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Conversion of energy equation for turbulent motion and its applications

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Abstract: Turbulent energy has developed revolutionary technology in the form of a portfolio of devices for the mixing, separation and the homogenization of liquids with liquids, liquids with gasses and gasses with gasses. The mixing technology may be applied to a wide variety including chemicals, pharmaceuticals, cosmetics, foods, agricultural, water treatment with purification and hybrid fuels. The paper reports the transformation of energy equation for turbulent flow in terms of correlation tensors of second order, where the correlation tensors are the functions of space coordinates, distance between two points and time. To reveal the relation of turbulent energy between two points, one point has been taken as the origin of the coordinate system. Correlation between pressure fluctuations and velocity fluctuations at the two points of flow field is applied to the turbulent energy equation. The applications of turbulent energy are discussed for the source of oceanic turbulence by means of Richardson number. A multiplication factor in terms of kinetic energy and potential energy is considered for finding the correlation between the multiplication factor and critical flux Richardson number and to signify the relative efficiency of mixing by Kelvin-Helmholtz billows and the critical flux Richardson number.

Keywords: Turbulent Energy, Turbulent Motion, Richardson Number, Two-Point Correlation, Correlation Tensor

1. Introduction

Turbulent flow is of central importance to many engineering applications such as the production of the composite materials, internal combustion engines, textile industry, aerospace industry, environmental engineering, chemical engineering, process engineering, paper making, and so on. The turbulence is maintained by the turbulent energy production, where the dissipation and the buoyancy flux act as sinks for the turbulent energy. Accurate estimation of the turbulence dissipation rate is important for the turbulent flows in the industry. The energy dissipation was measured by adjacent to an island by Osborn [1], where the turbulence was supported by the Reynolds stress working against the local mean shear. This mean shear would be time variable, largely due to internal waves and hence would grow and decay with time. Osborn [2] also estimated energetic of the current and balanced the turbulent energy equation to justify using as an estimation of the local production. Oakey [3] examined the rate of dissipation of turbulent energy from simultaneous temperature and velocity shear microstructure measurements. The dissipation rate was calculated from high-wavenumber cut-off of the temperature microstructure spectra and from velocity shear. The kinetic energy dissipation was also examined from batchelor curve fitting by Luketina and Imberger [4]. The kinetic energy dissipation rate for turbulent flow was compared between two ocean microstructure profilers by Moum et al. [5]. Large differences in dissipation rate were found between those two profilers, which appear to be greater in the meridional direction than the zonal direction.

The dynamics of the evolution of turbulence statistics depend on the structure of the turbulence. A systematic framework was introduced by Kassinos et al. [6] for exploring the role of turbulence structure in the evolution of one-point turbulence statistics. The one-point structure tensors were found to be useful descriptors of turbulence structure, and lead to a deeper understanding of some rather surprising observations from direct numerical solutions and experiments. Two new formulations such as an inviscid estimate for the viscous dissipation rate of turbulent kinetic energy and a mixing length estimate for the turbulent heat flus were examined from the measurements of energy-containing scales of turbulence in the ocean thermocline [7]. It was found that energy-containing scale estimates of both dissipation rate and heat flux compare favorably with dissipation scale estimates.

For dissipation rate estimation, a large eddy particle image velocimetry (PIV) method was proposed by Sheng [8] as PIV is capable of providing multi-point instantaneous measurements of a flow field. Nash and Moum [9] estimated the microstructure of turbulent salinity flux and the dissipation spectrum of salinity. Bhattacharya et al. [10] formulated a locally homogeneous representation for the two-point, second-order turbulent velocity fluctuation in terms of three linearly independent structure tensors. To evaluate the representation, a model correlation was constructed by fitting the representation to correlations calculated from direct numerical simulation (DNS) of homogeneous turbulence and channel flow. Carbone et al. [11] discussed the turbulent energy cascade in anisotropic magneto hydrodynamic turbulence. The occurrence of an energy cascade for turbulence in solar wind plasmas was historically addressed by using phenomenological arguments based to the sweeping of fluctuations by the large-scale magnetic field and the anisotropy of the cascade in wave vectors space.

There are some on-going researches on turbulent motion. An equation for turbulent motion was derived in terms of second order correlation tensors [12]. After injecting fibers in the same phenomena, an equation was developed in terms of second order correlation tensors by Ahmed and Sarker [13]. Two equations were also developed in presence of dust particles [14] and in a rotating system [15]. Derivation of turbulent energy was discussed with two-point correlation [16], and in presence of dust particles [17], in a rotating system [18] and in a rotating system with dust particles [19]. All the correlation tensors used in the developed equations were defined as the functions of space coordinates, distance between two points and time. In these view, it is important to develop a mathematical model for further analysis on turbulent energy. However, there are few studies relevant to the turbulent energy although it is prevalent in the industry. In view of all the work, the main aim of the present study is to develop an energy equation for turbulent flow in terms of second order correlation tensors and to thrash out the application of turbulent energy for the source of oceanic turbulence using Richardson number.

2. Mathematical Formulation

The equations of motion and continuity for turbulent flow of a viscous incompressible fluid are

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j\partial x_j} \tag{1}$$

$$\frac{\partial u_i}{\partial x_i}=0 \tag{2}$$

The energy equation for turbulent flow of a viscous incompressible fluid is given by

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j\partial x_j}-2\varepsilon_{ijl}\Omega_j u_l \tag{3}$$

Where u_i are the fluid velocity components; p is the unknown pressure field; υ is the kinematical viscosity of the fluid; ρ is the density of the fluid particle; ε_{ijl} the three-dimensional permutation symbol, where ε is the dissipation by turbulence per unit of mass; Ω_j, the rotation vector and t is the time.

We assume that the mean velocity $\overline{U_i}$ is constant throughout the region considered and independent of time, and we put

$$\left(U_i=\overline{U}_i+u_i\right)_A,\ \left(U_j=\overline{U}_j+u_j\right)_B.$$

The value of each term can be obtained by using the equations of motion for u_j at the point B and for u_i at the point A. The energy equation for u_i at the point A is obtained from equation (3),

$$\frac{\partial u_i}{\partial t}+\left(\overline{U}_k+u_k\right)\frac{\partial u_i}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_k\partial x_k}-2\varepsilon_{ikl}\Omega_k u_l \tag{4}$$

For an incompressible fluid $\left(u_i\frac{\partial u_k}{\partial x_k}\right)_A=0$ so that equation (4) can be written as

$$\begin{aligned}&\frac{\partial}{\partial t}(u_i)_A+\left[\overline{U}_k+(u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A+\left(u_i\frac{\partial u_k}{\partial x_k}\right)_A\\&=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A(u_i)_A-\left(2\varepsilon_{ikl}\Omega_k u_l\right)_A\end{aligned} \tag{5}$$

Multiplying equation (5) by $\left(u_j\right)_B$, we obtain

$$\begin{aligned}&\left(u_j\right)_B\frac{\partial}{\partial t}(u_i)_A+\left[\overline{U}_k+(u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A\left(u_j\right)_B\\&+(u_i)_A\left(\frac{\partial}{\partial x_k}\right)_A(u_k)_A\left(u_j\right)_B=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A\left(u_j\right)_B\\&+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A(u_i)_A\left(u_j\right)_B-\left(2\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B\end{aligned} \tag{6}$$

Where $\left(u_j\right)_B$ can be treated as a constant in a differential process at the point A.

Similarly, the energy equation for u_j at the point B is obtained as

$$\begin{aligned}&\frac{\partial u_j}{\partial t}+\left(\overline{U}_k+u_k\right)\frac{\partial u_j}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}\\&+\upsilon\frac{\partial^2 u_j}{\partial x_k\partial x_k}-2\varepsilon_{jkl}\Omega_k u_l\end{aligned}\tag{7}$$

For an incompressible fluid $\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B=0$ so that equation (7) becomes

$$\begin{aligned}&\frac{\partial}{\partial t}\left(u_j\right)_B+\left[\overline{U}_k+\left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B+\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B\\&=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\left(u_j\right)_B-\left(2\varepsilon_{jkl}\Omega_k u_l\right)_B\end{aligned}\tag{8}$$

Multiplying equation (8) by $\left(u_i\right)_A$, we get

$$\begin{aligned}&\left(u_i\right)_A\frac{\partial}{\partial t}\left(u_j\right)_B+\left[\overline{U}_k+\left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A\\&+\left(u_j\right)_B\left(\frac{\partial}{\partial x_k}\right)_B\left(u_k\right)_B\left(u_i\right)_A=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B\left(u_i\right)_A\\&+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A-\left(2\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A\end{aligned}\tag{9}$$

Where $\left(u_i\right)_A$ can be treated as a constant in a differential process at the point B.

Addition of equations (6) and (9) gives the result

$$\begin{aligned}&\frac{\partial}{\partial t}\left(u_i\right)_A\left(u_j\right)_B+\left[\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B+\left(\frac{\partial}{\partial x_k}\right)_B\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B\right]\\&+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A\left(u_j\right)_B+\left(\frac{\partial}{\partial x_k}\right)_B\left(u_i\right)_A\left(u_j\right)_B\right]\\&=-\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A\left(u_j\right)_B+\left(\frac{\partial}{\partial x_j}\right)_B p_B\left(u_i\right)_A\right]\\&+\upsilon\left[\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A+\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\right]\left(u_i\right)_A\left(u_j\right)_B\\&-2\left[\left(\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B+\left(\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A\right]\end{aligned}\tag{10}$$

To expose the relation of turbulent energy at the point B to those at point A, it will give no difference if we take one point as the origin of A or B of the coordinate system. Let us consider the point A as the origin. In order to differentiate between the effects of distance and location, we introduce as new independent variables [19],

$$\zeta_k=\left(x_k\right)_B-\left(x_k\right)_A$$

Then we obtain,

$$\left(\frac{\partial}{\partial x_k}\right)_A=-\frac{\partial}{\partial\zeta_k},\ \left(\frac{\partial}{\partial x_k}\right)_B=\frac{\partial}{\partial\zeta_k}$$

$$\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A=\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B=\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}$$

Using the above relations in equation (10) and taking ensemble average on both sides, equation (10) becomes

$$\begin{aligned}&\frac{\partial}{\partial t}\overline{\left(u_i\right)_A\left(u_j\right)_B}-\frac{\partial}{\partial\zeta_k}\overline{\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B}+\frac{\partial}{\partial\zeta_k}\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B}\\&=-\frac{1}{\rho}\left[-\frac{\partial}{\partial\zeta_i}\overline{p_A\left(u_j\right)_B}+\frac{\partial}{\partial\zeta_j}\overline{p_B\left(u_i\right)_A}\right]+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}\overline{\left(u_i\right)_A\left(u_j\right)_B}\\&-2\left[\overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B}+\overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A}\right]\end{aligned}\tag{11}$$

Equation (11) represents the mean motion for turbulent energy with pressure-velocity correlation.

It is noted that the coefficient of $\overline{U}_k$ has been vanished. The equation (11) describes the turbulent energy motion, where the motions with respect to a coordinate system moving with the mean velocity $\overline{U}_k$.

Equation (11) contains the double velocity correlation $\overline{\left(u_i\right)_A\left(u_j\right)_B}$, double correlations such as $\overline{p_A\left(u_j\right)_B}$, triple correlations such as $\overline{\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B}$ where all the terms apart from one another. The correlations $\overline{p_A\left(u_j\right)_B}$ and $\overline{p_B\left(u_i\right)_A}$ form the tensors of first order, because pressure is a scalar quantity and the triple correlations $\overline{\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B}$ and $\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B}$ form the tensors of third order. The double and triple correlations at the two points A and B in the flow field have been shown in Fig.1 and Fig.2 respectively, where r is the distance between two points A and B.

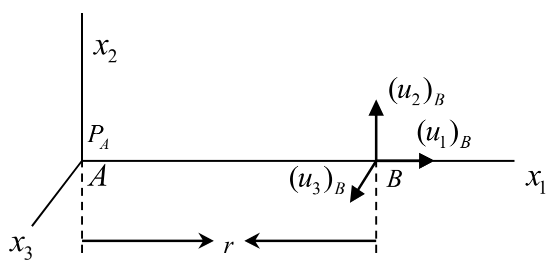

Fig. 1(a). *Double correlation between pressure at A and velocity components at B.*

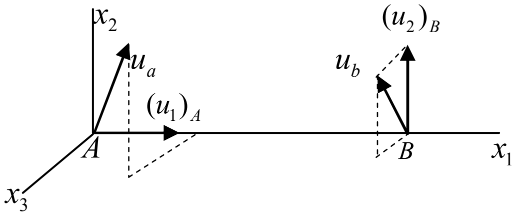

Fig. 1(b). *Double velocity correlation between the velocities* u_a *at* A *and* u_b *at* B.

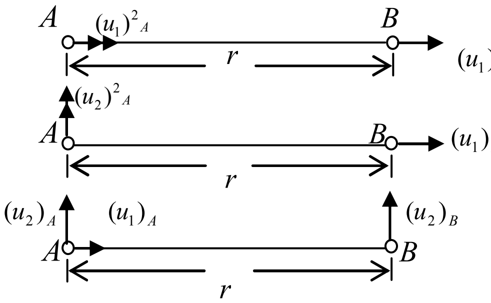

Fig. 2. *Triple velocity correlation among the velocities at the points* A *and* B.

We designate the first order correlations by $\left(k_{p,j}\right)_{A,B}$, second order correlations by $\left(Q_{i,j}\right)_{A,B}$ and third order correlations by $\left(s_{ik,j}\right)_{A,B}$.

Therefore, we set

$$\left(k_{i,p}\right)_{A,B}=\overline{\left(u_i\right)_A p_B},\left(k_{p,j}\right)_{A,B}=\overline{p_A\left(u_j\right)_B}$$
$$\left(Q_{i,j}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_j\right)_B},\left(s_{ik,j}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B}$$
$$\left(s_{i,kj}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B}.$$

Where, the index p indicates the pressure and is not a dummy index like i or j so that the summation convention does not apply to p.

Also the term $\overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B}$ and $\overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A}$ form the correlation tensors of second order, we designate these by $D_{i,j}$ and $H_{i,j}$ respectively.

Thus we set

$$\left(D_{i,j}\right)_{A,B}=\overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B},\left(H_{i,j}\right)_{A,B}=\overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A}.$$

If we use the above relations of first, second and third order correlations in equation (11) then we obtain

$$\begin{aligned}&\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}S_{ik,j}+\frac{\partial}{\partial\zeta_k}S_{i,kj}=-\frac{1}{\rho}\left(-\frac{\partial}{\partial\zeta_i}K_{p,j}+\frac{\partial}{\partial\zeta_j}K_{i,p}\right)\\&+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}-2\left[\left(D_{i,j}+H_{i,j}\right)\right]\end{aligned}\quad(12)$$

Where all the correlations refer to the two points A and B.

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is,

$$\left(k_{p,j}\right)_{A,B}=0,\ \left(k_{i,p}\right)_{A,B}=0.$$

In case of isotropy, the statistical features have no directional preference and perfect disorder persists. The velocity fluctuations are independent of the axis of reference, i.e. invariant to axis rotation and reflection. From the definition of isotropy, $\left(Q_{i,j}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_j\right)_B}=0$ for all $i\neq j$. In the rotating system in the flow field through 180^0 about x_1-axis must, because of isotropy, give

$$\overline{\left(u_1\right)_A\left(u_2\right)_B}=\overline{\left(u_1\right)_A[-\left(u_2\right)]_B}=-\overline{\left(u_1\right)_A\left(u_2\right)_B}$$

which can be true only when $\overline{\left(u_1\right)_A\left(u_2\right)_B}=0$.

The isotropic turbulence in a bounded domain is a model where in the turbulence is unaffected by the boundaries enclosing the fluid, and furthermore the statistical moments are spatially invariant and independent of orientation. Isotropic grid turbulence is a similar idealization, in that the turbulence is enclosed by wind tunnel walls and the homogeneity of the turbulence in the central region is known to be unaffected by the wall boundary layers.

In an isotropic turbulence it follows from the condition of invariance under reflection with respect to point A,

$$\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B}=-\overline{\left(u_k\right)_A\left(u_j\right)_A\left(u_i\right)_B}$$

or,

$$\left(s_{i,kj}\right)_{A,B}=-\left(s_{kj,i}\right)_{A,B}.$$

In absence of isotropic turbulence, physical properties will be different in different directions according to the direction of measurement. Anisotropic turbulence tends toward local isotropy, in that the statistics of velocity differences tend toward invariance under rotation as the distance between the velocities becomes smaller. For non-isotropic (anisotropic) turbulence, constant or non-constant average velocity of pressure field will not be zero. Anisotropy is the property of being directionally dependent. It can be defined as a difference, when measured along different axes, in a material's physical or mechanical properties (absorbance, refractive index, conductivity, tensile strength, etc.).

Thus equation (12) can be written as

$$\begin{aligned}&\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right)\\&=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}-2\left[\left(D_{i,j}+H_{i,j}\right)\right]\end{aligned}\quad(13)$$

The terms $\frac{\partial}{\partial \zeta_k}\left(S_{ik,j}+S_{kj,i}\right)$ and $\left(D_{i,j}+H_{i,j}\right)$ form the tensors of second order, we designate these by $S_{i,j}$ and $L_{i,j}$ respectively.

Thus we set,

$$S_{i,j}=\frac{\partial}{\partial \zeta_k}\left(S_{ik,j}+S_{kj,i}\right),\ L_{i,j}=\left(D_{i,j}+H_{i,j}\right).$$

Therefore equation (13) gives the result

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\left[\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}-L_{i,j}\right] \tag{14}$$

This is the energy equation for turbulent flow in terms of correlation tensors of second order.

If there are no effects of the dissipation ε by the turbulence per unit mass, $L_{i,j}=0$ so that the equation (14) takes the form

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} \tag{15}$$

This equation represents the turbulent motion in terms of correlation tensors of second order, which is the same as obtained by Hinze [12].

3. Application

The turbulent energy may be applied to the shear zone between the South Equatorial current and the Atlantic Equatorial undercurrent. The energy may be significant to some of the thick patches of turbulence found in other parts of the ocean. In steady case, the value of flux Richardson number R_f must be less than 1 for maintained turbulence in a shear flow. The shear flow is circulated by pressure velocity correlations. The latitude of a maximum value for R_f above which the turbulence cannot be maintained in steady state is very appealing. Such a value constitutes a critical flux Richardson number. At higher values of the flux Richardson number too much energy is going into buoyancy flux and turbulence will be suppressed. Britter's measurements suggest the critical value for R_f is 0.18-0.2. Ellison's theoretical prediction is that the critical value for flux Richardson number, $R_f crit \sim 0.15$. Since, the turbulence will be suppressed and too much the energy goes into the buoyancy flux at the higher values of R_f , we can apply the steady state value for critical flux Richardson number to a leisurely varying mean state.

If we define the eddy coefficient for density K_ρ by

$$K_\rho=\frac{R_f \varepsilon}{\left(1-R_f\right)N^2} \tag{16}$$

Where N is the vaisala frequency, ε is the dissipation. If $R_f \le R_f crit = 0.15$,

$$K_\rho \le \frac{0.15\varepsilon}{0.85N^2}<0.2\frac{\varepsilon}{N^2} \tag{17}$$

Equation (17) is applicable to thick patches of turbulence found in a variety of sections of the ocean. It also may be used in a steady shear zone to relate dissipation measurements to the buoyancy flux such as the Atlantic Equatorial Undercurrent. The functional dependence on ε and N^2 is determined by dimensional interpretation as soon as they are made the only available descriptors of the flux. To get hold of a better outcome of Kelvin-Helmholtz instabilities we consider thin features of scale 1 m. There are two sources of information on the mixing efficiency of Kelvin-Helmholtz billows. On the dimensions of Kelvin-Helmholtz billows, Thorpe comments that 10% of the energy removed from the mean field can go into mixing up and the energy up to 16% might be radiated away via internal waves. Koop defines the Richardson number by

$$R_i=\frac{g(\varDelta\rho/\rho)h_0}{(\varDelta u)^2} \tag{18}$$

Where g is the gravitational force and h_0 is the initial scale of the velocity shear. The Eq. (18) represents the ratio of the increase in potential energy to the decrease in kinetic energy as a function of gradient Richardson number.

The most efficient mixing occurred at small gradient Richardson numbers with the value of $\varDelta\mathrm{PE}/\varDelta\mathrm{KE}=0.25$, where $\varDelta PE$ is the potential energy and $\varDelta KE$ is the kinetic energy. Using such kind of data we get the range of values for the multiplication factor denoted by γ and defined as $\gamma=\frac{(\varDelta\mathrm{PE}/\varDelta\mathrm{KE})}{(1-\varDelta\mathrm{PE}/\varDelta\mathrm{KE})}$, which converts the dimensionally correct ratio $\frac{\varepsilon}{N^2}$ into an eddy coefficient for mass transport. Thus the factor $\frac{R_f}{\left(1-R_f\right)}$ derived from the concept of a critical flux Richardson number. The assumed mean method may be applied for finding the correlation between the critical flux Richardson number and the multiplication factor using different types of value of the multiplication factor γ are given in Table 1. The correlation may be defined as

$$r_{R_f\gamma}=\frac{n\sum d_{R_f}d_\gamma-\sum d_{R_f}\sum d_\gamma}{\sqrt{n\sum d_{R_f}{}^2-\left(\sum d_{R_f}\right)^2}\sqrt{n\sum d_\gamma{}^2-\left(\sum d_\gamma\right)^2}} \tag{19}$$

Where d_{R_f} and d_γ refers to deviations of R_f and γ respectively from an assumed mean; n , the total number of observations.

***Table 1.** Different types of value of the multiplication factor*

R_f	γ	R_f - 0.1267 d_{R_f}	γ - 0.1555 d_γ	$d_{R_f}^{\ 2}$	$d_\gamma^{\ 2}$	$d_{R_f} d_\gamma$
0.25	0.333	0.1233	0.1775	0.0152	0.0315	0.0219
0.20	0.250	0.0733	0.0945	0.0054	0.0089	0.0069
0.15	0.176	0.0233	0.0205	0.0005	0.0004	0.0005
0.10	0.111	-0.0267	-0.0445	0.0007	0.0020	0.0012
0.05	0.053	-0.0767	-0.1025	0.0059	0.0105	0.0079
0.01	0.010	-0.1167	-0.1455	0.0136	0.0212	0.0170
ΣR_f =0.76	$\Sigma\gamma$ =0.933	Σd_{R_f} =0.0002	Σd_γ =0	$\Sigma d_{R_f}^{\ 2}$ =0.0413	$\Sigma d_\gamma^{\ 2}$ =0.0745	$\Sigma d_{R_f} d_\gamma$ =0.0554

Simplification of equation (19) gives $r_{R_f\gamma} = 0.99875$ which implies that the correlation is positive higher degree correlation between the critical flux Richardson number and the multiplication factor. It indicates that as the critical flux Richardson number R_f increases, the factor γ also increases which is also exposed in Fig.3. The gradient Richardson number increases and we would have to think of this as the local value increasing everywhere, the factor γ becomes smaller is shown in Fig.4.

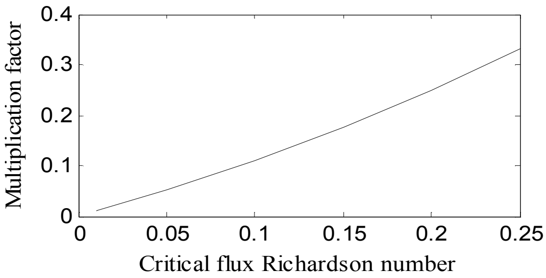

***Fig. 3.** Correlation between critical flux Richardson number and multiplication factor*

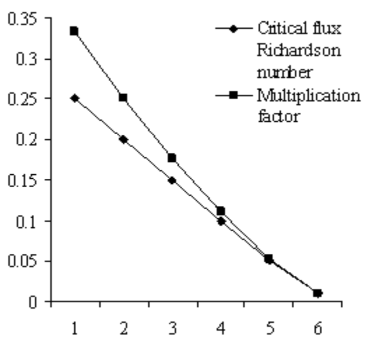

***Fig. 4.** Relative efficiency of mixing by Kelvin-Helmholtz billows and the critical flux Richardson number*

Thus to estimate an upper bound for diffusion from measurements of the local dissipation rate for the two sources of turbulence, the equation (18) is a realistic way.

4. Conclusions

Energy equation in terms of second order correlation tensor for turbulent flow has been developed by averaging procedure, which consists of the correlations between the pressure fluctuations and velocity fluctuations at the two points A and B of the flow field. The equation (14) stands for the energy equation of turbulent motion in terms of correlation tensors of second order. In the equation, all the terms $Q_{i,j}, S_{i,j}, L_{i,j}$ are the second order correlation tensors where, $Q_{i,j}$ and $S_{i,j}$ represents the velocity correlations at the two points A and B of the flow field; $L_{i,j}$ is the velocity correlation for turbulent energy. Equation (15) confers the turbulent motion in terms of correlation tensors of second order which was same as obtained by Hinze. The values of flux Richardson number has been used by maintained turbulence in a shear flow which is distributed by pressure velocity correlations. The equation (17) is germane to substantial patches of turbulence which is initiated in different expanses of the ocean. The eddy coefficient calculated by the equation (17) parameterizes the diffusion due to the small scale turbulence.

The model for K_ρ is used to calculate approximately the buoyancy flux associated with the local small scale turbulent fluctuations that are responsible for the dissipation. So, the buoyancy flux and therefore the derived eddy coefficient are associated with the small scale turbulence. The flux Richardson number has been considered the values less than 0.15 used in the derivation of equation (17). Dissipation that occurs in well-mixed portions of the water column is not allied with its proportionate share of mass transport. The positive correlation between the critical flux Richardson number and the multiplication factor is obtained from equation (19) which indicates that as the critical flux Richardson number R_f increases, the factor γ also increases is shown in Fig.3. The gradient Richardson number is as a function of the ratio of the increase in potential energy to the decrease in kinetic energy which has been derived in equation (18) and the Fig.4 states that the factor γ becomes smaller as the gradient Richardson number increasing everywhere.

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