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An easy computable approximate solution for a squeezing flow between two infinite plates by using of perturbation method

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Abstract: This article proposes Perturbation Method (PM) to find an approximate solution for the problem of an axis symmetric Newtonian fluid squeezed between two large parallel plates. After comparing figures between approximate and exact solutions, we will see that the proposed solutions besides of handy, are highly accurate and therefore that PM is efficient.

Keywords: Mixed Boundary Conditions, Nonlinear Differential Equation, Perturbation Method, Approximate Solutions

1. Introduction

Although the studies of squeezing flows have its origins in the 19th century, at present time, it is an issue of considerable importance due to its practical applications in different areas such as physical, biophysical, chemical engineering, and food industry, also are relevant in liquid metal lubrication theory, polymer processing, compression and injection molding, among many others.

The purpose of this job is the search for an approximate solution for the nonlinear problem, which describes a viscous, incompressible fluid, squeezed between two infinite parallel plates, separated instantaneously a distance $2l$so that the plates are moving towards each other with a certain velocity, say W(see Fig. 1).

As mentioned in [1] these fluids are of paramount importance in hydrodynamic lubrication theory. Thus, [2] and [3] analysed isothermal compressible squeeze films neglecting inertial effects; while [4] found an explicit solution, taking into account these effects. Also, some numerical solutions to these problems have been found, such as those provided in [5] and [6]. Additionally, [7] and [8] extended the previous investigations for the case of flow between rotating parallel plates.

The perturbation method (PM) is a widely known and established method; it is among the pioneer techniques to approach various kinds of nonlinear problems. This procedure was originated by S. D. Poisson and extended by J. H. Poincare. Although the method appeared in the early 19th century, the application of a perturbation procedure to solve nonlinear differential equations was performed later on that century. The most significant efforts were focused on celestial mechanics, fluid mechanics, and aerodynamics [9,10].

In a broad sense, it is possible to express a nonlinear differential equation in terms of one linear part and other nonlinear. The nonlinear part is considered as a small perturbation through a small parameter (the perturbation parameter). The assumption that the nonlinear part is small compared to the linear is considered as a disadvantage of the method. There are other modern alternatives to find approximate solutions to the differential equations that describe some nonlinear problems such as those based on: variational approaches [11-14], tanh method [15], exp-function [16-18], Adomian's decomposition method [19-24], parameter expansion [24], homotopy perturbation method [14,24,26-28,29-47], homotopy analysis method [48], homotopy asymptotic method [26],perturbation method [49,50], and (G'/G)-expansion method [51,52] among many others. Also, a few exact solutions to nonlinear differential equations have been reported occasionally [53].

Although the PM method provides in general, better results for small perturbation parameters $\varepsilon << 1$, we will see that our approximations, besides to be handy, have a good accuracy, even for relatively large values of the perturbation parameter [49,50].

This paper is organized as follows. In Section 2, we introduce the basic idea of the PM method. For Section 3, we provide the basic equation for the problem of the axisymmetric Newtonian fluid above mentioned. Section 4 discusses the main results obtained. Finally, a brief conclusion is given in Section 5.

2. Basic Idea of Perturbation Method

Let the differential equation of one dimensional nonlinear system be in the form

$$L(x)+\varepsilon N(x)=0, \tag{1}$$

where we assume that x is a function of one variable $x=x(t)$, $L(x)$ is a linear operator which, in general, contains derivatives in terms of t; $N(x)$ is a nonlinear operator, and ε is a small parameter.

Considering the nonlinear term in (1) to be a small perturbation and assuming that the solution for (1) can be written as a power series in the small parameter ε,

$$x(t)=x_0+\varepsilon x_1(t)+\varepsilon^2 x_2(t)+\cdots. \tag{2}$$

Substituting (2) into (1) and equating terms having identical powers of ε, we obtain a number of differential equations that can be integrated, recursively, to find the values for the functions:$x_0(t)$, $x_1(t)$, $x_2(t)$, ...

3. Approximate Solution for a Nonlinear Problem of Fluids

The objective of this section is employ PM, to find an analytical approximate solution for the nonlinear problem which describes an axisymmetric Newtonian fluid squeezed between two large parallel plates given by

$$d^4y(x)/dx^4+\varepsilon y(x)d^3y(x)/dx^3=0,$$

$$0\le x\le 1,\ y(0)=0,\ y''(0)=0,\ y(1)=1, y'(1)=0, \tag{3}$$

where, ε is a positive parameter of the fluid, related to its density and with instantaneous separation distance $2l$(see Fig. 1).

It is possible to find a handy solution by applying the LT-HPM method. Identifying terms

$$L(y)=y^{(4)}(x), \tag{4}$$

$$N(y)=y(x)y'''(x), \tag{5}$$

whereprime denotes differentiation with respect to x.

Identifyingε with PM parameter, we assume a solution for (3) in the form

$$y(x)=y_0(x)+\varepsilon y_1(x)+\varepsilon^2 y_2(x)+\varepsilon^3 y_3(x)+\varepsilon^4 y_4(x)+\cdots. \tag{6}$$

(see (2))

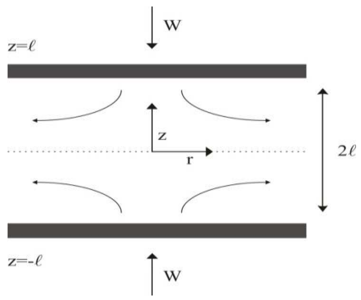

Figure 1. *Shows an axisymmetric fluid, squeezed between two infinite parallel plates.*

On comparing the coefficients of like powers of ε it can be solved for $y_0(x)$, $y_1(x)$, $y_2(x)$, $y_3(x)$,…, and so on. Later, we will see that a very good handy result is obtained by keeping a second order approximation.

$$\varepsilon 0)\ y0(4)=0,$$
$$y0(0)=0,\ y0''(0)=0,\ y0(1)=1,\ y0'(1)=0, \tag{7}$$

$$\varepsilon 1)\ y1(4)+y0y0'''=0,$$
$$y1(0)=0,\ y1''(0)=0,\ y1(1)=0,\ y1'(1)=0, \tag{8}$$

$$\varepsilon 2)\ y2(4)+y0y1'''+y1y0'''=0,$$
$$y2(0)=0, y2''(0)=0, y2(1)=0, y2'(1)=0, \tag{9}$$

Thus, the results obtained are

$$y0(x)=(3/2)x-(1/2)x3, \tag{10}$$

$$y1(x)=(19/560)x-(117/1680)x3+(3/80)x5-(1/560)x7, \tag{11}$$

$$y2(x)=-(137/10780)x-(443/517440)x3+$$
$$(17/2800)x5-(177/39200)x7+$$
$$(1/1680)x^9-(3/123200)x^{11}, \tag{12}$$

By substituting (10) thru (12) into (6) we obtain a second order approximation for the solution of (3).

Considering as cases study, the values of parameters ε=1 and ε=3; we obtain the following handy approximate solutions

$$y(x)=(859/560)x-(295199/517440)x^3-(247/39200)x^7+(61/1400)x^5-(3/123200)x^{11}-(1/1680)x^9-137/107800, \quad (13)$$

$$y(x)=(897/560)x-(24721/34496)x^3-(1803/39200)x^7+(117/700)x^5-(27/123200)x^{11}+(3/560)x^9-1233/107800. \quad (14)$$

As a matter of fact, we will see that (13) and (14) are also, highly accurate.

4. Discussion

The fact that the PM depends on a parameter, which is assumed to be small, suggests that the method is for a limited use. In this work, PM method has been applied, successfully to the problem of finding approximate solutions for nonlinear differential equation with mixed boundary conditions that describes an axisymmetric Newtonian fluid squeezed between two large parallel plates.

Fig. 2 and Fig. 3 shows the comparison between the approximate solutions (13) and (14) for differential equation (3) with a built-in numerical routine for BVP from Maple 15. It can be noticed that figures are in good agreement, showing the accuracy of proposed solutions. This proves the efficiency of PM method in this case, despite of the fact that it was only considered the second-order approximation to the equation to be solved. Therefore, accuracy can be increased considering higher order approximations.

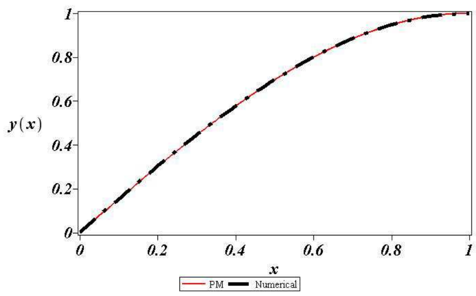

Figure 2. *Approximation for (3) and numerical comparison when ε =1.*

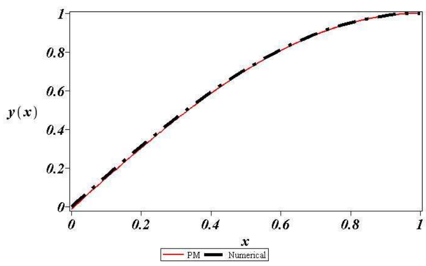

Figure 3. *Approximation for (3) and numerical comparison when ε =3.*

It is well known that PM method provides, in general, better results for small perturbation parameters ε << *1* (see (1)) and when are included the most number of terms from (2). To be precise, ε is a parameter of smallness, which measure how greater is the contribution of linear term *L(x)* than the one from *N(x)*in (1). Fig. 2 and Fig. 3 shows a noticeable fact, (13) and (14)turn out to be a good approximation of (3), although perturbation parameters ε=*1* and ε=3 are, indeed, large.

Finally, our approximate solutions do not depend on any adjustment parameter; thus they are, in principle, general expressions for the proposed problem.

5. Conclusions

In this study, PM was presented to construct analytical approximate solutions for nonlinear differential equations in the form of rapidly convergent series. In order to prove the versatility of this method, we proposed as an example the approximate solution for the nonlinear differential equation that describes a viscous, incompressible fluid, squeezed between two infinite parallel plates with mixed boundary conditions, obtaining acceptable results. The success of PM for this case, where we employed large values for perturbation parameter, must be considered as a real possibility to apply it in other nonlinear problems, instead of using other sophisticated and difficult methods. From Fig. 2 and Fig. 3, it is deduced that the proposed solutions have good precision.

References

[1] X.J. Ran, Q.Y. Zhu, and Y. Li,“An explicit series solution of the squeezing flow between two infinite plates by means of the homotopy analysis method”, Communications in Nonlinear Science and Numerical Simulation, vol. 14, pp. 119-132, 2009.

[2] W.E. Langlois, “Isothermal squeeze films”, Applied Mathematics, vol. 20, p. 131, 1962.

[3] E.O. Salbu, “Compressible squeeze films and squeeze bearings”, Journal of Basic Engineering, vol. 86, p. 355, 1964.

[4] J.F. Thorpe, Development in Theoretical and Applied Mathematics, W. A Shah, Ed., vol. 3, Pergamon Press, Oxford, UK, 1967.

[5] R.L. Verma, “A numerical solution for squeezing flow between parallel channels”, Wear, vol. 72, no. 1, pp. 89-95, 1981.

[6] P. Singh., V. Radhakrishnan, and K. A. Narayan, “Squeezing flow between parallel plates”, Ingenieur-Archiv, vol. 60, no. 4, pp. 274-281, 1990.

[7] K.R. Rajagopal and A. S. Gupta, “On a class of exact solutions to the equations of motion of a second grade fluid”, International Journal of Engineering Science, vol. 19, no. 7, pp. 1009-1014, 1981.

[8] B.S. Dandapat and A. S. Gupta, “Stability of a thin layer of a second-grade fluid on a rotating disk”, International Journal of Non-linear Mechanics, vo. 26, no. 3-4, pp. 409-417, 1991.

[9] T.L.Chow, Classical Mechanics. John Wiley and Sons Inc., USA, 1995.

[10] M.H. Holmes, Introduction to Perturbation Methods. Springer-Verlag, New York, 1995.

[11] L.M.B.Assas, “Approximate solutions for the generalized K-dV-Burgers’ equation by He’s variational iteration method”, Phys. Scr., vol. 76, pp. 161-164, DOI: 10.1088/0031-8949/76/2/008, 2007.

[12] J.H. He, “Variational approach for nonlinear oscillators”, Chaos, Solitons and Fractals, vol. 34, pp. 1430-1439. DOI: 10.1016/j.chaos.2006.10.026, 2007.

[13] M. Kazemnia, S.A. Zahedi, M. Vaezi, and N. Tolou, “Assessment of modified variational iteration method in BVPs high-order differential equations”, Journal of Applied Sciences, vol. 8, pp. 4192-4197, DOI:10.3923/jas.2008.4192.4197, 2008.

[14] R. Noorzad, A. Tahmasebi Poor and M. Omidvar, “Variational iteration method and homotopy-perturbation method for solving Burgers equation in fluid dynamics”, Journal of Applied Sciences, vol. 8, pp. 369-373, DOI:10.3923/jas.2008.369.373, 2008.

[15] D.J. Evans and K.R. Raslan, “The Tanh function method for solving some important nonlinear partial differential”, Int. J. Computat. Math., vol. 82, pp. 897-905. DOI: 10.1080/00207160412331336026, 2005.

[16] F.Xu, “A generalized soliton solution of the Konopelchenko-Dubrovsky equation using exp-function method”, ZeitschriftNaturforschung - Section A Journal of Physical Sciences, vol. 62, no. 12, pp. 685-688, 2007.

[17] J. Mahmoudi, N. Tolou, I. Khatami, A. Barari, and D.D. Ganji, “Explicit solution of nonlinear ZK-BBM wave equation using Exp-function method”, Journal of Applied Sciences, vo. 8, pp. 358-363.DOI:10.3923/jas.2008.358.363, 2008.

[18] H. Naher, F.A. Abdullah, M.A. Akbar, "New Traveling Wave Solutions of the Higher Dimensional Nonlinear Partial Differential Equation by the Exp-Function Method", Journal of Applied Mathematics, vol. 2012, pp. 1--14, doi:10.1155/2012/575387, 2012.

[19] G.Adomian, “A review of decomposition method in applied mathematics, Mathematical Analysis and Applications, vol.135, pp. 501-544, 1988.

[20] E.Babolian, and J. Biazar, “On the order of convergence of Adomian method”, Applied Mathematics and Computation, vol. 130, no. 2, pp. 383-387, DOI: 10.1016/S0096-3003(01)00103-5, 2002.

[21] A.Kooch, and M. Abadyan, “Efficiency of modified Adomian decomposition for simulating the instability of nano-electromechanical switches: comparison with the conventional decomposition method”, Trends in Applied Sciences Research, vol. 7, pp. 57-67, DOI:10.3923/tasr.2012.57.67, 2012.

[22] A.Kooch, and M. Abadyan, “Evaluating the ability of modified Adomian decomposition method to simulate the instability of freestanding carbon nanotube: comparison with conventional decomposition method”, Journal of Applied Sciences, vol. 11, pp. 3421-3428, DOI:10.3923/jas.2011.3421.3428, 2011.

[23] S.K.Vanani, S. Heidari, and M. Avaji, “A low-cost numerical algorithm for the solution of nonlinear delay boundary integral equations”, Journal of Applied Sciences, vol. 11, pp. 3504-3509, DOI:10.3923/jas.2011.3504.3509, 2011.

[24] S. H. Chowdhury, “A comparison between the modified homotopy perturbation method and Adomian decomposition method for solving nonlinear heat transfer equations”, Journal of Applied Sciences, vol. 11, pp. 1416-1420, DOI:10.3923/jas.2011.1416.1420, 2011.

[25] L.-N. Zhang and L. Xu, “Determination of the limit cycle by He’s parameter expansion for oscillators in a $u^3/1+u^2$potential”,ZeitschriftfürNaturforschung - Section A Journal of Physical Sciences, vol. 62, no. 7-8, pp. 396-398, 2007.

[26] V. Marinca and N.Herisanu, Nonlinear Dynamical Systems in Engineering, 1st edition,Springer-Verlag, Berlin Heidelberg, 2011.

[27] J.H.He, “A coupling method of a homotopy technique and a perturbation technique for nonlinear problems”, Int. J. Non-Linear Mech., vol. 351, pp. 37-43, DOI: 10.1016/S0020-7462(98)00085-7, 1998.

[28] J.H.He, “Homotopy perturbation technique”, Comput. Methods Applied Mech. Eng., vol. 178, pp. 257-262, DOI: 10.1016/S0045-7825(99)00018-3, 1999.

[29] J.H. He, “Homotopy perturbation method for solving boundary value problems”, Physics Letters A, vol. 350, no. 1-2, pp. 87-88, 2006.

[30] J.H.He, “Recent Development of the Homotopy Perturbation Method”, Topological Methods in Nonlinear Analysis, vol. 31, no. 2, pp. 205-209, 2008.

[31] A.Belendez, C. Pascual, M.L. Alvarez, D.I. Méndez, M.S. Yebra, and A. Hernández, “High order analytical approximate solutions to the nonlinear pendulum by He’s homotopy method”, PhysicaScripta, vol. 79, no. 1, pp. 1-24, DOI: 10.1088/0031-8949/79/01/015009, 2009.

[32] J.H.He, “A coupling method of a homotopy and a perturbation technique for nonlinear problems”, International Journal of Nonlinear Mechanics, vol. 35, no. 1, pp. 37-43, 2000.

[33] M. El-Shaed, “Application of He’s homotopy perturbation method to Volterra’sintegro differential equation”, International Journal of Nonlinear Sciences and Numerical Simulation, vol. 6, pp. 163-168, 2005.

[34] J.H.He, “Some Asymptotic Methods for Strongly Nonlinear Equations.International Journal of Modern Physics B”, vol. 20, no. 10, pp. 1141-1199, DOI: 10.1142/S0217979206033796, 2006.

[35] D.D.Ganji, H. Babazadeh, F Noori, M.M. Pirouz, and M. Janipour,“An Application of Homotopy Perturbation Method for Non linearBlasius Equation to Boundary Layer Flow Over a Flat Plate”, International Journal of Nonlinear Science, vol.7, no.4, pp. 309-404, 2009.

[36] D.D.Ganji, H. Mirgolbabaei, Me. Miansari, and Mo. Miansari, “Application of homotopy perturbation method to solve linear and non-linear systems of ordinary differential equations and differential equation of order three”, Journal of Applied Sciences, vol. 8, pp. 1256-126, DOI:10.3923/jas.2008.1256.1261, 2008.

[37] A.Fereidon, Y. Rostamiyan, M. Akbarzade, and D.D. Ganji, “Application of He’s homotopy perturbation method to nonlinear shock damper dynamics”, Archive of Applied Mechanics, vol. 80, no. 6, pp. 641-649, DOI: 10.1007/s00419-009-0334-x, 2010.

[38] P.R. Sharma and G. Methi, “Applications of homotopy perturbation method to partial differential equations”, Asian Journal of Mathematics & Statistics, vol. 4, pp. 140-150, DOI:10.3923/ajms.2011.140.150, 2011.

[39] A. Hossein, “Analytical Approximation to the Solution of Nonlinear Blasius Viscous Flow Equation”, LTNHPM. International Scholarly Research Network ISRN Mathematical Analysis, vol. 2012, Article ID 957473, 10 pages doi: 10.5402/2012/957473, 2011.

[40] H. Vazquez-Leal, U. Filobello-Niño, R. Castañeda-Sheissa, L. Hernandez Martinez, and A. Sarmiento-Reyes, “Modified HPMs inspired by homotopy continuation methods”, Mathematical Problems in Engineering, vol. 2012, Article ID 309123, DOI: 10.155/2012/309123, 20 pages, 2012.

[41] H.Vazquez-Leal, R. Castañeda-Sheissa, U. Filobello-Niño, A. Sarmiento-Reyes, and J. Sánchez-Orea, “High accurate simple approximation of normal distribution related integrals”, Mathematical Problems in Engineering, vol. 2012, Article ID 124029, DOI: 10.1155/2012/124029, 22 pages, 2012.

[42] U.Filobello-Niño, H. Vazquez-Leal, R. Castañeda-Sheissa, A. Yildirim, L. HernandezMartinez, D. PereyraDíaz, A. Pérez Sesma, and C. Hoyos Reyes, “An approximate solution of Blasius equation by using HPM method”, Asian Journal of Mathematics and Statistics, vol. 2012, 10 pages, DOI: 10.3923 /ajms.2012, ISSN 1994-5418, 2012.

[43] J. Biazarand H. Aminikhan, “Study of convergence of homotopy perturbation method for systems of partial differential equations”, Computers and Mathematics with Applications, vol. 58, no. 11-12, pp. 2221-2230, 2009.

[44] J. Biazar, and H. Ghazvini, “Convergence of the homotopy perturbation method for partial differential equations”, Nonlinear Analysis: Real World Applications, vol. 10, no. 5, pp. 2633-2640, 2009.

[45] U.Filobello-Niño, H. Vazquez-Leal, D. PereyraDíaz, A. Pérez Sesma, R. Castañeda-Sheissa, Y. Khan, A. Yildirim, L. Hernandez Martinez, and F. RabagoBernal,“HPM Applied to Solve Nonlinear Circuits: A Study Case”, Applied Mathematics Sciences, vol. 6, no. 87, pp. 4331-4344, 2012.

[46] D.D. Ganji, A.R. Sahouli, M. Famouri, “A New modification of He’shomotopy perturbation method for rapid convergence of nonlinear undamped oscillators”, Journal of Applied Mathematics and Computing, vol. 30, pp. 181-192, 2009.

[47] U.Filobello-Nino, H. Vazquez-Leal, Y. Khan, A. Perez-Sesma, A. Diaz-Sanchez, V.M. Jimenez-Fernandez, A. Herrera-May, D. Pereyra-Diaz, J.M. Mendez-Perez,and J. Sanchez-Orea, “Laplace transform-homotopy perturbation method as a powerful tool to solve nonlinear problems with boundary conditions defined on finite intervals”, Computational and Applied Mathematics, ISSN: 0101-8205, DOI= 10.1007/s40314-013-0073-z, 2013.

[48] T.Patel, M.N. Mehta, and V.H. Pradhan,“The numerical solution of Burger’s equation arising into the irradiation of tumour tissue in biological diffusing system by homotopy analysis method”, Asian Journal of Applied Sciences, Vol. 5, pp. 60-66, DOI:10.3923/ajaps.2012.60.66, 2012.

[49] U.Filobello-Niño, H. Vazquez-Leal, Y. Khan, A. Yildirim, V.M. Jimenez-Fernandez, A. L Herrera May, R. Castañeda-Sheissa, andJ.CervantesPerez,“Perturbation Method and Laplace-Padé Approximation to solve nonlinear problems”, Miskolc Mathematical Notes, vol. 14, no. 1, pp. 89-101, ISSN: 1787-2405, 2013.

[50] U.Filobello-Niño, H. Vazquez-Leal, K.Boubaker, Y. Khan, A. Perez-Sesma, A.Sarmiento Reyes, V.M. Jimenez-Fernandez, A Diaz-Sanchez, A. Herrera-May, J. Sanchez-Orea and K. Pereyra-Castro, “Perturbation Method as a Powerful Tool to Solve Highly Nonlinear Problems: The Case of Gelfand'sEquation”,Asian Journal of Mathematics and Statistics, vol. 2013, 7 pages, DOI: 10.3923 /ajms.2013, ISSN 1994-5418, 2013.

[51] H. Naher and F. Abdullah, "The Basic (*G′/G*)-Expansion Method for the Fourth Order Boussinesq Equation", Applied Mathematics, vol. 3, no. 10, pp. 1144-1152. doi: 10.4236/am.2012.310168, 2012.

[52] H. Naher and F.A. Abdullah, "New approach of (G′/G)-expansion method and new approach of generalized (G′/G)-expansion method for nonlinear evolution equation", vol. 3, no. 3, doi: 10.1063/1.4794947, 2013.

[53] U.Filobello-Niño, H. Vazquez-Leal, Y. Khan, A. Perez-Sesma, A. Diaz-Sanchez, A. Herrera-May, D. Pereyra-Diaz, R. Castañeda-Sheissa, V.M. Jimenez-Fernandez, and J. Cervantes-Perez, “A handy exact solution for flow due to a stretching boundary with partial slip”, Revista Mexicana de Física E, vol. 59, pp. 51-55. ISSN 1870-3542, 2013.