Applied and Computational Mathematics
2014; 3(6): 323-329
Published online December 29, 2014 (http://www.sciencepublishinggroup.com/j/acm)
doi: 10.11648/j.acm.20140306.16
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

An analytical treatment to fractional gas dynamics equation

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To cite this article:
Mohamed S. Al-luhaibi, Nahed A. Saker. An Analytical Treatment to Fractional Gas Dynamics Equation. *Applied and Computational Mathematics.* Vol. 3, No. 6, 2014, pp. 323-329. doi: 10.11648/j.acm.20140306.16

Abstract: In this paper, the new iterative method (NIM) is applied to solve nonlinear fractional gas dynamics equation. Further, a coupling of the Sumudu transform and Adomian decomposition (STADM) is used to get an approximate solution of the same problem. The results obtained by the two methods are found to be in agreement. Therefore, the NIM may be considered efficient method for finding approximate solutions of both linear and nonlinear fractional differential equations.

Keywords: Sumudu Transform Method, Adomian Decomposition Method, New Iterative Method, Fractional Gas Dynamics Equation

1. Introduction

In recent years, fractional differential equations have gained importance and popularity, mainly due to its demonstrated applications in numerous seemingly diverse fields of science and engineering. For example, the nonlinear oscillation of earthquake can be modeled with fractional derivatives and the fluid-dynamic traffic model with fractional derivatives can eliminate the deficiency arising from the assumption of continuum traffic flow. The fractional differential equations are also used in modeling of many chemical processes, mathematical biology and many other problems in physics and engineering [1–12]. There is a very comprehensive literature review in some new asymptotic methods for the search for the solitary solutions of nonlinear differential equations, nonlinear differential-difference equations, and nonlinear fractional differential equations; see [13]. The new iterative method (NIM) was first introduced by Gejji and Jafari [14]. The NIM was also studied by many authors to handle linear and nonlinear equations arising in various scientific and technological fields [15-18]. The Sumudu decomposition method [19,20] and variational iteration method (VIM) [21] have also been applied to study the various physical problems.

In this paper, we consider the following nonlinear time-fractional gas dynamics equation of the form

$$D_t^{\alpha}U(x,t)+\frac{1}{2}(U^2)_x-U(1-U)=0\,,\ t>0\ \ 0<\alpha\leq 1, \quad (1.1)$$

with the initial condition

$$\mathrm{U(x,0)=e^{-x}}. \quad (1.2)$$

where α is a parameter describing the order of the fractional derivative. The function $U(x,t)$ is the probability density function, t is the time, and x is the spatial coordinate. The derivative is understood in the Caputo sense. The general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses. In the case of $\alpha=1$ the fractional gas dynamics equation reduces to the classical gas dynamics equation. The gas dynamics equations are based on the physical laws of conservation, namely, the laws of conservation of mass, conservation of momentum, conservation of energy, and so forth. The nonlinear fractional gas dynamics has been studied previously by Das and Kumar [22]. Further, we apply the NIM and SDM to solve the nonlinear time-fractional gas dynamics equation. The objective of the present paper is to extend the application of the NIM to obtain analytic and approximate solutions to the time-fractional gas dynamics equation.

2. Basic Definitions of Fractional Calculus

In this section, we mention the following basic definitions of fractional calculus which are used further in the present work.

2.1. Definition

The Riemann-Liouvill fractional integral operator of order $\alpha > 0$; of a function $f(t) \in C_\mu$ and $\mu \geq -1$ is defined as :

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \ \alpha > 0, \tag{2.1}$$

$$I^0 f(t) = f(t). \tag{2.2}$$

For the Riemann-Liouville fractional integral, we have:

$$I^\alpha t^v = \frac{\Gamma(v+1)}{\Gamma(v+1+\alpha)} t^{v+\alpha}. \tag{2.3}$$

2.2. Definition

The fractional derivative of $f(t)$ in the Caputo sense is defined as:

$$D_t^\alpha f(t) = I_t^{m-\alpha} D^m f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^m(\tau) d\tau,$$
$$m-1 < \alpha \leq m, \ t > 0 \tag{2.4}$$

From properties of D_t^α; it is important to note that:

$$D_t^\alpha t^v = \frac{\Gamma(v+1)}{\Gamma(v+1-\alpha)} t^{v-\alpha}. \tag{2.5}$$

For the Riemann-Liouvill fractional integral and Caputo fractional derivative, we have the following relation:

$$I_t^\alpha D_t^\alpha f(t) = f(t) - \sum_{k=0}^{m-1} f^k(0_+) \frac{t^k}{k!}. \tag{2.6}$$

2.3. Definition

The Sumudu transform of the Caputo fractional derivative is defined as follows [23]:

$$S[D_t^\alpha f(t)] = u^{-\alpha} S[f(t)] - \sum_{k=0}^{m-1} u^{-\alpha+k} f^k(0_+), \ m-1 < \alpha \leq m \tag{2.7}$$

3. Basic Idea of New Iterative Method (NIM)

To describe the idea of the NIM, consider the following general functional equation [14-18]:

$$u(x) = f(x) + N(u(x)), \tag{3.1}$$

where N is a nonlinear operator from a Banach space $B \to B$ and f is a known function. We are looking for a solution u of (3.1) having the series form

$$u(x) = \sum_{i=0}^{\infty} u_i(x). \tag{3.2}$$

The nonlinear operator N can be decomposed as follows

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\}. \tag{3.3}$$

From Eqs. (3.2) and (3.3), Eq. (3.1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\}. \tag{3.4}$$

We define the recurrence relation:

$$u_0 = f, \tag{3.5a}$$

$$u_1 = N(u_0), \tag{3.5b}$$

$$u_{n+1} = N(u_0 + u_1 + \ldots + u_n) - N(u_0 + u_1 + \ldots + u_{n-1}), \quad n = 1,2,3,\ldots. \tag{3.5c}$$

Then:

$$(u_1 + \ldots + u_{n+1}) = N(u_0 + u_1 + \ldots + u_n), \quad n = 1,2,3,\ldots ,$$

$$u = \sum_{i=0}^{\infty} u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right). \tag{3.6}$$

If N is a contraction, i.e.

$$\|N(x) - N(y)\| \leq k\|x - y\|, \quad 0 < k < 1,$$

then:

$$\|u_{n+1}\| = \|N(u_0 + u_1 + \ldots + u_n) - N(u_0 + u_1 + \ldots + u_{n-1})\|$$
$$\leq k\|u_n\| \leq \ldots \leq k^n \|u_0\| \quad n = 0,1,2,\ldots, \tag{3.7}$$

and the series $\sum_{i=0}^{\infty} u_i$ absolutely and uniformly converges to a solution of (3.1) [24], which is unique, in view of the Banach fixed point theorem [25]. The k-term approximate solution of (3.1) is given by $u(x) = \sum_{i=0}^{k-1} u_i(x)$.

3.1. Reliable Algorithm of New Iterative Method for Solving the Linear and Nonlinear Partial Differential Equations

After the above presentation of the NIM, we introduce a reliable algorithm for solving nonlinear PDEs using the NIM. Consider the following nonlinear PDE of arbitrary order:

$$D_t^\alpha u(x,t) = A(u, \partial u) + B(x,t), \quad m-1 < \alpha \leq m, m \in IN \tag{3.8a}$$

with the initial conditions

$$\frac{\partial^k}{\partial t^k} u(x,0) = h_k(x), \quad k = 0,1,2,\ldots,m-1, \tag{3.8b}$$

where A is a nonlinear function of u and ∂u (partial derivatives of u with respect to x and t) and B is the source function. The initial value problem (3.8) is equivalent to the following integral equation

$$u(x,t)=\sum_{k=0}^{m-1}h_k(x)\frac{t^k}{k!}+I_t^{\alpha}B(x,t)+I_t^{\alpha}A=f+N(u), \quad (3.9)$$

where

$$f=\sum_{k=0}^{m-1}h_k(x)\frac{t^k}{k!}+I_t^{\alpha}B(x,t), \quad (3.10)$$

and

$$N(u)=I_t^{\alpha}A, \quad (3.11)$$

where I_t^n is an integral operator of n fold. We get the solution of (3.9) by employing the algorithm (3.5).

4. Sumudu Transform

A new integral transform, named Sumudu transform is defined over the set of functions.

$$A=\left\{ f(t) \,\middle|\, \exists M,\tau_1,\tau_2>0, |f(t)|<Me^{\left(\frac{|t|}{\tau_j}\right)},\ if \quad t\in(-1)^j\times[0,\infty) \right\} \quad (4.1)$$

by the following formula

$$\bar{f}(u)=S[f(t)]=\int_0^{\infty}f(ut)e^{-t}dt, \qquad u\in(-\tau_1,\tau_2) \quad (4.2)$$

This Sumudu transfirm is applied to the solution of ordinary differential equation in control engineering problem, for more details see Watugala [26]. Some of the properties of this transform were established in [27,28]. Further fundamental properties of this transform were also established ,see [29] . Similarly, this transform was applied to the one-dimensional neutron transport equation in [30]. In fact it was shown that there is a strong relationship between Sumudu and other integral transforms; see [31]. In particular the relation between Sumudu transform and Laplace transforms was proved in [32].

Further, in [33], the Sumudu transform was extended to the distributions and some of their properties were also studied in [34]. Recently, this transform is applied to solve the system of differential equations; see [35].

Another interesting fact about about Sumudu transform is that the original function and its Sumudu transform have the same Taylor coefficients except the factor n; see [36]. That is, if $f(t)=\sum_{n=0}^{\infty}a_nt^n$ then we have $F(u)=\sum_{n=0}^{\infty}n!a_nu^n$, see [31].

5. Basic Idea of Sumudu Transform and Adomian Decomposition Method (STADM)

To illustrate the basic idea of this method, we consider a general nonlinear non-homogeneous partial differential equation [19, 20]:

$$D_t^{\alpha}U(x,t)+RU(x,t)+NU(x,t)=g(x,t) \qquad (5.1a)$$

with initial conditions

$$U(x,0)=h(x), \quad U_t(x,0)=f(x), \qquad (5.1b)$$

where $D_t^{\alpha}U(x,t)$ is the Caputo fractional derivative of the function $U(x,t)$), R is the linear differential operator, N represents the general nonlinear differential operator, and $g(x,t)$ is the source term. Applying the Sumudu transform (denoted in this paper by S) on both sides of Eq. (5.1), we get

$$S[D_t^{\alpha}U(x,t)]+S[RU(x,t)]+S[NU(x,t)]=S[g(x,t)] \qquad (5.2)$$

Using the differentiation property of the Sumudu transform and above initial conditions, we have

$$S[U(x,t)]=u^{\alpha}S[g(x,t)]+h(x)+uf(x)-u^{\alpha}S[RU(x,t)+NU(x,t)] \quad (5.3)$$

Now, applying the inverse Sumudu transform on both sides of Eq. (5.3), we get

$$U(x,t)=G(x,t)-S^{-1}[u^{\alpha}S[RU(x,t)+NU(x,t)]] \qquad (5.4)$$

where $G(x,t)$ represents the term arising from the source term and the prescribed initial conditions. The second step in Sumudu decomposition method is that we represent solution as an infinite series given below

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x,t) \qquad (5.5)$$

and the nonlinear term can be decomposed as:

$$NU(x,t)=\sum_{n=0}^{\infty}A_n \qquad (5.6)$$

where A_n are Adomian polynomials [36] of $U_0,U_1,U_2,...,U_n$ and it can be calculated by formula

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_0^{\infty}\lambda^iU_i\right)\right]_{\lambda=0}, \qquad n=0,1,2,... \qquad (5.7)$$

Using Eq. (5.5) and Eq. (5.6) in Eq. (5.4), we get

$$\sum_{n=0}^{\infty}U_n(x,t)=G(x,t)-S^{-1}\left[u^{\alpha}S\left[R\sum_{n=0}^{\infty}U_n(x,t)+\sum_{n=0}^{\infty}A_n(U)\right]\right] \quad (5.8)$$

On comparing both sides of the Eq. (5.8), we get

$$U_0(x,t)=G(x,t),$$

$$U_1(x,t)=-S^{-1}[u^{\alpha}S[RU_0(x,t)+A_0]],$$

$$U_2(x,t)=-S^{-1}[u^{\alpha}S[RU_1(x,t)+A_1]] \qquad (5.9)$$

$$U_3(x,t)=-S^{-1}[u^{\alpha}S[RU_2(x,t)+A_2]]$$

In general the recursive relation is given by

$$U_0(x,t) = G(x,t),$$

$$U_{n+1}(x,t) = -S^{-1}\left[u^{\alpha} S\left[RU_n(x,t) + A_n\right]\right], \quad n \geq 0 \quad (5.10)$$

Now first of all applying the Sumudu transform of the right hand side of Eq.(5.10) then applying the inverse Sumudu transform, we get the values of $U_0, U_1, U_2, \ldots, U_n$ respectively.

6. Application

6.1. Solution of the Problem by NIM

In this subsection we present and illustrate the applicability and the effectiveness of the NIM to get an analytical solution to the nonlinear time-fractional gas dynamics equation of the form

$$D_t^{\alpha} U(x,t) + \frac{1}{2}(U^2)_x - U(1-U) = 0, \; 0 < \alpha \leq 1 \quad (6.1a)$$

with the initial condition

$$U(x,0) = e^{-x}. \quad (6.1b)$$

From (3.5a) and (3.10), we obtain $U_0(x,t) = e^{-x}$.

Therefore, from (3.9), the initial value problem (6.1) is equivalent to the following integral equation:

$$U(x,t) = e^{-x} - I_t^{\alpha}\left(\frac{1}{2}(U^2)_x - U(1-U)\right)$$

Taking

$$N(U) = -I_t^{\alpha}\left(\frac{1}{2}(U^2)_x - U(1-U)\right)$$

Therefore, from (3.5), we can obtain easily the following .first few components of the new iterative solution for the equation (6.1):

$$U_0(x,t) = e^{-x},$$

$$U_1(x,t) = e^{-x}\left(\frac{t^{\alpha}}{\Gamma(1+\alpha)}\right),$$

$$U_2(x,t) = e^{-x}\left(\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right),$$

$$U_3(x,t) = e^{-x}\left(\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right),$$

$$\vdots$$

and so on. The n-order term approximate solution, in series form, is given by:

$$U_n(x,t) = e^{-x}\left(1 + \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right) \quad (6.2)$$

In the special case, $\alpha = 1$; Eq. (6.2) becomes:

$$U(x,t) = e^{-x}\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) \quad (6.3)$$

In closed form, this gives:

$$U(x,t) = \sum_{n=0}^{\infty} U_n(x,t) = e^{t-x}$$

which is the exact solution for Eq. (6.1) in the special case $\alpha = 1$. The 3-order term approximate solution and the corresponding exact solution for Eq. (6.1) are plotted in Fig. 1(a), for $\alpha = 1/3$; in Fig. (1b), for $\alpha = 2/3$; in Fig. (1c), for $\alpha = 1$:and in Fig. 1(d) the exact solution. It is remarkable to note that the surface of the approximate solution converges to the surface of the exact solution as $\alpha \to 1$: It is evident that the efficiency of the NIM can be dramatically enhanced by computing further terms of $U(x,t)$.

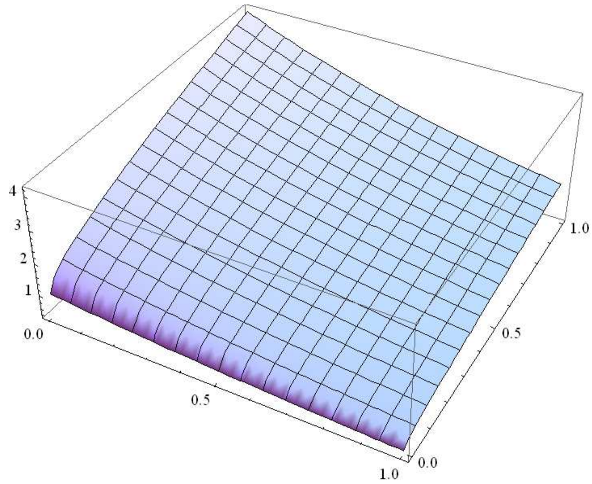

Fig. 1 (a). *Approximate solution for Eq. (6.1). in case* $x: 0 \to 1$, $t: 0 \to 1$, $\alpha = 1/3$

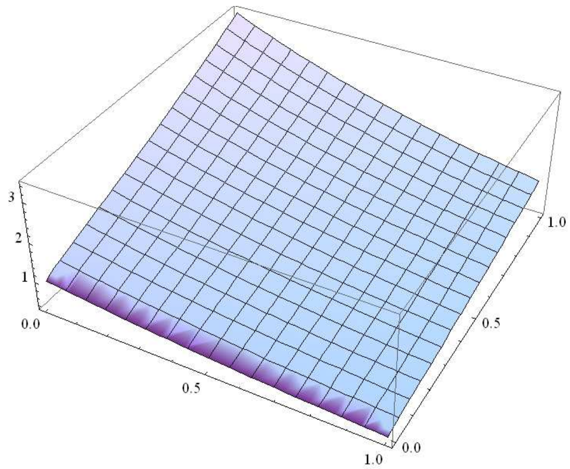

Fig. 1(b). *Approximate solution for Eq. (6.1). in case* $x: 0 \to 1$, $t: 0 \to 1$, $\alpha = 2/3$.

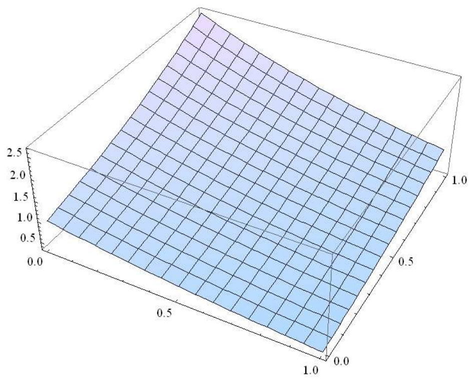

Fig. 1 (c). *Approximate solution for Eq. (6.1) in case* $x:0\to 1$, $t:0\to 1$, $\alpha=1$..

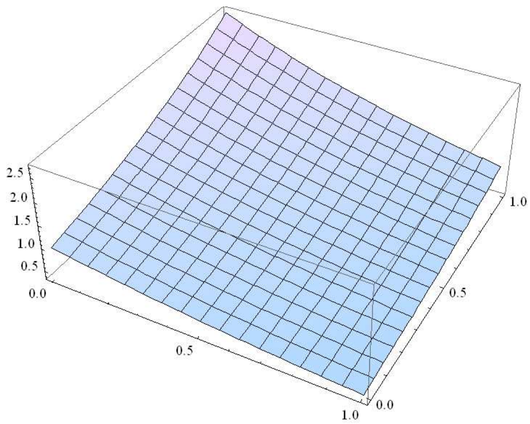

Fig. 1 (d). *Exact solution for Eq. (6.1), in case exact solution..*

6.2. Solution of the Problem by (STADM)

In this subsection we present and illustrate the applicability and the effectiveness of the SADM to get an analytical solution to the nonlinear time-fractional gas dynamics equation of the form

By taking Sumudu transform for (6.1), we obtain

$$SU(x,t)=e^{-x}+u^{\alpha}S\left(-\frac{1}{2}(U^2)_x+U-U^2)\right) \tag{6.4}$$

By applying the inverse Sumudu transform for (6.4), we get

$$U(x,t)=e^{-x}+S^{-1}\left[u^{\alpha}S\left(-\frac{1}{2}(U^2)_x+U-U^2)\right)\right] \tag{6.5}$$

Following the technique, if we assume an infinite series solution of the form (5.5) and (5.6), we obtain

$$\sum_{n=0}^{\infty}U_n(x,t)=e^{-x}+S^{-1}\left[u^{\alpha}S\left(-\frac{1}{2}\sum_{n=0}^{\infty}B_n(U)+U_n(x,t)-\sum_{n=0}^{\infty}A_n(U)\right)\right] \tag{6.6}$$

In (6.6), $A_n(U)$ and $B_n(U)$ are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$\sum_{n=0}^{\infty}A_n(U)=U^2\,,\ \sum_{n=0}^{\infty}B_n(U)=\left(U^2\right)_x \tag{6.7}$$

The few components of the Adomian polynomials are given as follows:

$$A_0=U_0^2,$$

$$A_1=2U_0U_1, \tag{6.8}$$

$$A_2=2U_0U_2+U_1^2,$$

$$\vdots$$

$$B_0=\left(U_0^2\right)_x,$$

$$B_1=\left(2U_0U_1\right)_x, \tag{6.9}$$

$$B_2=\left(2U_0U_2+U_1^2\right)_x,$$

$$\vdots$$

From the relationship in (5.10), we obtain

$$U_0(x,t)=G(x,t)=e^{-x},$$

$$U_1(x,t)=S^{-1}\left[u^{\alpha}S\left(-\frac{1}{2}B_0(U)+U_0(x,t)-A_0(U)\right)\right]=e^{-x}\left(\frac{t^{\alpha}}{\Gamma(1+\alpha)}\right),$$

$$U_2(x,t)=S^{-1}\left[u^{\alpha}S\left(-\frac{1}{2}B_1(U)+U_1(x,t)-A_1(U)\right)\right]=e^{-x}\left(\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right),$$

$$U_3(x,t)=S^{-1}\left[u^{\alpha}S\left(-\frac{1}{2}B_2(U)+U_2(x,t)-A_2(U)\right)\right]=e^{-x}\left(\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right),$$

$$\vdots$$

$$U_n(x,t)=e^{-x}\left(1+\frac{t^{\alpha}}{\Gamma(1+\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots\right), \tag{6.10}$$

which is the same solution as obtained by using NIM

In the special case, $\alpha=1$; Eq. (6.10) becomes:

$$U(x,t)=e^{-x}\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\cdots\right). \tag{6.11}$$

In closed form, this gives:

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x,t)=e^{t-x}. \tag{6.12}$$

Table 6.1. *Numerical results of nonlinear time-fractional gas dynamics equation via mathematica using NIM, STADM and the exact solution when* $\alpha=1$ *and x=0.1*

t	NIM	STADM	Exact solution
0.1	0.999996153	0.999996153	1
0.2	1.105108099	1.105108099	1.105170918
0.3	1.221078095	1.221078095	1.221402758
0.4	1.348810977	1.348810977	1.349858807
0.5	1.489211583	1.489211583	1.491824697
0.6	1.643184751	1.643184751	1.648721270

Table 6.2. *Numerical results of nonlinear time-fractional gas dynamics equation via mathematica using NIM, STADM and the exact solution when* $\alpha = 1$ *and t= 0.1*

x	NIM	STADM	Exact solution
0	1.105166666	1.105166666	1.105170918
0.2	0.904833937	0.904833937	0.904837418
0.4	0.740815370	0.740815370	0.740818220
0.6	0.606528326	0.606528326	0.606530659
0.8	0.496583393	0.496583393	0.496585303
1.0	0.406568095	0.406568095	0.406569659

Table 6.3. *Numerical results of nonlinear time-fractional gas dynamics equation via mathematica using NIM/STADM and the exact solution for different values of* α *and t = 0.1*

X	NIM / STADM $\alpha = 0.5$	NIM / STADM $\alpha = 0.75$	NIM / STADM $\alpha = 0.9$	NIM / STADM $\alpha = 1$
0	1.480613144	1.21948262	1.140829330	1.105166666
0.2	1.212223515	0.998427930	0.934032056	0.904833937
0.4	0.992484671	0.817443651	0.764720769	0.740815370
0.6	0.812577722	0.669266256	0.626100411	0.606528326
0.8	0.665282370	0.547948865	0.512607661	0.496583393
1.0	0.544687136	0.448622587	0.419687656	0.406569659

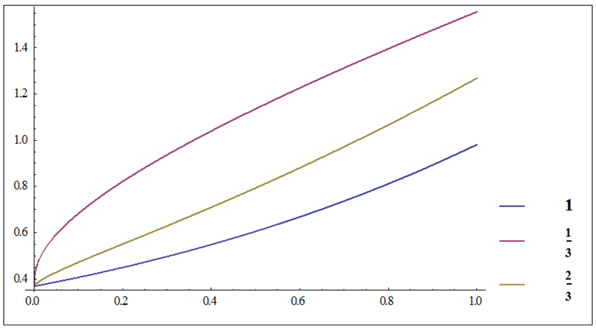

Fig. 2. *Plots of* $U(x,t)$ *versus* t *at* $x = 1$ *for different values of* α *.*

The obtained approximate semi-analytic solutions of fractional gas dynamics equation given in (6.1) is close at hand to the exact solution as it is seen from figures 1 and 2 and from tables 6.1-6.3. It is to be observed that only fourth - order term of the NIM and STADM are used to compute the approximate solutions. It is to be noted that the accuracy of the proposed method can be improved by computing more additional terms of the approximate solutions.

7. Conclusions

In this paper, the new iterative method (NIM) and a coupling of the Sumudu transform and Adomian decomposition method (STADM) are successfully applied for solving nonlinear time-fractional gas dynamics equation. The numerical solutions show that there is a good agreement between the two methods. Therefore, these two methods are very powerful and efficient techniques for solving different kinds of linear and nonlinear fractional differential equations arising in different fields of science and engineering. However, the (NIM) has an advantage over the (STADM) which is that it solves the nonlinear problems without using Adomian polynomials. In conclusion, the NIM and the (STADM) may be considered as a nice refinement in existing numerical techniques and might find the wide applications.

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