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The synchronization of identical Memristors systems via Lyapunov direct method

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Abstract: In this paper, we use Lyapunov direct method to analyze two identical Memristors systems and synchronization phenomena were discussed. The designed controllers were capable of making the time derivative of the Lyapunov's negative definite functions where these results give guarantees of stability of the error dynamics at the origin and proved the results in form of theoretical and numerical ways. As the result, in both cases, one can see the synchronization phenomena.

Keywords: Synchronization; 4-D chaotic System and Lyapunov Direct Method

1. Introduction

The general structure of electronics can be divided in three fundamental circuit elements, which are; the resistor, the inductor and the capacitor.

In 1971, Leon Chua produced a scientific foundation. It was a new circuit element has been named the memory resistor or Memristor [1].

In the last few decades, by discovery of chaotic phenomena, a new kind of oscillating system came to light and it has been named as the chaotic generator. As a result, the chaotic oscillators are found in many dynamical systems of various origins. Its behavior is characterized by instability and limited predictability in time.

The surprising synchronization phenomena generated between coupled chaotic systems has been discovered by Pecora and Carroll in 1990[2]. They proposed that "synchronization can be observed even in chaotic systems". Then, the synchronization of coupled chaotic systems has been investigated in relevant applications of physical systems, intensively [3, 4].

By the results of Pecora's and Carolle's working, there are many progress about synchronization methods in these researches. As we remark, there are linear feedback [3-5], adaptive synchronization [6] among these methods.

Lyapunov direct method is important to understand the stability of synchronization and construction of some synchronization methods.

However it has not been applied directly for synchronization. But in 2009, the results are obtained from Njah and Sunday's seminal working on synchronization of identical and non-identical 4-D chaotic systems via Lyapunov direct method [7]. In this paper, we applied the Lyapunov method directly to observe the synchronization phenomena between two identical memristor systems.

In section 2 we dwelled on the concept of memristor systems, in section 3 we dwelled on Synchronization of Memristors systems about finding the control functions which are important to make the trajectories of the state variables of systems, to track the trajectories of the drive systems and validated the results by using numerical simulations, and section 4 concludes our work.

2. The Memristor System

The general structure of electronics can be divided in three fundamental circuit elements, which are: the resistor, the inductor and the capacitor.

These elements can be explained via a nonlinear relationship between the charge and the magnetic flux as:

$$v = M(q)i \quad and\ i = W(\varphi)v \tag{1}$$

Where v and i are the device of terminal voltage and current, respectively. The nonlinear functions M and W called the Memristance and Memductance.

$$\begin{cases} M \triangleq \dfrac{d\varphi(q)}{dq} \\ W \triangleq \dfrac{dq(\varphi)}{d\varphi} \end{cases} \tag{2}$$

The equations (1) and (2) are taken from [8, 9, 10], we would obtain system from first-order differential equations as:

$$\begin{cases} C_1 \dfrac{dv_1}{dt} = i_3 - W(\varphi)v_1 \\ L\dfrac{di_3}{dt} = v_2 - v_1 \\ C_2 \dfrac{dv_2}{dt} = -i_3 + Gv_2 \\ \dfrac{d\varphi}{dt} = v_1 \end{cases} \tag{3}$$

where,

$$\begin{cases} \dfrac{dq_1}{dt} = i_1 = C_1 \dfrac{dv_1}{dt}, \\ \dfrac{dq_2}{dt} = i_2 = C_2 \dfrac{dv_2}{dt} \\ \dfrac{dq_3}{dt} = i_3 \\ \dfrac{dq_4}{dt} = i_4 = Gv_2 \end{cases} \tag{4}$$

$$\begin{cases} \dfrac{d\varphi_1}{dt} = v_1 \\ \dfrac{d\varphi_2}{dt} = v_2 \\ \dfrac{d\varphi_3}{dt} = v_3 = L\dfrac{di_3}{dt} \\ W(\varphi) = \dfrac{dq(\varphi)}{d\varphi} \end{cases} \tag{5}$$

The behavior of system (3) can be transformed to a first order differential equation as:

$$\begin{cases} \dfrac{dx}{dt} = \alpha(y - W(w)x) \\ \dfrac{dy}{dt} = z - x \\ \dfrac{dz}{dt} = -\beta y + \gamma z \\ \dfrac{dw}{dt} = x \end{cases} \tag{6}$$

Where

$x = v_1, y = i_3,$

$Z = v_2, w = \varphi,$

$\alpha = \dfrac{1}{C_1},$

$\beta = \dfrac{1}{C_2},$

$\gamma = \dfrac{G}{C_2},$

$L = 1.$

The function $W(w)$ can be defined in the form of:

$$W(w) = \frac{dq(w)}{dt} = \begin{cases} a & |w| < 1 \\ b & |w| > 1 \end{cases} \tag{7}$$

Where

$$q(w) = bw + 0.5(a-b)(|w+1| - |w-1|).$$

The parameters of equation (6) and (7) are α, β, γ, a and b, which are all positive.

The system (6) starts to behave as a chaotic attractors by setting $\alpha = 4$, $\beta = 1$, $\gamma = 0.65$, $a = 0.2$ and $b = 10$, as shown in figure (1).

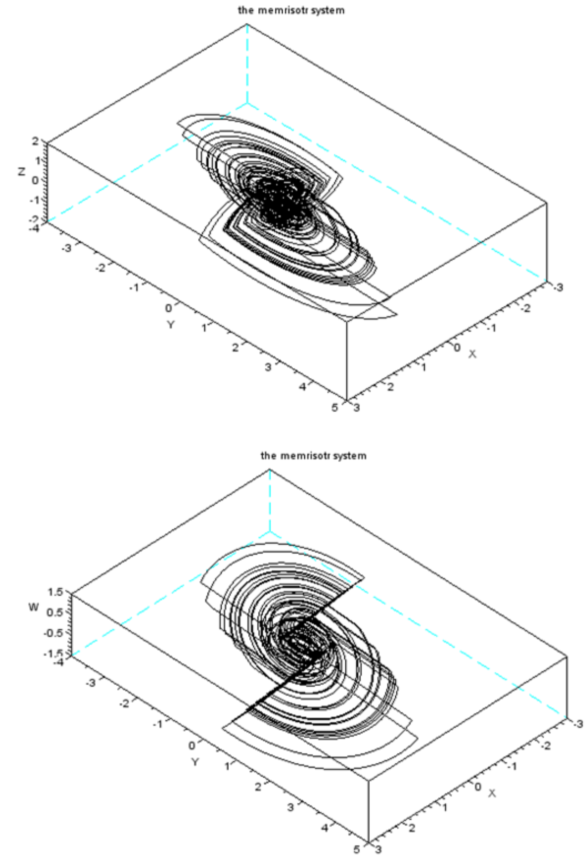


Figure 1. *Chaotic attractors of system (6).*

3. Synchronization of Memristors Systems

3.1. Synchronization between two Identical Memristors Circuits

It is clear in figure.1 that the Memristor system is chaotic and the problem of synchronization between two identical or non-identical systems depends on the control or coupling.

3.1.1. Theoretical Technique Study

Step 1. Fixed points stability

By setting α and $\beta > 0$, the fixed points of system (6) can be defined as the set A, such that $A = \{(x, y, z, w)\ s.t\ x = y = z = 0 \text{ and } w = c\}$, where c is an arbitrary constant. The jacobian matrix at the fixed point of system (6) can be defined as:

$$J = \begin{bmatrix} -\alpha W(w) & \alpha & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -\beta & \gamma & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The characteristic equation of J can be given as:

$$\lambda^4 + (\alpha W(w) - \gamma)\lambda^3 + \big(\beta + \alpha - \alpha\gamma W(w)\big)\lambda^2 + \alpha(\beta W(w) - \gamma)\lambda = 0 \tag{8}$$

Now let $\mu_3 = (\alpha W(w) - \gamma)$, $\mu_2 = \beta + \alpha - \alpha\gamma W(w)$, $\mu_1 = \alpha(\beta W(w) - \gamma)$.

The four eigenvalues $i = 1,2,3$ and 4 are unstable.

Step 2. Control Function Adaptation of (Smooth Function Coupling)

In general the behavior of synchronization chaos relies on the problem between two coupled systems. The control functions provide that the state variables control trajectories of the slave systems and they make possible that the master system tracks-in trajectories.

It is thought that the shortest and easiest technique has been found by [A. N. Njah and O.D. Sunday] in 2009 [7]. Their synchronization method relies on the Lyapunov direct method.

Let consider two identical Memristor systems, the first system being of our interest because of its chaotic behavior. It is given by:

$$\begin{cases} \dot{x}_1 = \alpha(y_1 - W(w_1)x_1) \\ \dot{y}_1 = z_1 - x_1 \\ \dot{z}_1 = -\beta y_1 + \gamma z_1 \\ \dot{w}_1 = x_1 \end{cases} \tag{9}$$

The second system which drives a similar Memristor system is given by:

$$\begin{cases} \dot{x}_2 = \alpha(y_2 - W(w_2)x_2) + v_1(h) \\ \dot{y}_2 = z_2 - x_2 + v_2(h) \\ \dot{z}_2 = -\beta y_2 + \gamma z_2 + v_3(h) \\ \dot{w}_2 = x_2 + v_4(h) \end{cases} \tag{10}$$

Where $v(t) = [v_1(t), v_2(t), v_3(t), v_4(t)]^T$ are control functions, which lead us to demonstrate the synchronizability of system (9) with system (10). We consider afterwards the error state or transfer state among the variables of the master-slave system as:

$$x_\perp = x_2 - x_1 ,\ y_\perp = y_2 - y_1, z_\perp = z_2 - z_1 \text{ and } w_\perp = w_2 - w_1 \tag{11}$$

$$x_2 = x_\perp + x_1, y_2 = y_\perp + y_1, z_2 = z_\perp + z_1 \text{ and } w_2 = w_\perp + w_1 \tag{12}$$

Subtracting equation (9) from equation (10) and using the notation (11) and (12), we obtain the dynamic error:

$$\begin{cases} \dot{x}_\perp = \alpha y_\perp - \alpha W(w_\perp + w_1) * (x_\perp + x_1) + \alpha W(w_1) * x_1 + v_1(h) \\ \dot{y}_\perp = z_\perp - x_\perp + v_2(h) \\ \dot{z}_\perp = -\beta\, y_\perp + \gamma\, z_\perp + v_3(h) \\ \dot{w}_\perp = x_\perp + v_4(h) \end{cases} \tag{13}$$

By using the Lyapunov direct method, we consider the Lyapunov function.

$$V(x_\perp, y_\perp, z_\perp, w_\perp) = \frac{1}{2}\sum_i^4 c_i X_\perp^2 \tag{14}$$

Where $c_i > 0$ *are constant coefficients and* $X_\perp = (x_\perp, y_\perp, z_\perp, w_\perp)$.

Here we need to find the best function $v(t)$, which satisfies the conditions of the strict Lyapunov function. This means $\dot{V} < 0$ at the fixed point of system (6).

However,

$$\dot{V}(x_\perp, y_\perp, z_\perp, w_\perp) = c_1 x_\perp \dot{x}_\perp + c_2 y_\perp \dot{y}_\perp + c_3 z_\perp \dot{z}_\perp + c_4 w_\perp \dot{w}_\perp \tag{15}$$

By putting (13) in (15) and establishing $v(t)$:

$$\begin{cases} v_1(h) = -\alpha y_\perp + \alpha\, W(w_\perp + w_1) * (x_\perp + x_1) - \alpha\, W(w_1)\, x_1 - x_\perp \\ v_2(h) = -z_\perp + x_\perp - y_\perp \\ v_3(h) = \beta\, y_\perp - z_\perp(\gamma + 1) \\ v_4(h) = -x_\perp - w_\perp \end{cases} \tag{16}$$

Now equation (15) becomes:

$$\dot{V}(x_\perp, y_\perp, z_\perp, w_\perp) = -c_1 x_\perp^2 - c_2 y_\perp^2 - c_3 z_\perp^2 - c_4 w_\perp^2 \tag{17}$$

As the result $\dot{V}$ is negative definite at the fixed point (0, 0, 0, 0).

Thus from (17), the Lyapunov function V is positive definite and the stability of system (13) will be guaranteed at the origin.

3.1.2. Numerical Technique Study of Bidirectional Two Coupled Memristor

We will present some numerical tests to verify the related functions between the two systems. Then, we use the fourth Runge-Kutta method to solve the system (9) and (10), which are associated with equation (17), by being placed in the initial points and by integrating the differential equations with a time step as well as the parameters of figure 1.

Step 1. Relation Function Adaptation

Figure 2 (a), (b), (c) and (d) present the results of two identical Memrisotr systems without affecting the related functions.

Figure 3 (a), (b), (c) and (d) demonstrate the onset of adaptation on related functions between two systems which means that all variables of the coupled chaotic subsystems converge. More clear, x_2 converges to x_1, y_2 converge to y_1, z_2 converge to z_1 and w_2 converge to w_1.

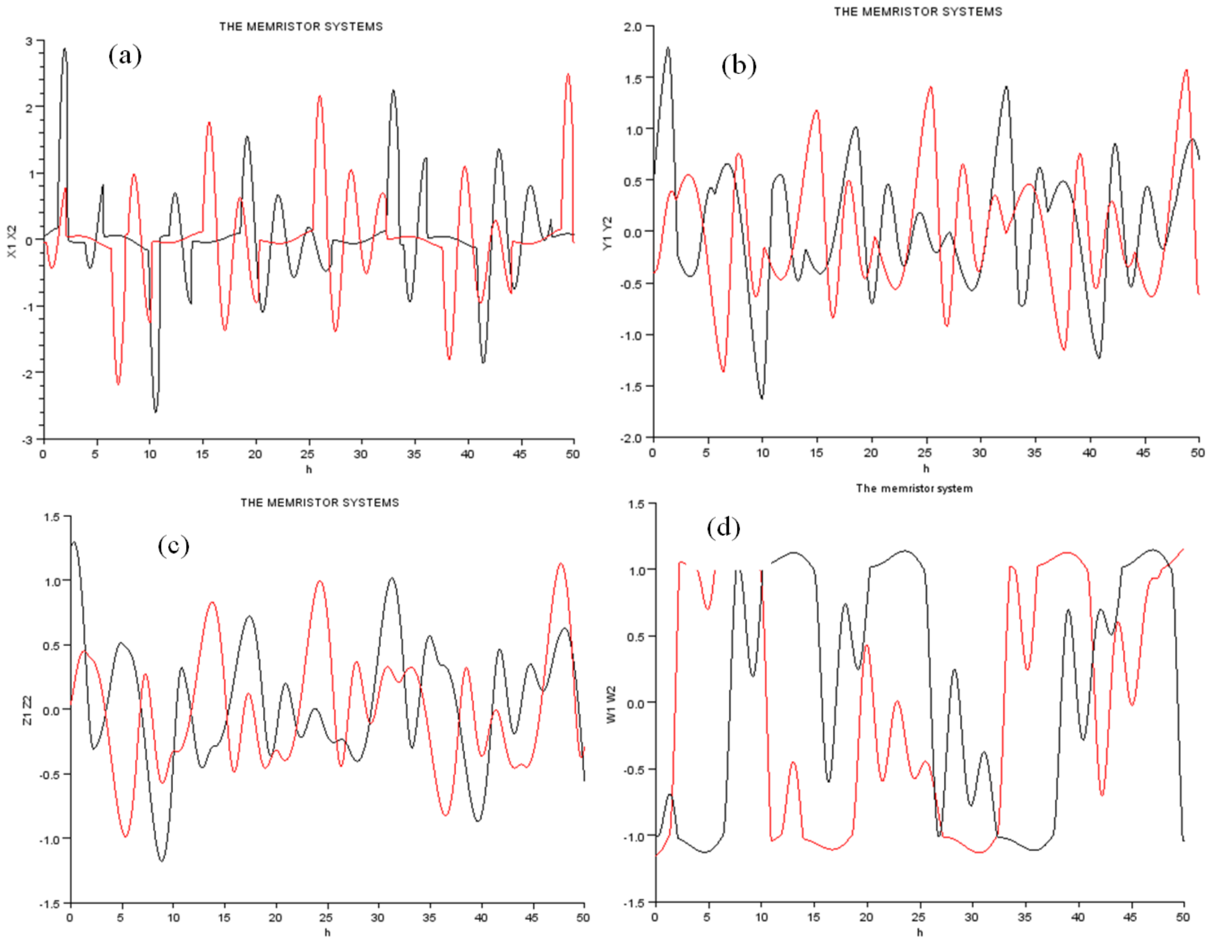
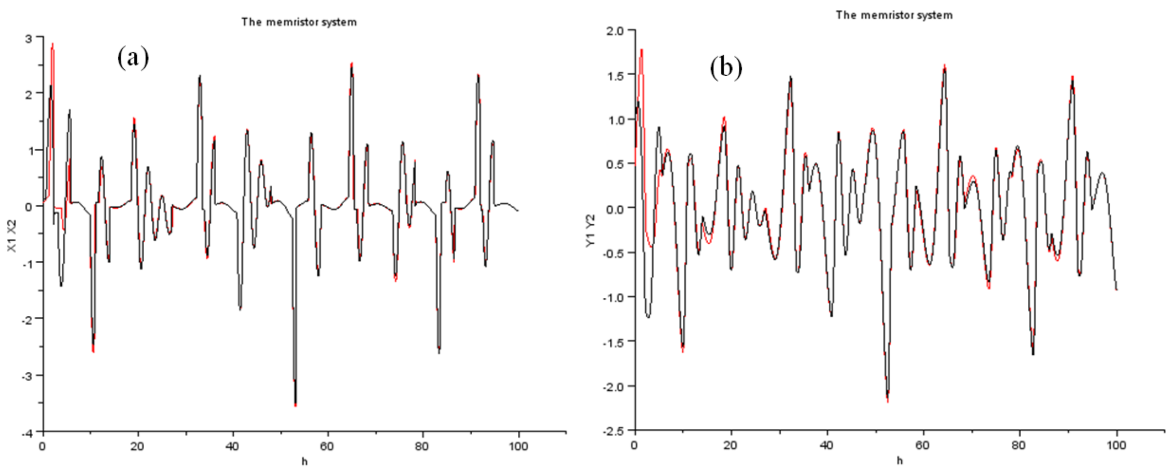


Figure 2. *The time series for variables system (9) and (10) without adaptive relation function.*



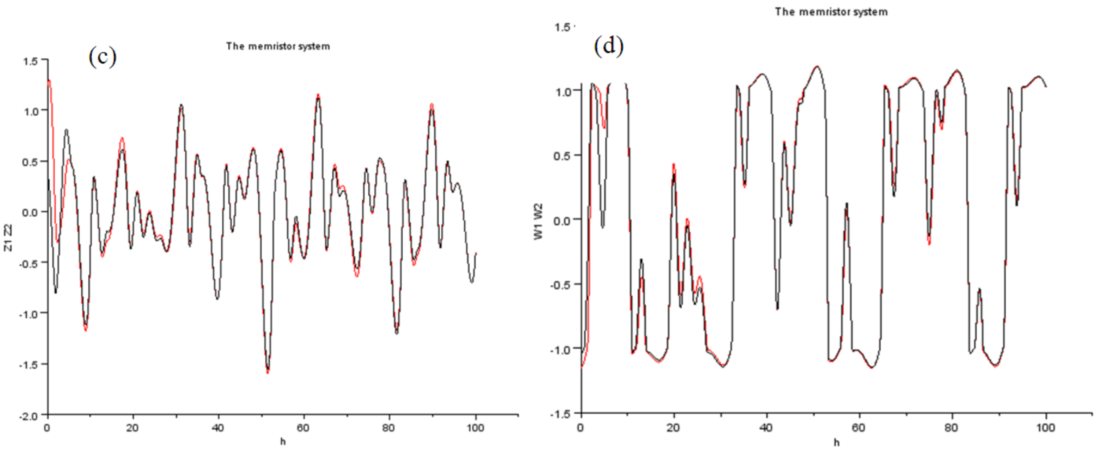


Figure 3. *The time series for variables system (9) and (10) with adaptive relation functions.*

Step 2. Active Synchronization of Chaos Memristor Systems

We try to prove the idea of synchronization by numerically testing the x_1 plotting against the x_2 plotting, and by taking different coupling strengths with time steps h=0.01.

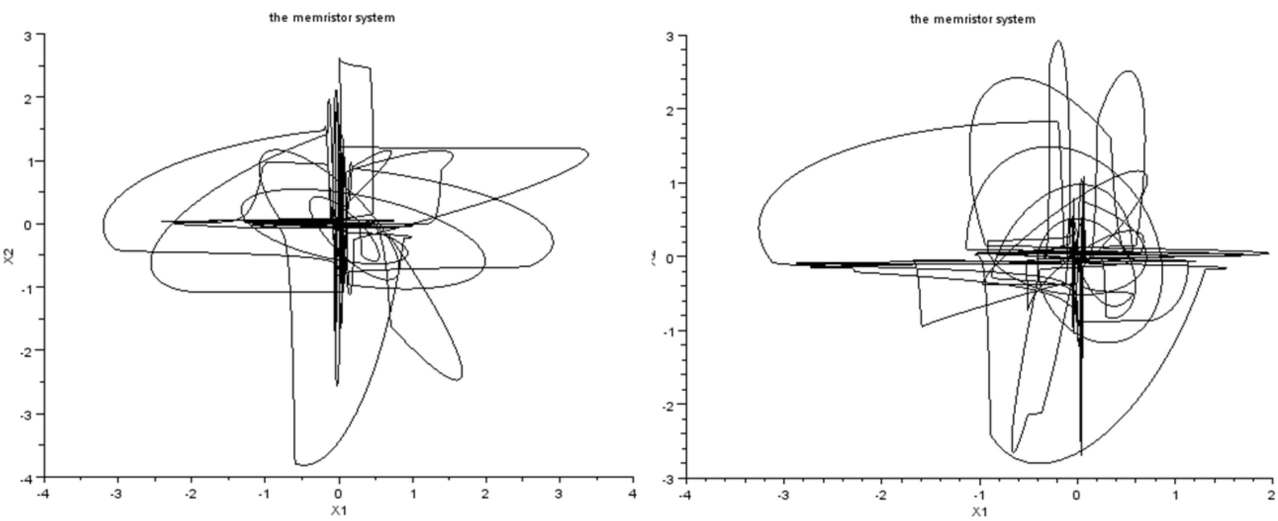
Our numerical process enables us to find the best value of coupling strengths, which yields evolution of synchronization phenomena.

We propose the two Memristors structure coupling as equation (18).

During our numerical test, we conclude that the synchronization phenomenon is reached for values of $k \geq 0.13$ as seen in figure 4:

$$\begin{cases} \dot{x}_1 = \alpha(y_1 - W(w_1)x_1) + k * (-\alpha(y_1 - y_2) + \alpha\, W(w_1) * (x_1) - \alpha\, W(w_2)\, x_2 - (x_1 - x_2)) \\ \dot{y}_1 = z_1 - x_1 + k * (-(z_1 - z_2) + (x_1 - x_2) - (y_1 - y_2)) \\ \dot{z}_1 = -\beta y_1 + \gamma z_1 + k * \left(\beta(y_1 - y_2) - (z_1 - z_2)(\gamma + 1)\right) \\ \dot{w}_1 = x_1 + k * \left(-(x_1 - x_2) - (w_1 - w_2)\right) \\ \dot{x}_2 = \alpha(y_2 - W(w_2)x_2) + k * \left(-\alpha(y_2 - y_1) + \alpha\, W(w_2) * (x_2) - \alpha\, W(w_1)\, x_1 - (x_2 - x_1)\right) \\ \dot{y}_2 = z_2 - x_2 + k * (-(z_2 - z_1) + (x_2 - x_1) - (y_2 - y_1)) \\ \dot{z}_2 = -\beta y_2 + \gamma z_2 + k * \left(\beta(y_2 - y_1) - (z_2 - z_1)(\gamma + 1)\right) \\ \dot{w}_2 = x_2 + k * (-(x_2 - x_1) - (w_2 - w_1)) \end{cases} \tag{18}$$

Where, k is positive feedback gain.



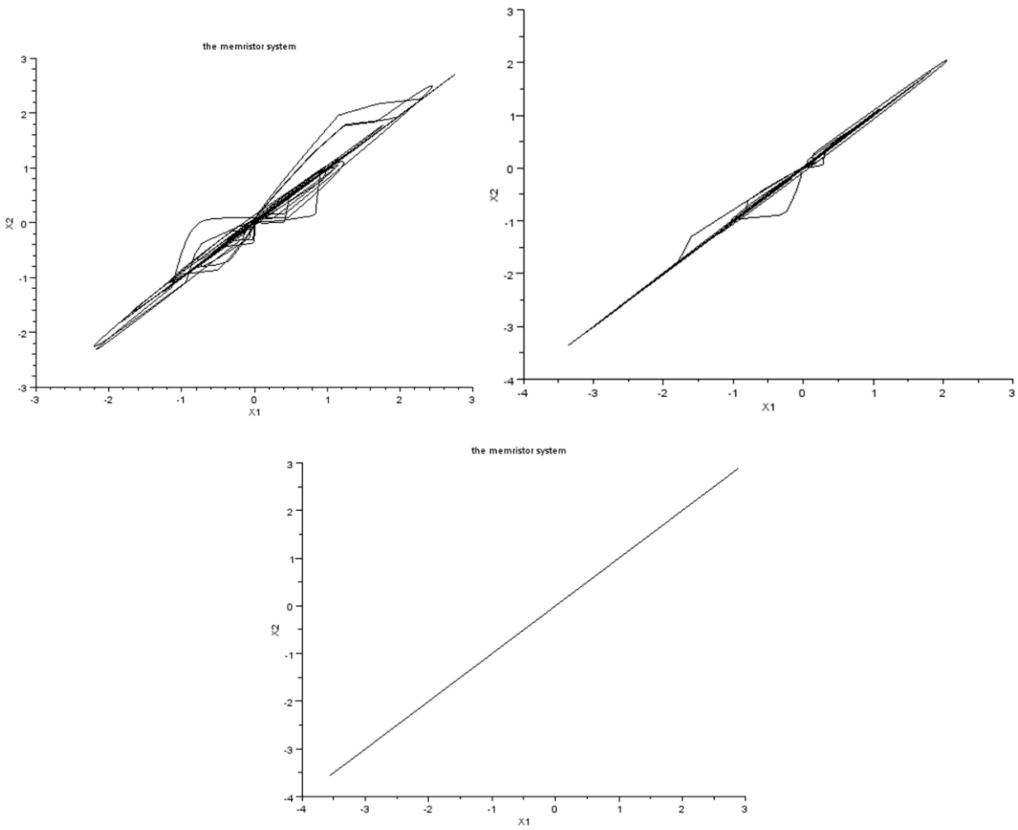


Figure 4. Synchronization and desynchronozation of different graph processes of system (50) for the value coupling strengths k=0.00001, 0.002, 0.1, 0.12, and 0.13.

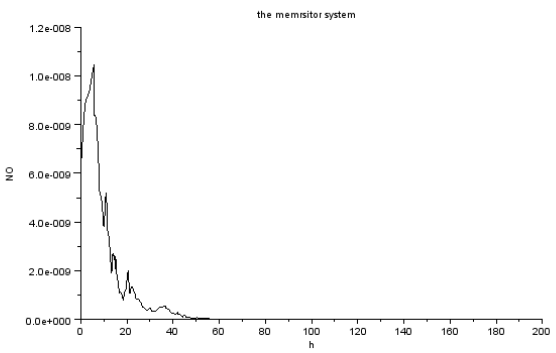


Figure 5 . *Plot k=0.13,* $N(X) = \| x_1 - x_2 \| + \| y_1 - y_2 \| + \| z_1 - z_2 \| + \| w_1 - w_2 \|$ *along (time) and show that the speed of the system(18) synchronization*

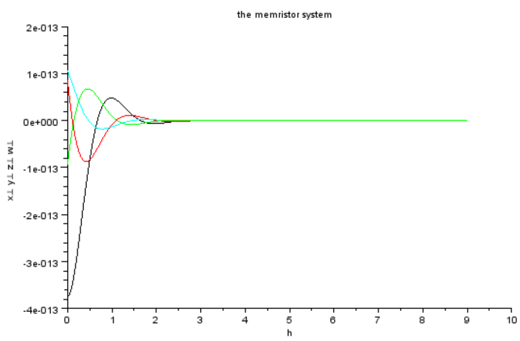


Figure 6. *The time series for the synchronization* $x_\perp$, $y_\perp$, $z_\perp$ *and* $w_\perp$ *of equation (13) with adaptive equation (16).*

4. Conclusion

This paper has presented the synchronization between two memristor sytems by using the Lyapunov methods directly. The control functions were capable of making the guarantees on stability of the dynamics systems. The Lyapunov direct method is clearer to carry out than other methods.

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