Multi-item EOQ model with demand dependent on unit price

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Abstract: A multi-item inventory model with demand dependent on unit cost without shortages is discussed in this paper. This paper presents a mathematical model of inventory control problem for determining the minimum total cost with limited storage space and investment. Apart from this, the warehouse space in the selling store is considered in volume. The model is solved using Kuhn-Tucker conditions method. The model is illustrated with a numerical example assuming unit price in fuzzy environment.

Keywords: Inventory, Rate Of Production, KKT Conditions, Demand Dependent On Unit Cost, Fuzzy Unit Cost, Triangular Fuzzy Number

1. Introduction


In any industry, the inventories are essential but they mean lockup of capital. The excess inventories are undesirable, which calls for controlling the inventories in the most profitable way. The different types of costs (ordering cost, carrying cost, etc) involved in inventory problems affect efficiency of a inventory control problem.

Silver [1985] designed the classical inventory problems by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature. But in practical situation unit price and demand rate of an item may be related to each other. When the demand of an item is high, an item is produced in large numbers fixed costs of production are spread over a large number of items. Hence the unit price of an item inversely related to the demand of that item.


The Kuhn-Tucker conditions [11] are necessary conditions for identifying stationary points of a non-linear constrained problem subject to inequality constraints. The development of this method is based on the Lagrangean method. These conditions are also sufficient if the objective function and the solution space satisfy the conditions in the following table 1.1.

<table>
<thead>
<tr>
<th>Sense of optimization</th>
<th>Required conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximization</td>
<td>Objective function</td>
</tr>
<tr>
<td></td>
<td>Solution space</td>
</tr>
<tr>
<td>Minimization</td>
<td>Concave</td>
</tr>
<tr>
<td></td>
<td>Convex Set</td>
</tr>
</tbody>
</table>

The conditions for establishing the sufficiency of the Kuhn-Tucker Conditions [4] are summarized in the
Assumptions and Notations

1. The basic assumptions about the model are:
   1. Replenishment is instantaneous.
   2. No back-order is allowed.
   3. Lead-time is zero.

2. We define the following notations in the model.

**Notations**

- **n**: number of items
- **B**: total investment cost for replenishment
- **l**: inside length of warehouse
- **b**: inside breadth of warehouse
- **h**: maximum height of shelf
- **V**: volume of warehouse
- **V_{vol}**: percentage of utilization of volume of warehouse.
- **D_i**: demand rate per unit time for the ith item.
- **Q_i**: inventory holding cost per unit item.
- **p_i**: price per unit item (fuzzy decision variable)
- **v_i**: volume of unit item i
- **D_{it}**: demand of the ith item.

**Assumptions**

1. Demand is related to unit price as:
   \[ D_i = \frac{A_i}{p_i^{\beta_i}} = \frac{A_i}{p_i}^{1-\beta_i} \]
   where \( A_i (> 0) \) and \( \beta_i (0 < \beta_i < 1) \) are constants and real numbers selected to provide the best fit of the estimated price function. \( A_i > 0 \) is an obvious condition since both \( D_i \) and \( p_i \) must be non-negative.

3. Formulation of Inventory Model

   The objective of the inventory model is to minimize the annual relevant total cost (i.e., the sum of production, setup and inventory carrying costs) which according to the basic assumptions of the EOQ model is:
   \[
   \text{Total cost} = \text{Production cost} + \text{Setup cost} + \text{Holding cost}
   \]
   Total average cost of the ith item is
   \[
   \text{Min} TC(p_i, Q_i) = \left[ \frac{A_i p_i^{1-\beta_i}}{Q_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{1}{2} \left[ 1 - \frac{A_i p_i^{-\beta_i}}{d_i} \right] Q_i H_i \right]
   \]

   for \( i = 1, 2, 3, \ldots, n \)

   There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

2. **Formulation of Inventory Model**

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   \[
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   \]

   for \( i = 1, 2, 3, \ldots, n \)

4. Membership Function

   The membership function for the triangular fuzzy variable \( p_i \) is defined as follows
\[ \mu_p(X) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{x - a_1}{a_2 - a_1}, & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \]

5. Numerical Example

The model is illustrated for one item (i=1) and also the common parametric values assumed for the given model are:

For i = 1, 
- \( A_1 = 113, S_1 = $100, H_1 = $1400, l_1 = 2m, b_1 = 3m, h_1 = 4m, \)
- \( l = 10m, b = 12m, h = 30m, d_1 = 300 \text{ units} \) and \( p = (3, 15, 27) \).

From the given values \( v_1 = 24 \text{ cubic m} \) and \( V = 3600 \text{ cubic m} \).

The proposed model is solved by Karush-Kuhn-Tucker conditions and the optimal results are presented in the table 5.1.

**Table 5.1**

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( p_1 )</th>
<th>( Q_1 )</th>
<th>( D_1 )</th>
<th>( \mu_{p_1} )</th>
<th>( V_w )</th>
<th>Expect Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.82</td>
<td>4.47</td>
<td>83.34</td>
<td>33.10</td>
<td>0.12</td>
<td>55.56</td>
<td>224.75</td>
</tr>
<tr>
<td>0.85</td>
<td>8.63</td>
<td>62.66</td>
<td>18.10</td>
<td>0.47</td>
<td>41.8</td>
<td>214.44</td>
</tr>
<tr>
<td>0.88</td>
<td>16</td>
<td>46.004</td>
<td>9.85</td>
<td>0.92</td>
<td>30.67</td>
<td>201.26</td>
</tr>
<tr>
<td>0.92</td>
<td>42.46</td>
<td>27.18</td>
<td>3.59</td>
<td>0</td>
<td>18</td>
<td>179.16</td>
</tr>
</tbody>
</table>

From the above table it follows that 16 has the maximum membership value 0.92.

Hence the required optimum solution is \( p_1 = 16, Q_1 = 46 \).

Minimum expected Total cost = $201.26

6. Conclusion

This paper is dedicated to solve a multi-item inventory problem for determining the total annual cost using Karush-Kuhn-Tucker conditions method. The decision variables namely the unit price and demand are calculated by varying the values of the parameter \( \beta \). The solution table shows that as the parametric value \( \beta \) increases, the unit price also increases whereas the demand, lot size and the percentage of utilization of volume decreases. This work can be extended by varying the constraints like limited budgetary, setup cost etc.

References


