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Using maple to study the double integral problems

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Abstract: This paper uses the mathematical software Maple as the auxiliary tool to study the evaluation of two types of double integrals. We can find the closed forms of these two types of double integrals by using Euler's formula and finite geometric series. On the other hand, we propose four examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

Keywords: Double Integrals, Euler's Formula, Finite Geometric Series, Closed Forms, Maple

1. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Mozart, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, [1-7] can be adopted as references.

In calculus and engineering mathematics curricula, determining the surface area, the volume under a surface, and the center of mass of a lamina requires using double integrals. Therefore, both the evaluations and numerical calculations of double integrals possess significance, and can be studied based on [8-14]. This paper mainly studies the following two types of double integrals

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\frac{1-y\cos x-y^{n+1}\cos(n+1)x+y^{n+2}\cos nx}{1-2y\cos x+y^2}dxdy \quad (1)$$

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\frac{y\sin x-y^{n+1}\sin(n+1)x+y^{n+2}\sin nx}{1-2y\cos x+y^2}dxdy \quad (2)$$

where n is any non-negative integer, and

$$x_1, x_2, y_1, y_2$$

are real numbers such that

$$(k\pi,(-1)^k)\notin[x_1,x_2]\times[y_1,y_2]$$

for all integers k. These two types of double integrals are related to double integrals studied in [15-20], but our methods are different from the methods used in these papers. The main methods used in [15-20] to determine the double integrals including integration term by term, geometris series, power series expansions, and so on. And the answers of these double integrals obtained are presented in infinite series forms. In this study, we can determine the closed forms of the double integrals (1) and (2) by using the

Euler's formula and the finite geometric series; these are the main results of this study (i.e., Theorems 1 and 2). On the other hand, we propose four double integral examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce the notations and formulas used in this paper.

Notations:

Let $z = a + ib$ be a complex number, where a, b are real numbers. We denote a the real part of z by $\operatorname{Re}(z)$, and b the imaginary part of z by $\operatorname{Im}(z)$.

Euler's Formula:

$e^{ix} = \cos x + i \sin x$, where x is any real number.

Finite Geometric Series:

$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$, where r is a real number, $r \neq 1$ and n is any non-negative integer.

Before deriving the first result of this paper, we need a lemma.

Lemma 1. Suppose that x, y are real numbers, n is any non-negative integer, and $(x, y) \neq (k\pi, (-1)^k)$ for all integers k. Then

$$\frac{1 - y\cos x - y^{n+1}\cos(n+1)x + y^{n+2}\cos nx}{1 - 2y\cos x + y^2} = \sum_{k=0}^{n} y^k \cos kx \tag{3}$$

Proof.

$$\begin{aligned}
&\frac{1 - y\cos x - y^{n+1}\cos(n+1)x + y^{n+2}\cos nx}{1 - 2y\cos x + y^2} \\
&= \frac{(1 - y\cos x)[1 - y^{n+1}\cos(n+1)x] + y^{n+2}\sin(n+1)x \cdot \sin x}{(1 - y\cos x)^2 + (y\sin x)^2} \\
&= \operatorname{Re}\left\{\frac{[1 - y^{n+1}\cos(n+1)x] - iy^{n+1}\sin(n+1)x}{(1 - y\cos x) - iy\sin x}\right\} \\
&= \operatorname{Re}\left\{\frac{1 - y^{n+1} \cdot e^{i(n+1)x}}{1 - y \cdot e^{ix}}\right\} (\text{by Euler's Formula}) \\
&= \operatorname{Re}\left\{\sum_{k=0}^{n} (y \cdot e^{ix})^k\right\} (\text{by Finite Geometric Series}) \\
&= \sum_{k=0}^{n} y^k \cos kx (\text{by Euler's Formula})
\end{aligned}$$

The following is our first result, we obtained the closed form of the double integral (1).

Theorem 1. Assume that n is any non-negative integer, and x_1, x_2, y_1, y_2 are real numbers such that $(k\pi, (-1)^k) \notin [x_1, x_2] \times [y_1, y_2]$ for all integers k. Then

$$\begin{aligned}
&\int_{y_1}^{y_2}\int_{x_1}^{x_2} \frac{1 - y\cos x - y^{n+1}\cos(n+1)x + y^{n+2}\cos nx}{1 - 2y\cos x + y^2} dxdy \\
&= (x_2 - x_1)(y_2 - y_1) + \\
&\sum_{k=1}^{n} \frac{1}{k(k+1)} (\sin kx_2 - \sin kx_1)(y_2^{k+1} - y_1^{k+1})
\end{aligned} \tag{4}$$

Proof.

$$\begin{aligned}
&\int_{y_1}^{y_2}\int_{x_1}^{x_2} \frac{1 - y\cos x - y^{n+1}\cos(n+1)x + y^{n+2}\cos nx}{1 - 2y\cos x + y^2} dxdy \\
&= \int_{y_1}^{y_2}\int_{x_1}^{x_2} \left(\sum_{k=0}^{n} y^k \cos kx\right) dxdy (\text{by Lemma } 1) \\
&= \int_{y_1}^{y_2}\int_{x_1}^{x_2} 1 dxdy + \sum_{k=1}^{n} \int_{y_1}^{y_2}\int_{x_1}^{x_2} y^k \cos kx dxdy \\
&= (x_2 - x_1)(y_2 - y_1) + \\
&\sum_{k=1}^{n} \frac{1}{k} (\sin kx_2 - \sin kx_1) \cdot \int_{y_1}^{y_2} y^k dy \\
&= (x_2 - x_1)(y_2 - y_1) + \\
&\sum_{k=1}^{n} \frac{1}{k(k+1)} (\sin kx_2 - \sin kx_1)(y_2^{k+1} - y_1^{k+1})
\end{aligned}$$

Next, we derive the second result of this paper, and also need a lemma.

Lemma 2. The assumptions are the same as Lemma 1, then

$$\frac{y\sin x - y^{n+1}\sin(n+1)x + y^{n+2}\sin nx}{1 - 2y\cos x + y^2} = \sum_{k=1}^{n} y^k \sin kx \tag{5}$$

Proof.

$$\begin{aligned}
&\frac{y\sin x - y^{n+1}\sin(n+1)x + y^{n+2}\sin nx}{1 - 2y\cos x + y^2} \\
&= \frac{(y\sin x)[1 - y^{n+1}\cos(n+1)x] - (1 - y\cos x) \cdot y^{n+1}\sin(n+1)x}{(1 - y\cos x)^2 + (y\sin x)^2} \\
&= \operatorname{Im}\left\{\frac{[1 - y^{n+1}\cos(n+1)x] - iy^{n+1}\sin(n+1)x}{(1 - y\cos x) - iy\sin x}\right\} \\
&= \operatorname{Im}\left\{\frac{1 - y^{n+1} \cdot e^{i(n+1)x}}{1 - y \cdot e^{ix}}\right\} (\text{by Euler's Formula}) \\
&= \operatorname{Im}\left\{\sum_{k=0}^{n} (y \cdot e^{ix})^k\right\} \\
&= \sum_{k=1}^{n} y^k \sin kx (\text{by Euler's Formula})
\end{aligned}$$

The following is our second result, we determined the closed form of the double integral (2).

Theorem 2. If the assumptions are the same as Theorem 1, then

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\frac{y\sin x-y^{n+1}\sin(n+1)x+y^{n+2}\sin nx}{1-2y\cos x+y^2}dxdy = -\sum_{k=1}^{n}\frac{1}{k(k+1)}(\cos kx_2-\cos kx_1)(y_2^{k+1}-y_1^{k+1}) \tag{6}$$

Proof.

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\frac{y\sin x-y^{n+1}\sin(n+1)x+y^{n+2}\sin nx}{1-2y\cos x+y^2}dxdy$$

$$=\int_{y_1}^{y_2}\int_{x_1}^{x_2}\left(\sum_{k=1}^{n}y^k\sin kx\right)dxdy$$

(by Lemma 2)

$$=\sum_{k=1}^{n}\int_{y_1}^{y_2}\int_{x_1}^{x_2}y^k\sin kxdxdy$$

$$=\sum_{k=1}^{n}\frac{-1}{k}(\cos kx_2-\cos kx_1)\cdot\int_{y_1}^{y_2}y^k dy$$

$$=-\sum_{k=1}^{n}\frac{1}{k(k+1)}(\cos kx_2-\cos kx_1)(y_2^{k+1}-y_1^{k+1})$$

3. Examples

In the following, we aimed at the two types of double integrals we explored, to propose four examples, and use Theorems 1 and 2 to determine their solutions. On the other hand, we employ Maple to calculate the approximations of these double integrals and their closed forms for verifying our answers.

3.1. Example 1

By Theorem 1, we determined the following double integral

$$\int_{2}^{7}\int_{\pi/4}^{\pi/2}\frac{1-y\cos x-y^5\cos 5x+y^6\cos 4x}{1-2y\cos x+y^2}dxdy = \frac{5\pi}{4}+\sum_{k=1}^{4}\frac{1}{k(k+1)}\left(\sin\frac{k\pi}{2}-\sin\frac{k\pi}{4}\right)(7^{k+1}-2^{k+1}) \tag{7}$$

Using Maple to calculate the approximations of this double integral and its closed form as follows.

>evalf(Doubleint((1-y*cos(x)-y^5*cos(5*x)+y^6*cos(4*x))/(1-2*y*cos(x)+y^2),x=Pi/4..Pi/2,y=2..7),18);

-384.603717853869732

>evalf(5*Pi/4+sum(1/(k*(k+1))*(sin(k*Pi/2)-sin(k*Pi/4))*(7^(k+1)-2^(k+1)),k=1..4),18);

-384.603717853869731

3.2. Example 2

Also by Theorem 1, we can evaluate the following double integral

$$\int_{-6}^{-2}\int_{\pi/3}^{5\pi/6}\frac{1-y\cos x-y^{101}\cos 101x+y^{102}\cos 100x}{1-2y\cos x+y^2}dxdy = 2\pi+\sum_{k=1}^{100}\frac{1}{k(k+1)}\left(\sin\frac{5k\pi}{6}-\sin\frac{k\pi}{3}\right)[(-2)^{k+1}-(-6)^{k+1}] \tag{8}$$

Then we use Maple to verify our answer.

>evalf(Doubleint((1-y*cos(x)-y^101*cos(101*x)+y^102*cos(100*x))/(1-2*y*cos(x)+y^2),x=Pi/3..5*Pi/6,y=-6..-2),1);

$-8.47342453683191052\cdot 10^{73}$

>evalf(2*Pi+sum(1/(k*(k+1))*(sin(5*k*Pi/6)-sin(k*Pi/3))*((-2)^(k+1)-(-6)^(k+1)),k=1..100),18);

$-8.47342453683191051\cdot 10^{73}$

3.3. Example 3

Using Theorem 2, we can obtain the following double integral

$$\int_{-1/2}^{1/4}\int_{-5\pi/9}^{4\pi/9}\frac{y\sin x-y^{21}\sin 21x+y^{22}\sin 20x}{1-2y\cos x+y^2}dxdy = -\sum_{k=1}^{20}\frac{\frac{1}{k(k+1)}\left(\cos\frac{4k\pi}{9}-\cos\frac{5k\pi}{9}\right)}{\left[\left(\frac{1}{4}\right)^{k+1}-\left(-\frac{1}{2}\right)^{k+1}\right]} \tag{9}$$

Next, we employ Maple to calculate this double integral and its closed form.

>evalf(Doubleint((y*sin(x)-y^21*sin(21*x)+y^22*sin(20*x))/(1-2*y*cos(x)+y^2),x=-5*Pi/9..4*Pi/9,y=-1/2..1/4),18);

0.0283497926020995993

>evalf(-sum(1/(k*(k+1))*(cos(4*k*Pi/9)-cos(5*k*Pi/9))*((1/4)^(k+1)-(-1/2)^(k+1)),k=1..20),18);

0.0283497926020995995

3.4 Example 4

Also using Theorem 2, we determined the double integral

$$\int_{-2/3}^{1/6}\int_{3\pi/4}^{7\pi/4}\frac{y\sin x-y^{11}\sin 11x+y^{12}\sin 10x}{1-2y\cos x+y^2}dxdy = -\sum_{k=1}^{10}\frac{\frac{1}{k(k+1)}\left(\cos\frac{7k\pi}{4}-\cos\frac{3k\pi}{4}\right)}{\left[\left(\frac{1}{6}\right)^{k+1}-\left(-\frac{2}{3}\right)^{k+1}\right]} \tag{10}$$

We employ Maple to verify our result.

>evalf(Doubleint((y*sin(x)-y^11*sin(11*x)+y^12*sin(10*x))/(1-2*y*cos(x)+y^2),x=3*Pi/4..7*Pi/4,y=-2/3..1/6),18);

0.268559849052527921

```
>evalf(-sum(1/(k*(k+1))*(cos(7*k*Pi/4)-cos(3*k*Pi/4))*((
1/6)^(k+1)-(-2/3)^(k+1)),k=1..10),18);
```

0.268559849052527921

4. Conclusion

As mentioned, the Euler's formula and the finite geometric series play significant roles in the theoretical inferences of this study. These two methods are different from those used in earlier works. In fact, the application of these two methods is extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications.

On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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