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Multi-item EOQ model with demand dependent on unit price

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Abstract: A multi-item inventory model with demand dependent on unit cost without shortages is discussed in this paper. This paper presents a mathematical model of inventory control problem for determining the minimum total cost with limited storage space and investment. Apart from this, the warehouse space in the selling store is considered in volume. The model is solved using Kuhn-Tucker conditions method. The model is illustrated with a numerical example assuming unit price in fuzzy environment.

Keywords: Inventory, Rate Of Production, KKT Conditions, Demand Dependent On Unit Cost, Fuzzy Unit Cost, Triangular Fuzzy Number

1. Introduction

The constrained multi-item inventory model had been treated by many researchers. Maloney and Klein [5] discussed constrained multi-item inventory systems: an implicit approach using algorithms method. Abou-Ei-Ata and Kotb [1] developed a crisp inventory model under two restrictions. The pioneer work began by Cheng [3], who studied an EOQ model with demand dependent unit cost of single-item using geometric programming.

In any industry, the inventories are essential but they mean lockup of capital. The excess inventories are undesirable, which calls for controlling the inventories in the most profitable way. The different types of costs [ordering cost, carrying cost, etc] involved in inventory problems affect efficiency of a inventory control problem.

Silver [1985] designed the classical inventory problems by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature. But in practical situation unit price and demand rate of an item may be related to each other. When the demand of an item is high, an item is produced in large numbers fixed costs of production are spread over a large number of items. Hence the unit price of an item inversely related to the demand of that item.

Zadeh [1965] first gave the concept of fuzzy set theory. Later on, Bellman and Zadeh [1970] used the fuzzy set theory to the decision-making problem. Tanaka [1974] introduced the objectives as fuzzy goals over the α-cut of a fuzzy constraint set and Zimmermann [1976] gave the concept to solve multi-objective linear programming problem. Park [1987] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Hence we may impose warehouse space, cost parameters, number of orders, production cost etc, in fuzzy environment.

The Kuhn-Tucker conditions [11] are necessary conditions for identifying stationary points of a non-linear constrained problem subject to inequality constraints. The development of this method is based on the Lagrangean method. These conditions are also sufficient if the objective function and the solution space satisfy the conditions in the following table 1.1.

Table 1.1

Sufficient conditions for Optimization:

Sense of optimization	Required conditions	
	Objective function	Solution space
Maximization	Concave	Convex Set
Minimization	Convex	Convex Set

The conditions for establishing the sufficiency of the Kuhn-Tucker Conditions [4] are summarized in the following table 1.2

Table 1.2

Kuhn-Tucker conditions:

Problem	Kuhn-Tucker conditions
1.Max z = f(X) subject to $h^i(X) \leq 0$ $X \geq 0$, i = 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \leq 0, i = 1,2,....m$ $\lambda_i \geq 0, i = 1,2,......m$
2.Min z = f(X) subject to $h^i(X) \geq 0$ $X \geq 0$, i= 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \geq 0, i = 1,2,....m$ $\lambda_i \geq 0, i = 1,2,......m$

In this paper, multi-item fuzzy inventory model is considered under three constraints such as warehouse space constraint, investment amount constraint and the third one is the percentage of utilization of volume of the warehouse space. Here the volumes of the unit items are taken for calculations. The demand is dependent on unit cost and the unit cost is taken in fuzzy environment. The unit cost and lot size are decision variables. The problem is solved using Karush-Kuhn-Tucker Conditions method.

2 .Assumptions and Notations

We define the following notations in the model.

Notations

n = number of items
B = Total investment cost for replenishment
l = inside length of warehouse
b = inside breadth of warehouse
h = maximum height of shelf
V = Volume of warehouse
V_w = Percentage of utilization of volume of warehouse.
For i^{th} item: (i= 1,2,………..n)
$D_i = D_i(p_i)$ demand rate[function of unit cost price]
Q_i = lot size (decision variable)
S_i = Set-up cost for the i^{th} item
H_i = Inventory holding cost per unit item
p_i = price per unit item (fuzzy decision variable)
l_i = Length of unit item i
b_i = breadth of unit item i
h_i = height of unit item i
v_i = Volume of unit item i
d_i= the annual rate of production for the i^{th} item
TC(p_i, Q_i) = expected annual total cost

Assumptions

The basic assumptions about the model are:

1. Replenishment is instantaneous.
2. No back-order is allowed.
3. Lead-time is zero.
4. demand is related to the unit price as:

$$D_i = \frac{A_i}{p_i^{\beta_i}} = A_i p_i^{-\beta_i}$$

where A_i (>0) and β_i($0 < \beta_i < 1$) are constants and real numbers selected to provide the best fit of the estimated price function. $A_i>0$ is an obvious condition since both D_i and p_i must be non-negative.

3. Formulation of Inventory Model

The objective of the inventory model is to minimize the annual relevant total cost (i.e the sum of production, setup and inventory carrying costs) which according to the basic assumptions of the EOQ model is,

Total cost = Production cost + Set up cost + Holding cost

Total average cost of the ith item is

$$MinTC(p_i, Q_i) = p_i D_i + \frac{S_i D_i}{Q_i} + \frac{1}{2}\left[\; - \frac{D_i}{d_i}\right] Q_i H_i$$

$$= A_i p_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{1}{2}\left[1 - \frac{A_i p_i^{-\beta_i}}{d_i}\right] Q_i H_i$$

for i=1,2,3,………n

Min $TC(p_i, Q_i) =$

$$\sum_{i=1}^{n}\left[A_i p_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{1}{2}\left[1 - \frac{A_i p_i^{-\beta_i}}{d_i}\right] Q_i H_i\right]$$

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

(i) The limitation on the available warehouse space in the store

$$\sum_{i=1}^{n} v_i Q_i \leq V$$

(ii) The upper limit of the total amount investment

$$\sum_{i=1}^{n} p_i Q_i \leq B$$

where p_i, Q_i > 0 (i=1,2….n)

(iii) Percentage of utilization of volume of the warehouse

$$\frac{VXV_W}{\left(\sum_{i=1}^{n} v_i Q_i\right)100} = 1, \quad 0 \leq V_W \leq 100$$

4. Membership Function

The membership function for the triangular fuzzy variable p_i is defined as follows

$$\mu_{p_i}(X) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{x-a_3}{a_2-a_3}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

5. Numerical Example

The model is illustrated for one item (i=1) and also the common parametric values assumed for the given model are

For i = 1,

i = 1, A_1 = 113, S_1 = \$100, H_1 = \$1,,B = \$1400, l_1 = 2m, b_1 = 3m, h_1 = 4m,

l = 10m, b = 12m, h = 30m d_1 = 300 units and p = (3,15,27)

From the given values v_1 = 24 cubic m and V = 3600 cubic m.

The proposed model is solved by Karush-Kuhn-Tucker conditions and the optimal results are presented in the table 5.1

Table 5.1

Optimal solution table:

β_1	p_1	Q_1	D_1	μ_{P1} value	V_W	Expected Total cost
0.82	4.47	83.34	33.10	0.12	55.56	224.75
0.85	8.63	62.66	18.10	0.47	41.8	214.44
0.88	16	46.004	9.85	0.92	30.67	201.26
0.92	42.46	27.18	3.59	0	18	179.16

From the above table it follows that 16 has the maximum membership value 0.92.

Hence the required optimum solution is p_1 = 16, $Q1$ = 46

Minimum expected Total cost = \$201.26

6. Conclusion

This paper is dedicated to solve a multi-item inventory problem for determining the total annual cost using Karush-Kuhn-Tucker conditions method. The decision variables namely the unit price and demand are calculated by varying the values of the parameter β. The solution table shows that as the parametric value β increases, the unit price also increases whereas the demand, lot size and the percentage of utilization of volume decreases. This work can be extended by varying the constraints like limited budgetary, setup cost etc.

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