

# Compactified dimensions as produced by quantum entanglement, the four dimensionality of Einstein's smooth spacetime and 'tHooft's 4- $\epsilon$ fractal spacetime

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**Abstract:** We show how Einstein's four dimensionality of spacetime arises via a Hardy quantum entanglement form of compactification acting on Veneziano bosonic strings space. In turn this quantum entanglement mechanism is directly connected to the transfinite version of Heterotic string's dimensional hierarchy, i.e. 26, 16, 10, 6, 4 and leads directly to 'tHooft-Veltman-Wilson fractal spacetime of dimensional renormalization from which the missing 95.5% dark energy density of the cosmos may be accurately determined. Furthermore we predict the existence of a quasi dimensional regularization quasi particle with a topological mass charge equal to twice that of Hardy's entanglement.

**Keywords:** Quantum Entangled Dimensions, Einstein's Smooth Space, 'tHooft's Fractal Spacetime, Cosmic Expansion, Dark Energy, Feigenbaum Turbulence, New Elementary Particle form Dimensional Regularization, Dimensional Renormalization

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## 1. Introduction

The notion that the curling up of extra spacetime dimensions could be due to quantum entanglement is an attractive intuitive picture which makes the various compactification procedures used in super strings [1,2] and related theories more natural and credible.

In the present short paper we recount how the above initial hunch unfolded into a surprisingly coherent theory connecting various corners of high energy physics and cosmology into a monolithic picture. Thus starting from Veneziano's 26 dimensional bosonic string theory of strong interaction [3,4] and applying Hardy's celebrated results of quantum entanglement to it [5,6] we see how almost 22 dimensions shrink by curling up leaving only four dimensionality of Einstein's spacetime in disentangled form intact. Furthermore when the fractal fine structure of Veneziano spacetime is taken into account, we retrieve 'tHooft-Veltman-Wilson dimension  $D = 4 - \epsilon$  as a Hausdorff dimension corresponding to a fractal renormalization spacetime where  $\epsilon = 2\phi^5$ , while  $\phi^5$  is Hardy's quantum entanglement and  $\phi = 2/(1 + \sqrt{5})$  [7,8]. Finally we show that the preceding result follows directly

from Weyl scaling [9-14] of the inverse electromagnetic fine structure constant for a Cooper pair  $\bar{\alpha}_o/2$  and is part and parcel of the Heterotic superstring hierarchy. We conclude by emphasizing the unity of the transfinite structure of high energy physics and cosmology which constitutes a number theoretical mathematical symphony leading to a remarkable result for the cosmic dark energy density, namely  $1 - (\epsilon/D) = 5\phi^2/2$  which is almost 95.5% of the total energy in the universe in full agreement with all modern astrophysical measurements, observation and analysis [9-14].

## 2. Einstein's D = 4 from Veneziano's Bosonic Transfinite Strings Spacetime D = 26 + k = 26

We recall that the bosonic string theory of the strong interaction was rather successful in reproducing a substantial part of the experimental facts proving that it reflects an equally substantial portion of reality. This theory which goes

back to the work of Regge, Nambu and Veneziano presupposes a 26 dimensional spacetime [3,4]. Adding fine structure to this space it was shown in an earlier work that a consistent theory of this kind which can harmonize with Gross' Heterotic string theory needs to have  $26 + k = 26.18033989$  fractal bosonic dimensions [10-13]. It is this dimension which we will probe mathematically by adding Hardy's quantum entanglement to it [5]. Since Hardy's entanglement is  $\phi^5$  for two particles, it follows that each particle contributes  $\phi^5/2$  [10,11]. Consequently following Noether's theorem and the equivalence between particle-like states and internal dimensions [13], we find for the  $26 + k$  dimensions that the entanglement reduction is

$$\begin{aligned} D(\text{entangled}) &= (26 + k)(\phi^5 / 2) \\ &= (2 + 2k)/2 \\ &= 1 + k \\ &= 1 + (2\phi^5) \\ &= 1.18033989. \end{aligned}$$

From the above, the non-entangled rest is clearly

$$D(\text{disentangled}) = (26 + k) - (1 + k) = 25.$$

The corresponding scaling which we will identify as the density of dark energy for obvious reasons is thus [6-11]

$$\lambda(\text{dark}) = \frac{D(\text{disentangled})}{D(\text{total})} = \frac{25}{26 + k} = 0.954915028.$$

The entangled scaling is consequently

$$\begin{aligned} \lambda(O) &= 1 - \lambda(D) \\ &= 1 - 0.954915028. \\ &= 0.04508497197 \\ &= 1/22 + k \end{aligned}$$

which could be readily interpreted as the density of the ordinary measurable energy in the cosmos in full agreement with experiments [14-16]. Consequently entanglement causes a reduction from  $26 + k$  to  $22 + k$ . That means the observed dimension of spacetime is precisely what remains after the compactification quasi magnetic effect of quantum entanglement:

$$\begin{aligned} D(\text{topological}) &= (26 + k) - (22 + k) \\ &= D(\text{Einstein}) \\ &= 4 \end{aligned}$$

which represents the large "tangible" spacetime dimensions [13-17].

### 3. The 'tHooft-Veltman-Wilson Fractal Renormalization Spacetime Dimension

Instead of starting from the fractal  $26 + k$  strong interaction model of spacetime, we start this time from the crisp ordinary value 26 exactly as in the original theory and we proceed from there to discover undreamed of unity, connection and consistency with various exciting branches of elementary particles high energy physics [1,3,4,10,11]. Proceeding in an exactly analogous way to the preceding section, one finds exactly as before. However instead of  $D$  which corresponds to  $D(\text{Einstein}) = 4$  one now finds a remarkable value, namely a slightly smaller value resembling that of dimensional regularization: [7,8]

$$D(\text{entangled}) = (26)(\phi^5 / 2) = 1.172209271.$$

Therefore we have

$$\lambda(\text{dark}) = \frac{26 - 1.172209271}{26} = \frac{1}{22 + k}$$

$$\begin{aligned} D(\text{'tHooft}) &= 26 - (22 + k) \\ &= 4 - k \\ &= 4 - \epsilon \\ &= 4 - 2\phi^5. \end{aligned}$$

We definitely do not jump to conclusions when we assert that this is the living proof that 'tHooft-Veltman-Wilson's dimensional regularization [7] is by no means a mathematically clever manoeuvre or artifact to get rid of unwanted singularities but a physically motivated, meaningful real result testifying to the fractality of spacetime at extreme energy scales related to the Planck length and by Witten's T-duality, also related to the Hubble length of cosmology [10,15]. In fact  $4 - \epsilon$  was found in previous analysis [18] as the exact solution of quadratic form  $x^2 - 30x + 100 = 0$  leading to and two solutions,

$$x_1 = 26 + k \text{ and } x_2 = 4 - k \text{ where } k = 2\phi^5 = \epsilon.$$

### 4. 'tHooft's Fractal Dimension $4 - \epsilon$ as Part of Weyl Scaling Symphony

We recall that a reconstruction of the inverse electromagnetic fine structure constant  $\bar{\alpha}$  of E-infinity theory which is consistent with the exact renormalization groups equation of the golden quantum field theory is given by [12,13]

$$\bar{\alpha}_o = (1/\phi)\bar{\alpha}_1 + (\bar{\alpha}_2 = \bar{\alpha}_1/2) + \bar{\alpha}_3 + (\bar{\alpha}_4 = \bar{\alpha}_o)$$

where  $\bar{\alpha}_1 = 60$ ,  $\bar{\alpha}_2 = 30$ ,  $\bar{\alpha}_3 = 9$  and  $\bar{\alpha}_4 = 1$ . This leads to

[12,13]

$$\begin{aligned} \bar{\alpha}_o &= (20)(1/\phi)^4 \\ &= 137.082039325 \approx 137 \end{aligned}$$

where  $\phi^4$  is the Hardy self entanglement of a single quantum particle with counterfactual Cantorian spacetime surrounding it and is also equivalent to an Unruh thermal bath [15]. For a Cooper pair, bosonizing  $\bar{\alpha}_o$  leads to  $\bar{\alpha}_o/2$ . It was shown in numerous previous publications that a Weyl scaling of  $\bar{\alpha}_o/2$  which is equivalent to a quasi differentiation, i.e. scaling down  $\bar{\alpha}_o/2$  via the golden mean exponent  $\phi$  leads to the following Heterotic strings hierarchy [12,13]

$$\begin{aligned} (\bar{\alpha}_o/2)(\phi)^n &\xrightarrow{n=1} 42+k \approx 42 \\ &\xrightarrow{n=2} 26+k \approx 26 \\ &\xrightarrow{n=3} 16+k \approx 16 \\ &\xrightarrow{n=4} 10 \\ &\xrightarrow{n=5} 6+k \approx 6 \\ &\xrightarrow{n=6} 4-k \approx 4. \end{aligned}$$

We may remind ourselves again that  $4-k$  is one of two solutions of a fundamental quadratic equation  $x^2 - 30x + 100 = 0$  where  $\bar{\alpha}_1 = 30$  and  $\bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\alpha}_3 + \bar{\alpha}_4 = 100$  [18]. The second solution is the vital  $26+k$  of Veneziano-Nambu spacetime. Note that  $4-k$  is the exact value of 'tHooft-Veltman-Wilson dimensional regularization found in [7] which is the accurate way of understanding the meaning of it all or at least grasping some of the puzzling questions of science as far as the nature of fractal spacetime is concerned [1,16,18]. Finally the existence of a dimensional regularization quasi particle with a topological masse charge equal to twice Hardy's quantum entanglement.

## 5. A New Renormalization Elementary Particle - Connection to Feigenbaum's Turbulence and Outlook

From the preceding analysis and interpretation many important results follow. Here we consider only a few. The most important points are related to a new renormalization quasi elementary high energy particle as well as a deep connection to the Feigenbaum period doubling scenario to turbulence and the associated universalities [19, 20]. We will be quite brief:

a) Since it is a well known fact that the Feigenbaum golden mean renormalization group is homomorphic to the skeleton of the K-Theory behind E-Infinity Cantorian Spacetime, it comes as no surprise that the accumulation point of the Feigenbaum period doubling given by the Eigen value.  $\lambda_\infty = 3.8784$  is extremely close to 'tHooft regularization fractal dimension  $4-K = 3.81966$  [19].

Consequently  $4-k$  marks the global disorder associated with chaotic disintegration of the vacuum i.e, the phase transition from a smooth 4D Einstein space to a 'tHooft-Veltman-Wilson fractal Spacetime [21-29].

b) From point (a) it follows that  $k = 2\phi^5$  may be seen as the union of two Hardy "Entangelon" particles with topological mass charge  $m = \phi^5$  forming one 'tHooft renormalization particle with topological mass charge equal  $m = 2\phi^5 = k$  [21 - 29]. More will be discussed in a forthcoming paper.

## 6. Conclusion

Against the initial belief of the pioneers of dimensional regularization themselves,  $D = 4 - \epsilon$  is far from being merely a neat mathematical trick [7,18]. In fact it tells us profound things about the true fractal-Cantorian non-smooth nature of our spacetime. The parameter  $\mathcal{E}$  is equal to  $2\phi^5$  where  $\phi^5$  is Hardy's quantum entanglement so that  $2\phi^5$  corresponds in a sense to 4 particle-like states which in turn correspond to Einstein's "tangible" spacetime dimensions. We see that quantum entanglement, dark energy, 'tHooft dimensional regularization [7] as well as the Heterotic superstrings dimensional hierarchy are different facets of the same physical reality behind dark energy and accelerated cosmic expansion [7,8]. Finally we propose the existence of a dimensional renormalization particle with a topological mass charge equal to twice Hardy's quantum entanglement.

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