,k} c_{f,m,k} + \sum_{f,k,l} c_{f,k,l} - \Omega_u - U_s z_m
\]

The objected function is

\[
\min \mu \min \sum_m c_{m}^{WH} w_m + \sum_k c_k^{DH} D_k + \sum_{i,j} c_{ij}^{DI} \psi_{ij} \\
+ \sum_i c_i^{RH} (\sum_j Q_{jm}) + \sum_k c_k^{RH} (\sum_s Q_{ks}) \\
+ \sum_{f,j,m} c_{j,m} f_m + \sum_{f,m,k} c_{f,m,k} + \sum_{f,k,l} c_{f,k,l}
\]

(50)

\[
\forall f, k, l
\]

\[
+ \sum_{s \in S} \psi_s c_s \rho_s z_m) + \sum_{s \in S} \psi_s \delta_s
\]

Decisions vary corresponding to different \( \Omega \). Decisions are conservative when \( \Omega \) is smaller, conversely, decisions are aggressive. So it is helpful for decision makers to make selections for their preference.

4. Case Study

We use the second case of Tsiakis et al. [2]. All models were implemented in Lingo 11.0 on an Intel Core 3.50 GHz/4G RAM platform. The case contains three plants, three warehouses, three distribution centers and eighteen customers with fourteen products demands. Besides three given scenarios, we will increase eighteen scenarios, that the number of scenarios is twenty-one.

4.1. Two-Stage Stochastic Model

We first suppose customer demands of added scenarios are normal distribution, mean values are the same to scenario 2 of Tsiakis et al. [2], variances are 5% of mean values. We assume that all twenty-one scenarios have equally probability, i.e., \( \psi_s = 1/21 \) (\( s = \ldots 21 \)). Without risk management constraints, the risk under expected cost is shown in Figure 1. For the cost of each scenario, its corresponding value shows the risk level. In Figure 1, the dotted line is the expected cost 1957.046 (k£/week) in the two-stage stochastic programming, but it's risk is 35%, that is to say, there exists 35% probability that the real cost building the supply chain network exceeds
the expected value. Therefore financial risk management constraints should be added to help decision makers with their different risk attitudes.

4.2. Financial Risk Management Analysis

To illustrate the usefulness of the risk management constrains, three different cost targets $\Omega$ are used. A set of risks with hypothetical solutions at cost targets with $\rho=100$ are depicted in Figure 2. The operational decisions of warehouses are shown respectively in Table 1.

After managing financial risk, risk at each cost will typically result in better performance around the specific target $\Omega$. For example, From Figure 2, at the cost $= 1950$, the risk of $\Omega = 1900$, 2000, and 2100 are 35%, 25%, and 10% respectively, the risk of the two-stage stochastic model is 50%. It is up to the decision maker to choose $\Omega$ and $\rho$ accordingly with his/her risk preference. When $\Omega = 2000$, there is no risk when expected cost larger than 2250, but the risk is up to 50% when the expected cost is 1800. Therefore downside risk constraints are introduced to measure the risk integral between $\Omega$ and expected costs.

4.3. Downside Risk Analysis

Downside risk management can reduce the risk of specific target $\Omega$, as the objective of the two-stage stochastic model is 1957.046, we assume $\Omega = 1950$. The solution is shown in Figure 3. The full line is the risk of the two-stage stochastic model, the dotted line is the risk with downside risk management constraints. Although objectives of both models are approximately equal, risk for 1950 reduces from 50% to 10%. It is helpful for a risk-averse investor having low risk for some conservative profit aspiration level.

5. Conclusion

Supply chain networks design is a hard task because of the intrinsic complexity and interactions with outer cooperators, as well as the considerable uncertainty in product demands. This paper proposes a detailed two-stage stochastic programming formulation with risk constraints which aims to minimize the risk level at certain expected cost. Under demand uncertainty, the trade-off between risk and total cost is analyzed using downside risk as measurement. The result is able to provide a full spectrum of solutions for decision makers to make choices accordingly with their risk preference.

Notation

- $C$ ——— Cost
- $D$ ——— Distribution center capacity
- Dem ——— Product demand
- N ——— Expected profit selection
- NR ——— Amount of transportation flows
- P ——— Production capacity
- Q ——— Transportation amount
- S ——— Scenario
- U ——— Calculated cost
- W ——— Warehouse capacity
- X, Y, Z ——— Binary variables
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**Superscript**

- $\min$ —— Minimum value
- $\max$ —— Maximum value
- DH —— Distribution center
- PH —— Plant
- WH —— Warehouse

**Subscripts**

- $e$ —— Manufacturing resource
- $f$ —— Product family
- $i$ —— Product
- $j$ —— Plant
- $k$ —— Distribution center
- $l$ —— Customer
- $m$ —— Warehouse
- $n$ —— Weight of certain expected cost
- $r$ —— Discount range of transportation flow cost
- $s$ —— Scenario

**Greek letters**

- $\alpha, \beta, \rho$ —— Index
- $\delta$ —— Risk
- $\xi$ —— Cost
- $\psi$ —— Probability
- $\Omega$ —— Expected cost

**References**


