Applied and Computational Mathematics
2016; 5(2): 46-50
http://www.sciencepublishinggroup.com/j/acm
doi: 10.11648/j.acm.20160502.12
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

Oscillation of Second Order Nonlinear Differential Equations with a Damping Term

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To cite this article:
Xue Mi, Ying Huang, Desheng Li. Oscillation of Second Order Nonlinear Differential Equations with a Damping Term. *Applied and Computational Mathematics*. Vol. 5, No. 2, 2016, pp. 46-50. doi: 10.11648/j.acm.20160502.12

Received: February 5, 2016; **Accepted**: March 7, 2016; **Published**: March 25, 2016

Abstract: A class of second-order nonlinear differential equations with a damping term is investigated in this paper. By using the Riccati transformation technique and general weight functions, we obtain some new sufficient conditions for the oscillation of the equation. Our results improve and extend some known results. Two examples are given to illustrate the main results.

Keywords: Oscillation, Second Order Nonlinear Differential Equation, Damping Term, Riccati Transformation Technique, Weight Function

1. Introduction

In this paper we are concerned with the problem of oscillation of the nonlinear second order differential equation with a damping term

$$(r(t)(x'(t))^{\alpha})' + p(t)(x'(t))^{\alpha} + q(t)f(x(t)) = 0 \qquad (1)$$

$$t \ge t_0 > 0$$

Several assumptions are as follow:

(I) $r(t) \in C^1([t_0,\infty),(0,\infty))$, $p(t), q(t) \in C([t_0,\infty),(0,\infty))$;

(II) $f(x) \in C(R,R)$, and $f(x)/x^{\beta} \ge k$, for some $k > 0$ and for all $x(t) \ne 0$. $\alpha \ge 1$, $\beta \ge 1$, and they are both quotients of odd positive integers.

Let $D = \{(t,s): t_0 \le s \le t < +\infty\}$, $D_0 = \{(t,s): t_0 \le s < t < +\infty\}$. We say the function $H \in C(D,[0,+\infty))$ belongs to a class W_{α} if:

(i) $H(t,t) = 0$ for all $t \ge t_0$, $H(t,s) > 0$ in D_0;

(ii) H has a continuous and non-positive partial derivative in D_0 with respect to the second variable satisfying the condition

$$\frac{\partial}{\partial s} H(t,s) = -h(t,s)(H(t,s))^{\alpha/\alpha+1},$$

for some function $h \in L_{loc}(D,R)$.

We shall consider the solutions of Equation (1) which are defined for all large t. A solution of Equation (1) is said to be oscillatory if it has arbitrarily large zeros, otherwise it is said to be non-oscillatory. Equation (1) is called oscillatory if all its solutions are oscillatory.

Recently, there are many authors who have investigated the oscillation for second order differential equations with a damping term, see [3-16] and the references are cited therein.

Wong [10] has studied the equation

$$x'' + p(t)x' + q(t)f(x) = 0. \qquad (2)$$

Rogovchenko and Tuncay [7], M. Kirane and Yu. V. Rogovchenko [8], Yan [12] have obtained oscillation criteria of the following equation:

$$(r(t)x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0. \qquad (3)$$

Theorem A [8]. Assume that the function f satisfies $\frac{f(x)}{x} \ge K > 0$ for some constant K and for all $x \ne 0$. Suppose further that the functions h, $H \in C(D,(-\infty,+\infty))$ are such that H belongs to the class P and

$$\frac{\partial}{\partial s}H(t,s) = -h(t,s)(H(t,s))^{1/2}, \text{ for all } (t,s) \in D_0.$$

Assume that there exists a function $g \in C^1([t_0,\infty);(0,\infty))$ such that

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)\psi(s) - \frac{a(s)r(s)}{4}Q^2(t,s)\right]ds = \infty,$$

where $a(s) = \exp\left(-\int^s g(u)du\right)$,

$$\psi(s) = a(s)\left(Kq(s) - p(s)g(s) - (r(s)g(s))' + r(s)g^2(s)\right)$$

and $Q(t,s) = h(t,s) + p(s)(r(s))^{-1}(H(t,s))^{1/2}$,

Then Eq.(3) is oscillatory.

More recently, Li et al [9] investigated oscillation criteria for the following equation:

$$(r(t)(x'(t))^{\gamma})' + p(t)(x'(t))^{\gamma} + q(t)f(x(t)) = 0, \quad (4)$$

where $\gamma \ge 1$ is a quotient of odd positive integers and $f(x)/x^{\gamma} \ge \mu$ for some $\mu > 0$.

Theorem B [9]. Suppose that there exists a function $\rho \in C^1((t_0,+\infty),R)$ such that, for some $\beta \ge 1$ and for some $H \in W_{\gamma}$,

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]ds = \infty$$

where

$$\psi(s) = v(s)\left(\mu q(s) - p(s)\rho(s) - (r(s)\rho(s))' + r(s)\rho^{\gamma+1/\gamma}(s)\right),$$

$$v(s) = \exp\left(-(\gamma+1)\int_{t_0}^{t}\left[\rho^{1/\gamma}(s) - \frac{p(s)}{(\gamma+1)r(s)}\right]ds\right)$$

and $Q(t,s) = h(t,s) + p(s)(r(s))^{-1}(H(t,s))^{1/2}$.

Then Eq. (4) is oscillatory.

Theorem C [9]. Suppose that there exists a function $H \in W_{\gamma}$,

$\rho \in C^1((t_0,+\infty),R)$ and $\phi \in C((t_0,+\infty),R)$ such that, for some $\beta \ge 1$ and for all $T \ge t_0$,

$$0 < \inf_{s\ge t_0}\left[\liminf_{t\to\infty}\frac{H(t,s)}{H(t,t_0)}\right] \le +\infty$$

and

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)\psi(s) - \frac{\beta^{\gamma}}{(\gamma+1)^{\gamma+1}}v(s)r(s)h^{\gamma+1}(t,s)\right]ds \ge \phi(T)$$

where $\psi(s)$ and $v(s)$ are as in theorem B. If

$$\int_{t_0}^{+\infty}\left(\frac{\phi_+(s)}{v(s)r(s)}\right)^{1/\gamma}ds = +\infty,$$

where $\phi_+(t) = \max\{\phi(t),0\}$, then Eq.(4) is oscillatory.

It is obvious that (2), (3) and (4) are special cases of Eq. (1).

Motivated by the idea of Li [9], in this paper we obtain, by using a generalized Riccati technique due to Li [9], several new interval criteria for oscillation, that is, criteria given by the behavior of equation (1) on $[t_0,\infty)$. Our results improve and extend the results of Li [9], Rogovchenko [3, 7, 8], and Grace [16]. Finally, several examples are inserted to illustrate the main results.

2. Lemmas

Lemma 1. Let $\lambda \ge 1$ be a ratio of two odd numbers. Then,

$$A^{1+1/\lambda} - (A-B)^{1+1/\lambda} \le \frac{B^{1/\lambda}}{\lambda}[(\lambda+1)A - B]. \quad (5)$$

Lemma 2. Let $C \ne 0$, $D > 0$, $u > 0$ and $\lambda > 0$, then

$$Du - Cu^{\frac{\lambda+1}{\lambda}} \le \left(\frac{\lambda^{\lambda}}{(\lambda+1)^{\lambda+1}}\right)\frac{D^{\lambda+1}}{C^{\lambda}}. \quad (6)$$

3. Conclusions

Theorem 1. Suppose that there exists a function $g(t) \in C^1([t_0,\infty),R)$ such that, for some $m \in [0,1]$ and for some $H \in W_{\alpha}$,

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)\psi(s) - \frac{h^{\alpha+1}(t,s)r(s)v(s)}{(\alpha+1)^{\alpha+1}}\right]ds = \infty \quad (7)$$

where $v(t) = \exp\left\{\int_{t_0}^{t}\left[\frac{p(s)}{r(s)} - (\alpha+1)g^{\frac{1}{\alpha}}(s)\right]ds\right\}$ and

$$\psi(t) = m^{\beta-\alpha}kv(t)q(t) + g^{\frac{\alpha+1}{\alpha}}(t)v(t)r(t) - g(t)p(t)r(t) - v(t)(r(t)g(t))',$$

Then, equation (1) is oscillatory.

Proof. To obtain a contradiction, suppose that $x(t)$ is a non-oscillatory solution of Eq. (1) and let $t_1 \ge t_0$ such that $x(t) \ne 0$ for all $t \ge t_1$. Without loss of generality, we may assume that $x(t) > 0$ for all $t \ge t_1$ since the similar argument holds also for $x(t)$ eventually negative. We define a generalized Riccati substitution by

$$u(t) = v(t)r(t)\left[\left(\frac{x'(t)}{x(t)}\right)^{\alpha} + g(t)\right], \ t \ge t_1 \quad (8)$$

Differentiating (8) and using (1), we obtain

$$u'(t)=\frac{v'(t)}{v(t)}u(t)+v(t)\big(r(t)g(t)\big)'-\frac{p(t)u(t)}{r(t)}+p(t)g(t)v(t)$$

$$-v(t)q(t)\frac{f(x(t))}{(x(t))^{\alpha}}-\alpha v(t)r(t)\left(\frac{x'(t)}{x(t)}\right)^{\alpha+1}-\alpha v(t)r(t)\left(\frac{x'(t)}{x(t)}\right)^{\alpha+1}$$

$$\le\frac{v'(t)}{v(t)}u(t)+v(t)\big(r(t)g(t)\big)'-\frac{p(t)u(t)}{r(t)}+p(t)g(t)v(t)$$

$$-kv(t)q(t)(x(t))^{\beta-\alpha}-\alpha v(t)r(t)\left(\frac{u(t)}{v(t)r(t)}-g(t)\right)^{\frac{\alpha+1}{\alpha}} \quad , \quad (9)$$

where $x(t)$ is a continuous function and $x(t)>0$, so there exist $t_2\ge t_1$ and $m\in[0,1]$ such that $x(t)\ge m$, for all $t\ge t_2$. By lemma 1, let $A=\frac{u(t)}{v(t)r(t)}$, $B=g(t)$, then

$$\left[\frac{u(t)}{v(t)r(t)}-g(t)\right]^{\frac{\alpha+1}{\alpha}}\ge\left(\frac{u(t)}{v(t)r(t)}\right)^{\frac{\alpha+1}{\alpha}}$$

$$-\frac{g^{\frac{1}{\alpha}}(t)}{\alpha}\left[(\alpha+1)\frac{u(t)}{v(t)r(t)}-g(t)\right] \quad (10)$$

Thus, (9) and (10) yield

$$u'(t)\le-\psi(t)-\alpha\left(\frac{u^{\alpha+1}(t)}{v(t)r(t)}\right)^{\frac{1}{\alpha}} \quad (11)$$

Multiplying the both sides of (11) by $H(t,s)$ and integrating the inequality from t_2 to t, we obtain, for all $t\ge t_2$,

$$\int_{t_2}^{t}H(t,s)\psi(s)ds+\alpha\int_{t_2}^{t}\frac{H(t,s)}{(r(s)v(s))^{\frac{1}{\alpha}}}u^{\frac{\alpha+1}{\alpha}}(s)ds$$

$$-\int_{t_2}^{t}h(t,s)H^{\frac{\alpha}{\alpha+1}}(t,s)u(s)ds$$

$$\le H(t,t_2)u(t_2)-2\int_{t_2}^{t}h(t,s)H^{\frac{\alpha}{\alpha+1}}(t,s)u(s)ds. \quad (12)$$

Let $C=\alpha\frac{H(t,s)}{(r(s)v(s))^{\frac{1}{\alpha}}}$, $D=h(t,s)H^{\frac{\alpha}{\alpha+1}}(t,s)$, $u=u(t)$ and $\lambda=\alpha$, by lemma 2, we have

$$\int_{t_2}^{t}H(t,s)\psi(s)ds-\int_{t_2}^{t}\frac{h^{\alpha+1}(t,s)v(s)r(s)}{(\alpha+1)^{\alpha+1}}ds\le H(t,t_2)|u(t_2)|$$

$$\le H(t,t_0)|u(t_2)|.$$

Thus,

$$\int_{t_0}^{t}\left[H(t,s)\psi(s)-\frac{h^{\alpha+1}(t,s)v(s)r(s)}{(\alpha+1)^{\alpha+1}}\right]ds$$

$$\le H(t,t_0)\left[|u(t_2)|+\int_{t_0}^{t}|\psi(s)|ds\right].$$

Hence,

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)\psi(s)-\frac{h^{\alpha+1}(t,s)r(s)v(s)}{(\alpha+1)^{\alpha+1}}\right]ds$$

$$\le|u(t_2)|+\int_{t_0}^{t}|\psi(s)|ds<\infty,$$

which contradicts (7). The proof is complete.

Theorem 2. Suppose that there exist functions $H\in W_{\alpha}$, $g(t)\in C^1([t_0,\infty),R)$, and $\phi(t)\in C([t_0,\infty),R)$ such that, for all $T\ge t_0$,

$$0<\inf_{s\ge t_0}\left[\liminf_{t\to\infty}\frac{H(t,s)}{H(t,t_0)}\right]\le+\infty \quad (13)$$

and

$$\limsup_{t\to\infty}\frac{1}{H(t,T)}\int_{T}^{t}\left[H(t,s)\psi(s)-\frac{h^{\alpha+1}(t,s)r(s)v(s)}{(\alpha+1)^{\alpha+1}}\right]ds\ge\phi(T) \quad (14)$$

where ψ and v are as in Theorem 1. If

$$\int_{t_0}^{\infty}\frac{\phi_+(s)h(t,s)}{(H(t,s))^{\frac{1}{\alpha+1}}}ds=\infty \quad (15)$$

where $\phi_+(t)=\max\{\phi(t),0\}$, then equation (1) is oscillatory.

Proof. As in Theorem 1, without loss of generality we may assume that there exists a solution $x(t)$ of Eq. (1) such that $x(t)>0$ on $[t_1,\infty)$ for some $t_1\ge t_0$. Defining again the function $u(t)$ by (8), we arrive at (12) which implies, for all $t>t_2$,

$$\int_{t_2}^{t}H(t,s)\psi(s)ds-\int_{t_2}^{t}\frac{h^{\alpha+1}(t,s)v(s)r(s)}{(\alpha+1)^{\alpha+1}}ds$$

$$\le H(t,t_2)u(t_2)-2\int_{t_2}^{t}h(t,s)H^{\frac{\alpha}{\alpha+1}}(t,s)u(s)ds.$$

By (14), we have

$$\limsup_{t\to\infty}\frac{1}{H(t,t_2)}\int_{t_2}^{t}\left[H(t,s)\psi(s)-\frac{h^{\alpha+1}(t,s)r(s)v(s)}{(\alpha+1)^{\alpha+1}}\right]ds\ge\phi(t_2).$$

Thus for all $t>t_2$,

$$\phi(t_2) \le u(t_2) - 2\liminf_{t\to\infty} \frac{1}{H(t,t_2)} \int_{t_2}^{t} h(t,s) H^{\frac{\alpha}{\alpha+1}}(t,s) u(s) ds .$$

Consequently,

$$2\liminf_{t\to\infty} \frac{1}{H(t,t_2)} \int_{t_2}^{t} h(t,s) H^{\frac{\alpha}{\alpha+1}}(t,s) u(s) ds \le u(t_2) - \phi(t_2) < +\infty \tag{17}$$

Assume that

$$\int_{t_2}^{+\infty} \frac{u(s)h(t,s)}{(H(t,s))^{\frac{1}{\alpha+1}}} ds = +\infty . \tag{18}$$

By (13), there exists a $\lambda > 0$ such that

$$\liminf_{t\to\infty} \frac{H(t,s)}{H(t,t_0)} > \lambda . \tag{19}$$

By (18), for any positive constant γ, there exists a $t_3 > t_2$ such that for all $t > t_3$,

$$\int_{t_2}^{+\infty} \frac{u(s)h(t,s)}{(H(t,s))^{\frac{1}{\alpha+1}}} ds \ge \frac{\gamma}{\lambda} . \tag{20}$$

Then

$$\frac{1}{H(t,t_2)} \int_{t_2}^{t} h(t,s) H^{\frac{\alpha}{\alpha+1}}(t,s) u(s) ds$$
$$= \frac{1}{H(t,t_2)} \int_{t_2}^{t} \frac{u(s)h(t,s)}{H^{\frac{1}{\alpha+1}}(t,s)} H(t,s) ds$$
$$= \frac{1}{H(t,t_2)} \int_{t_2}^{t} H(t,s) d\left[\int_{t_2}^{s} \frac{u(\xi)h(t,\xi)}{H^{\frac{1}{\alpha+1}}(t,\xi)} d\xi\right]$$
$$= \frac{1}{H(t,t_2)} \int_{t_2}^{t} \left[\int_{t_2}^{s} \frac{u(\xi)h(t,\xi)}{H^{\frac{1}{\alpha+1}}(t,\xi)} d\xi\right]\left(-\frac{\partial H(t,s)}{\partial s}\right) ds$$
$$\ge \frac{1}{H(t,t_2)} \frac{\gamma}{\lambda} \int_{t_3}^{t} \left(-\frac{\partial H(t,s)}{\partial s}\right) ds = \frac{\gamma}{\lambda} \frac{H(t,t_3)}{H(t,t_2)} \ge \frac{\gamma}{\lambda} \frac{H(t,t_3)}{H(t,t_0)} .$$

By (19), there exists a $t_4 > t_3$ such that, for all $t > t_4$,

$$\frac{H(t,t_3)}{H(t,t_0)} > \lambda ,$$

which implies that

$$\frac{1}{H(t,t_2)} \int_{t_2}^{t} h(t,s) H^{\frac{\alpha}{\alpha+1}}(t,s) u(s) ds > \gamma , \quad t > t_4 .$$

Since γ is an arbitrary positive constant,

$$\phi(t_2) \le u(t_2) , \tag{16}$$

and

$$2\liminf_{t\to\infty} \frac{1}{H(t,t_2)} \int_{t_2}^{t} h(t,s) H^{\frac{\alpha}{\alpha+1}}(t,s) u(s) ds = +\infty ,$$

and that contradicts (15).

Thus

$$\int_{t_2}^{+\infty} \frac{u(s)h(t,s)}{(H(t,s))^{\frac{1}{\alpha+1}}} ds < +\infty ,$$

and by (16),

$$\int_{t_0}^{\infty} \frac{\phi_+(s)h(t,s)}{(H(t,s))^{\frac{1}{\alpha+1}}} ds \le \int_{t_0}^{\infty} \frac{u(s)h(t,s)}{(H(t,s))^{\frac{1}{\alpha+1}}} ds < \infty ,$$

which contradicts (15). This complete the proof.

4. Examples

Example 1. Consider the following equation

$$\left(\frac{1}{t}(x'(t))^{\alpha}\right)' + t(x'(t))^{\alpha} + \left(1 - \frac{1}{t} g^{\frac{\alpha+1}{\alpha}}(t) + \frac{1}{t} g(t)\right.$$
$$\left. + \frac{1}{t} g'(t) - \frac{1}{t^2} g(t)\right)(x(t))^{\beta} = 0 , \quad t \ge 1 \tag{21}$$

where $\alpha \ge 1, \beta \ge 1$ are both quotients of odd positive integers, $k = 1, m = 1$ and $g(t) = \left(\frac{t^2 - t^{-1}}{\alpha+1}\right)^{\alpha}$.

Let $H(t,s) = (t-s)^2$. Then $h(t,s) = 2(t-s)^{(1-\alpha)/(1+\alpha)}$, $v(t) = t$ and $\psi(t) = t$. We have

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} \left[H(t,s)\psi(s) - \frac{h^{\alpha+1}(t,s) r(s) v(s)}{(\alpha+1)^{\alpha+1}}\right] ds$$

$$= \limsup_{t\to\infty} \frac{1}{(t-1)^2} \int_{1}^{t} \left[(t-s)^2 s - \frac{2^{\alpha+1}(t-s)^{1-\alpha}}{(\alpha+1)^{\alpha+1}}\right] ds = \infty$$

By theorem 1, Eq. (21) is oscillatory.

Example 2. Consider the following equation

$$\left(\sin t (x'(t))^{\alpha}\right)' + \frac{\sin t}{t}(x'(t))^{\alpha} + (x(t))^{\beta} = 0 , t \ge 1 \tag{22}$$

where $\alpha \geq 1, \beta \geq 1$ are both quotients of odd positive integers, $k=1$ and $m=1$. Let $H(t,s)=(t-s)^2$ and $g(t)=0$. Then $h(t,s)=2(t-s)^{(1-\alpha)/(1+\alpha)}$, $v(t)=t$ and $\psi(t)=t$.

We have

$$\limsup_{t\to\infty}\frac{1}{H(t,T)}\int_T^t\left[H(t,s)\psi(s)-\frac{h^{\alpha+1}(t,s)r(s)v(s)}{(\alpha+1)^{\alpha+1}}\right]ds$$

$$=\limsup_{t\to\infty}\frac{1}{(t-T)^2}\int_T^t\left[(t-s)^2 s-\frac{2^{\alpha+1}(t-s)^{(1-\alpha)}s\sin s}{(\alpha+1)^{\alpha+1}}\right]ds$$

$$\geq \limsup_{t\to\infty}\frac{1}{(t-T)^2}\int_T^t(t-s)^2 sds=\phi(T).$$

It is easy to verify that (15) is satisfied. Hence, Eq. (22) is oscillatory by theorem 2.

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