An Alternative Model of Rotation Curve that Explains Anomalous Orbital Velocity, Mass Discrepancy and Structure of Some Galaxies

Paul Cadelina Rivera1, Marco Immanuel Bayle Rivera2

1Astro-Metocean Department, Hymetocean Peers Company, Antipolo City, Philippines
2National Institute of Physics, University of the Philippines-Diliman, Quezon City, Philippines

Email address:
paulcrivera2@gmail.com (P. C. Rivera)

To cite this article:

Received: November 13, 2019; Accepted: December 7, 2019; Published: December 19, 2019

Abstract: A new model of galaxy rotation based on the cyclostrophic model of vortices found in nature is developed. The model is tested using the SPARC dataset of 175 galaxies and a smaller dataset comprising of 60 galaxies. Analysis of the datasets showed that galactic rotation can be adequately described using the observed surface brightness of galaxies and the newly developed cyclostrophic velocity model. The use of the luminosity and the inverse mass-to-light ratio in lieu of the surface brightness, also yield a very good fit of the observed and computed galaxy rotation velocity. Evidently, galactic rotation greatly depends on the cyclostrophic balance of the pressure gradient and the centrifugal forces and the seismic-induced radial expansion occurring in various stars. This is the most probable origin of the action of a single force law that has been overlooked in previous studies. Therefore, the need for a super-massive black hole at the center of galaxies or hidden dark matter can be eliminated. Attractive gravitational force can occur even without a massive black hole at the center of galaxies. There appears to be a pressure gradient force between the center and the outer parts of galaxies that sustains attraction. The cyclostrophic model appears to be the physical basis of the Tully-Fisher relation. Furthermore, the missing mass problem associated with galactic rotation can be attributed to the orbital expansion of celestial objects perturbed by seismic-induced forces. In addition, massive tremors or starquakes may create a domino effect in perturbing nearby stars along the axis of the seismic-induced force and this could result in the formation of elliptical galaxies as the orbits of seismic-perturbed neighboring stars become larger.

Keywords: Cyclostrophic Rotation, Gravitational Weakening, Tully-Fisher Relation, Surface Brightness, Mass Discrepancy

1. Introduction

There is an apparent mass discrepancy with existing models of rotational velocities of galaxies since galactic rotation was discovered over a century ago. Galactic rotation curves are mostly based on the Tully-Fisher relation and its modifications which generally assumes a Newtonian gravitational potential from visible baryonic and dark matter that attract galactic matter towards the center. A number of studies have shown that the Tully-Fisher relation cannot fully describe the galactic rotation as it gives underestimated velocity profiles in larger distances [4, 5-7]. There appears to be a missing mass that holds stars in large distances preventing them from flying away from the galactic center. Such a missing mass discrepancy has always been attributed to dark matter supposedly composed of weakly interacting massive particles (WIMP). In another study, [12] showed that rotation curves and mass distribution in the Milky Way and nearby spiral galaxies showed very striking relationship. The Modified Newtonian Dynamics (MOND) does not resort to dark matter to explain galactic motion and appears to solve the missing mass problem but it lacks the essential physical basis in its derivation and differentiates between weak and strong gravitational fields. This study aims to introduce an alternative way to explain the observed rotation curves of different galaxies.
2. Methods and Theory

The diversity of rotation curves of observable galaxies may be due to the action of a single force law that has been overlooked. In particular, the impact of the pressure gradient force analogous to a frictionless rotating system like twisters may play an important role in galactic motion in the universe. The cyclostrophic model for vortices such as tornadoes occurring in nature is a simple balance between the inward pressure gradient force and the outward centrifugal force [1]. In the absence of Coriolis effect and negligible friction, tornadoes were observed to obey this basic rule. In a similar manner, if we assume the presence of a low pressure area at the center of a galactic ‘vortex’, then the balance of forces is given by the relation

$$\frac{V^2}{R} = -\frac{1}{\rho} \frac{\partial P}{\partial R}$$  \hspace{1cm} (1)

where $V$ is the rotation (tangential) velocity, $R$ is radial distance, $P$ is pressure and $\rho$ is baryonic density. We can assume that this holds for rotating galaxies where the central pressure could be lower than the pressure outside and that the effect of friction in outer space can be neglected. This further assumes the absence of any massive black hole at the galactic center since the ‘eye’ of a tornado is also devoid of any matter like clouds and yet rotates at a tremendous velocity. Hence, similar to a tornado, the present galactic model assumes a central pressure $P_c$ that is relatively lower than the surrounding pressure and this is given by

$$P = P_c - \frac{4\sigma T^4}{c}$$  \hspace{1cm} (2)

where $P_c$ is a constant external pressure, $\sigma$ is the Stefan-Boltzmann constant, $T$ is the galactic temperature, and $c$ is the speed of light [11]. The negative pressure (second term in Equation 2) can also be assumed to be proportional to $2n_k kT$ where $n_k$ is the number density and $k$ is the Boltzmann constant. Differentiating this with respect to $R$ and substituting the result in Equation (1) yields a new circular velocity model for galactic rotation as

$$V = \sqrt{\frac{16\sigma T^3}{\rho} \frac{\partial T}{\partial R}}$$  \hspace{1cm} (3)

The term inside the parenthesis is a temperature gradient ‘force’ which emerges from the pressure gradient force that balances the centrifugal force. With very low temperature at the center, it must be noted that galaxies may possess a temperature gradient from the center and outward to the intergalactic medium. There is generally an increasing radial temperature profile in the innermost portion of galaxies as shown in [9] and Equation 3 should adequately describe the tangential velocities of galaxies when their radial temperature profiles are known. However, if the temperature profile is not known (i.e. they are not readily available), information on the surface brightness or luminosity from galactic data can be used. Using the integrated Eddington flux, the temperature gradient in stars is proportional to negative luminosity as in

$$\frac{16\sigma T^3}{\rho} \frac{\partial T}{\partial R} = -\frac{2k}{4\pi R^2} L$$ \hspace{1cm} where $L$ is the luminosity and $k$ is opacity [3]. This is true for individual stars where the core is too hot compared to the outer parts. Unlike in individual stars however, the temperature at the center of galaxies is too low (approximately equal to the Hawking temperature) and generally increases radially outwards [9]. Hence, the temperature gradient is positive and is directly proportional to luminosity. Here, we introduce a parameter $\beta$ ($\sim \frac{2k}{4\pi}$) which is proportional to the ratio of the opacity to the speed of light. Its exact value may vary for each galaxy and is hereby assumed as a free parameter. Now, since the luminosity and surface brightness $\mu$ are directly proportional ($L = 4\pi R^2 \mu$), the surface brightness can be used as a measure of the temperature gradient term so that Equation 3 can be simplified as

$$V = \sqrt{R\beta \mu_o}$$  \hspace{1cm} (4)

where $\mu_o$ is the effective surface brightness of the galactic disk. Moreover, it was shown in [11] that seismic activities can result in radial orbital expansion where $R = R_o (1+5\times10^{-6} Q_o)$ in which $Q_o$ is the number of quakes and $R_o$ is the orbital radius. Therefore, Equation (4) becomes

$$V = \sqrt{R_o (1+5\times10^{-6} Q_o) \beta \mu_o}$$  \hspace{1cm} (5)

This now assumes that a large number of massive quakes occur in stars of each galaxy so that their orbital distances also expand due to gravitational weakening as massive tremors occur. This gives an important correction to $R$ since any underestimation would lead to underestimated velocity curve. It should be noted that the ‘missing mass’ problem in galactic motion could be due to underestimated radial distance as $V$ also depends on $R$ and not just on galactic matter or surface brightness. The dimensional parameter $\beta$ may vary for each galaxy but its dimension should give units of $V$ in km/s.

2.1. Data Analysis

There are two sets of data used in the present study namely; the recently obtained SPARC (Spitzer Photometry & Accurate Rotation Curves) which covers about 175 galaxies [13] and another dataset comprising of 60 galaxies [6]. The SPARC dataset is available online at astroweb.cwru.edu/SPARC. The 60-galaxy dataset was retrieved from Table 1 in the same article. For both data sets, all physical parameters (effective radius, circular velocity and surface brightness etc.) were subject to a 10-point running average to minimize any spurious oscillations.

3. Results and Discussion

Using the new cyclostrophic model of galactic rotation given by Equation (5), the results of the analysis for the SPARC dataset are shown below. It was assumed that there were about 50,000 quakes per year occurring in each galaxy. Furthermore, the dimensional parameter $\beta$ is set at 25 and
300 for the SPARC dataset and the 60-galaxy dataset in [6], respectively, to get $V$ in km/s. Using Excel Data analysis tool, the F-test was performed for each the two datasets. The results are shown in Tables 1-2 with very small probability values of about $1.21 \times 10^{-10}$ and 0.000155 for both datasets, respectively. This shows that there is a very small probability that the computed velocities could have happened by chance.

**Table 1. F-test for the SPARC dataset.**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Variance</th>
<th>Observations</th>
<th>df</th>
<th>$F$</th>
<th>$P (F\leq f)$ one-tail</th>
<th>$F$ Critical one-tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>118.8798</td>
<td>4093.153</td>
<td>174</td>
<td>173</td>
<td>2.67265</td>
<td>1.21E-10</td>
<td>1.285054</td>
</tr>
<tr>
<td>Variance</td>
<td>121.6315</td>
<td>1531.496</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>174</td>
<td>174</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>173</td>
<td>173</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P (F\leq f)$ one-tail</td>
<td>1.21E-10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F$ Critical one-tail</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. F-test for the 60-Galaxy dataset of [6].**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Variance</th>
<th>Observations</th>
<th>df</th>
<th>$F$</th>
<th>$P (F\leq f)$ one-tail</th>
<th>$F$ Critical one-tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>142.322</td>
<td>3059.705</td>
<td>59</td>
<td>58</td>
<td>2.63631</td>
<td>0.000155</td>
<td>1.545768</td>
</tr>
<tr>
<td>Variance</td>
<td>154.0854</td>
<td>1160.504</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>59</td>
<td>59</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>58</td>
<td>58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P (F\leq f)$ one-tail</td>
<td>0.000155</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F$ Critical one-tail</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The observed and computed velocities for 175 galaxies from the SPARC data [13] are plotted in Figure 1. It can be seen that a good fit exists between the observed and computed tangential velocities. A scatter-plot of the observed and computed velocities is also shown in the figure. The 60-galaxy dataset [6] also showed very significant correlation (Figure 2). A scatter plot of the computed and observed tangential velocities of this dataset is also shown in Figure 2.

**Figure 1.** Observed and computed velocity for 175 galaxies from SPARC data [13] using surface brightness. A scatter plot of the observed and computed velocities is also shown.

Further analysis of the SPARC dataset showed that the mass-to-light ratio $\Upsilon$ can be used in lieu of the surface brightness $\mu_o$ since $L = M/\Upsilon$ where $M$ is the galactic mass. The new rotation velocity equation now becomes

$$V = \frac{R_o(1+5x10^{-6}d_2)\beta}{\Upsilon}$$

(6)

Here, the value of the dimensional parameter is not equal to $\beta$ in Equation 5. It should be noted that the use of the mass-to-light ratio in the denominator gives a new relation between $\Upsilon$ and $V$. This gives a consistent relation analogous to the Hawking temperature and mass since the temperature $T$ in Equation 3 has been replaced by the inverse of mass in Equation 6.

Using $\beta = 4000$ in Equation 6, the computed and observed rotation velocity is plotted in Figure 3. A scatter plot of this is
also shown in the figure. It appears that the use of the mass-to-light ratio in our new model (Eq. 6) yields a very good fit to the observed rotation velocity. Again, it was assumed that there were about 50,000 quakes per year occurring in each galaxy.

A good approximation to the velocity curve can also be obtained using the surface brightness in a 1/3-power law as in

\[ V = \left[ R_o (1 + 5 \times 10^{-6} Q_a) \beta \mu_o \right]^{1/3} \]  

(7)

Using Equation 7 in the SPARC dataset with \( \beta = 2.5 \times 10^3 \), the computed velocity is also compared with the observed velocity in Figure 4. A scatter plot is also shown in the figure. Apparently, there is a good fit between the computed and observed velocities.

The mass-to-light ratio \( \Upsilon \) can also be used in a 1/3-power law as in

\[ V = \left[ \frac{R_o (1 + 5 \times 10^{-6} Q_a)}{\Upsilon} \right]^{1/3} \]  

(8)

With \( \beta = 5 \times 10^3 \), the computed and observed velocities also appear to be in good agreement (Figures 5). In all the exercises, the number of quakes was assumed to be 50,000 per year. This could be an underestimation but the absence of galacto-seismology data precludes accurate determination of the number of quakes and the \( \beta \)-parameter as well.

Rewriting Eq. 7-8 in a 1/4-power law still lead to a good fit between computed and observed velocities as long as the \( \beta \)-parameters are adjusted. Such a 1/4-power law is analogous to
the Baryonic Tully-Fisher relation where the total baryonic mass of galaxies is related to their flat rotation velocity [6, 5]. It should be noted that a correct orbital distance $R$ also leads to a correct determination of the circular velocity with the conventional Newtonian formula

$$V = \sqrt{R_o(1 + 5 \times 10^{-6} Q_o) \frac{\partial\phi_{tot}}{\partial R}}$$

(9)

where $\phi_{tot}$ is the total gravitational potential of the galactic mass. The apparent mass discrepancy problem can be attributed to underestimated orbital distances and not on the weak gravitational potential from visible baryonic matter. The galaxy rotation should indeed be very well correlated with visible mass as in [7].

### 3.1. The Physical Basis of the Tully-Fisher Relation

Using Equation 5 with the square-root of the surface brightness, the new equation becomes;

$$V = \sqrt{R_o(1 + 5 \times 10^{-6} Q_o) \beta \mu_o^{0.5}}$$

(10)

Figure 6. Observed and computed velocity for 175 galaxies from SPARC data [13] using the square root of the surface brightness. A scatter plot of the observed and computed velocities is also shown.

We perform the same analysis with the SPARC data and the results showed very good correlation (with $\beta = 192$). The computed and observed velocities are shown in Figure 6 while the scatter-plot is also shown in the figure. The same analysis was undertaken using the square root of the inverse mass-to-light ratio. Similarly, the correlation is shown to be very good (Figure 7).

We can simplify Equation 10 and obtain

$$V^4 = [R_o(1 + 5 \times 10^{-6} Q_o)]^2 \beta \mu_o$$

(11)

This is analogous to the Tully-Fisher relation but uses only the surface brightness. The use of the luminosity or the inverse mass-to-light ratio can be re-written in this manner.

For the luminosity $L$, the circular velocity becomes

$$V^4 = [R_o(1 + 5 \times 10^{-6} Q_o)]^2 \beta L$$

(12)

With $\beta = 560$, the result of the analysis is plotted in Figure 8 and the corresponding scatter plot is also shown. Similar to the Tully-Fisher relation, there is a very tight correlation between the luminosity and the circular velocity with a correlation coefficient ($r^2 = 0.88$).

Using $L = M/\Upsilon$, then Equation 12 becomes completely dependent on the galactic mass $M$ as in

$$V^4 = [R_o(1 + 5 \times 10^{-6} Q_o)]^2 \beta \frac{M}{\Upsilon}$$

(13)

Without the need for dark matter, this model should give an accurate mass-velocity curve relation when the number of quakes is determinedly accurately. Even with a constant $Q_o = 50,000$ (which may be an underestimation), analysis of the SPARC dataset also yields a very tight correlation between the circular velocity and $M/\Upsilon$. The results are shown in Figure 9 below.
From these analyses, it can be stated that surface brightness, luminosity, or mass-to-light ratio, can be used to quantify the fourth-power of the circular velocity. We can conclude therefore, that galactic rotation greatly depends on the cyclostrophic balance of the pressure gradient force and the centrifugal force plus the seismic-induced orbital expansion occurring in (seismic-perturbed) galactic stars and planets. This is the most probable origin of the action of a single force law that has been overlooked in previous studies. Therefore, the need for a super-massive black hole at the center of galaxies and dark matter halo as in [2, 10] can be eliminated. Attractive gravitational force can occur even without a massive black hole at the center of galaxies or dark matter. The pressure gradient force that occurs due to the difference in pressure between the center and the outer parts of the galaxies and the rotation of stars is what sustains attraction, and prevents the stars from flying away from the galactic disk.

3.2. Effect on the Structure of Galaxies

There appears to be a very strong impact of seismic perturbation on galactic motion that the missing mass problem can be attributed to the expected increase in the orbital distance $R$ of stars in every galaxy undergoing massive tremors. Using the orbital expansion model described in [11], the individual stars which undergo massive tremors or starquakes must create a domino effect in perturbing nearby stars along the axis of the seismic-induced force. Due to associated gravitational weakening problem, the orbits of stars undergoing tremors must increase. This could easily result in the formation of elliptical galaxies as the orbit of seismic-perturbed neighboring stars should become larger.

The seismic-driven increase in $R$ appears to be the root cause of the mysterious mass problem in many, if not all, galaxies in the universe. It is not really mysterious after all, when seismic-induced radial expansion is considered in the calculation of the orbital velocity. This appears to be the main problem in previous estimations of the galactic rotation curve. The underestimated velocity could be due largely to underestimated radial distance and not on the missing mass.

4. Conclusions

The analysis showed that the rotation curve of some galaxies can be described using a cyclostrophic balance of pressure gradient force and centrifugal force similar to vortices occurring in nature. This implies that stars and gases in galaxies naturally rotate and obey the cyclostrophic balance even in the absence of a massive black hole. Furthermore, the mystery of the missing mass of dark matter associated with galaxies is explained by the present model to be largely due to the action of massive quakes on the radial orbital expansion. Due to associated gravitational weakening problem, the orbits of celestial objects including stars undergoing tremors, must increase. As $R$ expands during quakes in galaxies, the rotation velocity of farthest objects in those galaxies must also increase. There is therefore no mass
discrepancy but a discrepancy in the orbital distances of stars in each galaxy observed. The total mass $M_t$ can be determined precisely from the orbital expansion due to seismic-driven gravitational weakening as in

$$M_t = \frac{V^2 R_0 (1 + 5 \times 10^{-7} Q_a)}{g}$$ \hspace{1cm} (14)$$

where $G$ is the universal constant of gravitation. While this should give a good estimate of the total galactic mass, this requires a good inventory of the number of quakes in each galaxy. This can yield an accurate determination of the total galactic mass if $Q_a$ is correctly determined. Assuming that a particular galaxy is quite seismically active with $Q_a = 200,000$, Equation 14 can easily yield a total mass which is 2 times the quake-less mass. A more accurate determination of the total mass can be obtained using Equation 13 as long as $V, Y$ and $\beta$ are accurately determined and calibrated.

With the new galaxy rotation given by the new model presented here, the action of a single effective force law identified in previous studies [5] could just be a cyclostrophic balance of the pressure gradient and centrifugal forces acting in various galaxies. The mass, structure and fate of galaxies can be explained when orbital expansion due to seismic-driven perturbation and concomitant gravitational weakening are included in the dynamics of galactic motions.

References


