Numerical Study of Convective Heat Transfer on the Power Law Fluid over a Vertical Exponentially Stretching Cylinder

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Abstract: The present paper is the study of boundary layer flow and heat transfer of Power law fluid flowing over a vertical exponentially stretching cylinder along its axial direction. The governing partial differential equations and the associated boundary conditions are reduced to nonlinear ordinary differential equations after using the boundary layer approximation and similarity transformations. The obtained system of nonlinear ordinary differential equations subject to the boundary conditions is solved numerically with the help of Fehlberg method. The effects of Power law index $n$, Reynolds number $Re$, Prandtl number $Pr$, the natural convection parameter $\lambda$ and local Reynolds number $Re_a$ are presented through graphs. The skin friction coefficient and Nusselt number are presented through tables for different parameters.

Keywords: Boundary Layer Flow, Exponential Stretching, Vertical Cylinder, Power Law Fluid, Natural Convection Heat Transfer, Fehlberg Method

1. Introduction

Large amount of work has been done on laminar boundary layer flow over stretching sheet. For example in extrusion processes such as polymer extrusion from a dye and wire driling, drawing, tunning and annealing of copper wires, the cooling of a metallic plate in a cooling bath and so on. Crane (1970) was the first who studied the stretching sheet problem. After Crane many researchers have extended this work (D. R. Jeng at al., 1986; F. Labropulu at al., 2010; E. Magyari and B. Keller, 1999; R. Ellahi, 2019; M. Y. Malik at al., 2013). The above mentioned studies are about linear stretching but in many practical situations involves non linear stretching such as exponential stretching. Many authors vary velocity of sheet exponentially with distance from slit. Elbashesh (2001) was the first who studied the exponentially stretching sheet problem. He take a perforated sheet and notice the effect of wall mass suction on the flow and heat transfer over an exponentially stretching surface using similarity transformation.

Later on, Sanjayan and Khan (2005,2006) extended the work on exponential stretching. They studied a similar kind of problem considering viscoelastic fluid model under viscousdissipation effects. The non-Newtonian fluids are very useful in industrial and engineering applications. Schowalter (1960) studied the applications of boundary layer using power law fluid. Similarity solutions for non Newtonian power law fluids were obtained by Kapur and Srivastave (1963) and Lee et al. (1966). The power law fluids over a continous moving flat plate with constant surface velocity and temperature distribution was given by Fox et al. (1969). Anderson and Dandapat (1991) extended the pioneeery work of Crane (1970) for a non-Newtonian power law fluids. Later on Hassanin (1998) extended the work for heat transfer analysis. Abel et al. (2009) studied the power law fluid over a vertical stretching sheet with variable thermal conductivity and non uniform heat source. Few relavent intresting works concerning the stretching flowes are cited in (S. Nadeem et al., 2009; Abdul Rehman et al., 2013; M.Naseer et al., 2014; C.Y.Wang and Z Angew, 1989; A. Ishak et al., 2008; I.A. Hassaniern et al., 1998; S. Nadeem and Anwar Hussain, 2010; A. Ishak et al., 2011; C. Y. Wang, 2012; Abdul Rehman et al., 2013). In this paper we have studied the flow and heat transfer of a power law fluid over a vertical exponentially stretching cylinder.
2. Formulation

Consider the problem of natural convection boundary layer flow of a power law fluid flowing over a vertical circular cylinder of radius $a$. The cylinder is assumed to be stretched exponentially along the axial direction with velocity $U_a$. The temperature at the surface of the cylinder is assumed to be $T_a$ and the uniform ambient temperature is taken as $T_e$ such that the quantity $T_u - T_e > 0$ in case of the assisting flow, while $T_u - T_e < 0$ in case of the opposing flow, respectively. Under these assumptions the boundary layer equations of motion and heat transfer are

$$u_r + u/m + w_z = 0,$$  \hspace{1cm} (1)

$$uw_z + w(w_z) = k \left[ \frac{w_r^2}{r} + n w_r^{n-1} u_r \right] + g \beta (T - T_e),$$  \hspace{1cm} (2)

$$uT_r + wT_z = \alpha (T_r + \frac{1}{r} T_z),$$  \hspace{1cm} (3)

where the velocity components along the $(r,z)$ axes are $(u, w)$, $\rho$ is fluid density, $k$ is the consistency coefficient, $\rho$ is pressure, $g$ is the gravitational acceleration along the $z$-direction, $\beta$ is the coefficient of thermal expansion, $T$ is the temperature, $\alpha$ is the thermal diffusibility. The corresponding boundary conditions for the problem are

$$u(a,z) = 0, \quad w(a,z) = U_a, \quad w(r,z) \rightarrow 0 \text{ as } r \rightarrow \infty,$$  \hspace{1cm} (4)

$$T(a,z) = T_a, \quad T(r,z) \rightarrow T_e \text{ as } r \rightarrow \infty,$$  \hspace{1cm} (5)

where $U_a = 2a e^{z/a}$ is the fluid velocity at the surface of the cylinder.

3. Solution of the Problem

Introduce the following similarity transformations:

$$u = \frac{1}{2} U_a f(\eta), \quad w = U_a f'(\eta),$$  \hspace{1cm} (6)

$$\theta = \frac{T - T_e}{T_u - T_e}, \quad \eta = \frac{r^2}{a^2},$$  \hspace{1cm} (7)

Where the characteristic temperature difference is calculated from the relations $T_u - T_e = c e^{z/a}$. With the help of transformations (6) and (7), Eqs. (1) to (3) take the form

$$(n+1)\eta^{2n} (f^{n})^2 + 2n\eta^{n-1} f'^n (f^n)^{n-2}$$

$$+ Re_s (f^{n+1} - f^n) + Re_s \lambda \theta = 0$$  \hspace{1cm} (8)

$$\eta\theta'' + \theta' + \frac{1}{2} \Re Pr (f'' - f') = 0,$$  \hspace{1cm} (9)

In which $\lambda = g \beta a(T_e - T_u)/U_a$ is the natural convection parameter, $\Pr = k/\rho c$ is the Prandtl number, $Re_s = \rho a^2 U_a^{2+\alpha} / k$ is the local Reynolds number and $Re = a \rho U_a / 4k$ is the Reynolds number. The boundary conditions in non dimensional form become

$$f(1) = 0, \quad f'(1) = 1, \quad f' \rightarrow 0, \text{ as } \eta \rightarrow \infty,$$  \hspace{1cm} (10)

$$\theta(1) = 1, \quad \theta \rightarrow 0, \text{ as } \eta \rightarrow \infty.$$  \hspace{1cm} (11)

The important physical quantities such as the shear stress at the surface $\tau_u$, the skin friction coefficient $c_f$, the heat flux at the surface of the cylinder $q_u$ and the local Nusselt number $Nu$ are

$$\tau_u = \tau_u |_{r=a}, \quad q_u = -k T_r |_{r=a},$$  \hspace{1cm} (12)

$$c_f = \frac{\tau_u}{\rho U_a^2}, \quad Nu = \frac{ae^{z/a} q_u}{k(T_u - T_e)}$$  \hspace{1cm} (13)

The solution of the present problem is obtained by using Fehlberg Method.

4. Results and Discussion

The problem of natural convection boundary layer flow of a Power law fluid over an exponentially stretched cylinder is studied in this paper. The cylinder is assumed to be stretched exponentially along its axial direction. The exponential stretching velocity at the surface of the cylinder is assumed to be $U_a = 2a e^{z/a}$. The solution of the problem is obtained numerically with the help of Fehlberg Method. The effect of the various parameters such as the Reynolds number $Re$, the local Reynolds number $Re_s$, the power law index $\eta$, the Prandtl number $Pr$ and the natural convection parameter $\lambda$ over the non dimensional velocity and temperature profiles are presented graphically and in the form of tables. Fig.1 shows the effects of natural convection parameter $\lambda$ on the velocity profile $f'$ when $n = 1$. From Fig.1 it is observed that by increasing the values of natural convection parameter $\lambda$ the velocity profile increases. Fig.2 Shows the influence of local Reynolds number $Re_s$ over the velocity profile $f'$ when $n = 1$. From Fig.2 it is clear that by increasing the values of local Reynolds number $Re_s$ the velocity profile $f'$ decreases. Figs.3 and 4 shows the effects of Prandtl number $Pr$ and Reynolds number $Re$ on temperature profile $\theta$ when $n = 1$. Similar characteristics are observed for Prandtl number $Pr$ and Reynolds number $Re$ in Figs.3 and 4 , by increasing the values of these numbers temperature profile decreases. Fig.5 shows the effects of natural
convection parameter $\lambda$ on the velocity profile $f'$ when
$n = 2$. The velocity profile $f'$ decreases by increasing the
values of natural convection parameter $\lambda$. Fig.6 shows
opposite behavior of velocity profile $f'$ when $n = 2$, the
velocity profile increases by increasing local Reynolds
number $Re_a$. In Figs.7 and 8 temperature profiles are
presented for $n = 2$. The temperature profiles behave just like
for $n = 1$. Table 1 shows the boundary derivatives for the
velocity profile at the surface of the cylinder that corresponds
to the skin friction coefficient at the surface tabulated for
different values of $\lambda$ and $Re_a$. From the Table 1 it is
observed that the magnitude of the boundary derivative
increases with increase in both $\lambda$ and $Re_a$. Table 2 shows
the values for local Nusselt numbers calculated for different
values of Re and Pr. From entries in the Table 2 it is
noticed that with increase in both $Re$ and $Pr$, the Local
Nusselt number $Nu$ decreases.

**Table 1.** $[- f'(1)]$ skin friction coefficient at the surface.

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<th>$\lambda \times Re_a$</th>
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<th>0.1</th>
<th>0.2</th>
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<th>0.4</th>
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<tr>
<td>5</td>
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<td>1.4755</td>
<td>1.5065</td>
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<td>1.5972</td>
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<tr>
<td>10</td>
<td>1.9274</td>
<td>1.9809</td>
<td>2.0499</td>
<td>2.1505</td>
<td>2.3941</td>
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<tr>
<td>15</td>
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<td>2.3968</td>
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<td>2.9537</td>
</tr>
</tbody>
</table>

**Table 2.** $[- \theta'(1)]$ local Nusselt numbers.

<table>
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**Fig. 1.** The influence of natural convection parameter on velocity profile.

**Fig. 2.** The influence of local Reynolds number on velocity profile.

**Fig. 3.** The influence of Prandtl number on temperature profile.

**Fig. 4.** The influence of Reynolds number on temperature profile.
Fig. 5. The influence of natural convection parameter on velocity profile.

Fig. 6. The influence of local Reynolds number on velocity profile.

Fig. 7. The influence of Prandtl number on temperature profile.

Fig. 8. The influence of Reynolds number on temperature profile.

References


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