

# Weibull Transformation Approach to Formulation of Reliability Model for Analysis of Filth Formation Using Zenith Grinding Machine

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**Abstract:** It is one of the major concerns of production industries to keep sustain quality products through maintenance of reliability goals which is capable of attaining to high demand of the competitive products in the societies. This is one of the motivations of using Weibull method to formulate a reliability model for grinding calcite and barite in production industries. The uniqueness of this work centers on the transformation of the Weibull cumulative function into a linear model which was used to check the level of reliability of the grinded chemicals using zenith grinding machines in manufacturing industries and to design the level of reliability suitable for further productions. These assumptions are in line with the linear transformation model following the aim of ascertaining efficiency of the grinding machines. The Weibull Cumulative distribution function was used to compare with a simple regression model to ascertain the parameter estimates which reflects the reliability levels of the production industries. When the Weibull transformation was compared to the linear model, the shape and scale parameters were estimated and used to establish the level of reliability. This research work described what happened at the various levels of production before felts started forming and developed a reliability model for the prevention of filth formation in grinding calcite and barite with zenith grinding machine in paper producing industries and other industries of similar products.

**Keywords:** Reliability, Scale Parameter, Survival Probability, Parameter Estimates, Regression Model, Filth Formation

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## 1. Introduction

Reliability is one of the main considerations influencing the performance of a production system in producing industries. It is not just because machine breakdowns affect the production rate, the stress brought about by these breakdowns lead to scheduling issues which diminish the productivity of the whole manufacturing operations. The unfavorable impacts of machine breakdowns are felt distinctively in diverse manufacturing circumstances due to the changing way of manufacturing system. While, profoundly computerized, large scale manufacturing system is most sensitive to the reliability changes, grinding process are more adaptable in managing machine failures.

The efficient operation of these systems often required an attempt in the optimization of various indices such as machine voltage, machine maintenance, adequate monitoring and evaluation, quality control, operational control and time which measure the performance of the system. Gradually, more products and services are sold together with the previous satisfaction and value added service that reflects functions earlier carried out by clients. The industrial input in terms of machine voltage, time, filths reduction, machine maintenances and fine quality as outputs are quantified and represented as algebraic variables. Then the values of these variables must be analyzed to find the appropriate trend suitable for estimating future grinding and reliability. In production processes, there may be lost materials, time, due

to errors explained or unexplained caused by the formation of these filths. These errors were taken into account in order to test for the reliability of production design and analysis used during the study. Reliability consists of a wide range of sequence generating methods, for example linear, discrete, continuous, Poisson, Weibull and so on. The application of reliability is a method of optimization that covers a variety of fields in statistics [1]. In the course of this study we restricted ourselves to Weibull transformation to linear transformation model, for it was at this stage that we could determine the equilibrium time, the minimum and maximum minutes within 99 percent failure and survival rates. The research was carried in a paper producing industries using Cassa Paper Industries Limited as a case study. Design and analysis of the experiment was done to examine the reliability of the grinded calcite and barite through the determination of time to failure record in hours and later converted to minutes.

Whenever a model is purely approximated as a straight-line plot of non-dependent factors, it is known as a linear model of the Weibull transformation. In other words, if a response is solely represented by a simple or straight-line function of the self-sufficient variables, it implies that the approximating function is the linear model known as the Weibull representation. This model is generally referred to as the easiest and the simplest model since it is transformed from Weibull exponential to simple regression. It is also the basis of other models. Significant Level shows how likely a result is due to chance, 0.95 means a finding has 95% of being true. When probability calculated is less than probability of critical region, we say that a test is significant. Designs for fitting linear models have various approaches and categories. The range starts from the preparation of the data to suit Weibull analysis. This is followed by linear representation of the various times to failure into ranks. It is at this juncture that the transformation of the ranks forms the dependent variables and the various times to failure are used as the independent values.

The probability that an item will perform its intended function for a specific interval under stated conditions is reliability. The process of determining the level of reliability of a program, item or process is not as simple as the definition seems to appear. In the definition, there are three major aspects which a reliability analyst is not expected to overlook. These are required functions, stated conditions and defined period. It is bases on these objectives that the design and analysis of the calcite and barite grinding industries were carried out in order to understand the components that can be validated as reliable. Reliability is also affected by the environmental conditions of the grinding machines, workers and time of production. For example, stress to the equipment due to vibration could also decrease the reliability [2]. So these conditions should also be taken in account while calculating reliability otherwise such calculations do not have real meaning. Many machines do not have a good reliability function and they do not have any special program that directly defines the problems regarding reliability. Some sections like quality and maintenance management program clarified that equipment reliability is an issue. But still these

sections do not have specific programs that could improve reliability and in the end industry faces different challenges regarding production, maintenance, scheduling, and so on.

In the work of [3], it is necessary that every industry should identify some key goals regarding their production and maintenance plan to overcome these kinds of problems. That could help an industry to overcome these kinds of problems and also helps to find out their current state. Measuring equipment reliability is very much important if you want to improve it. As most equipment failure events are not random. Most happen soon after equipment is maintained. There is a big drawback of only mean time, because it is not showing the value of that level of reliability. By doing this, industries may spend lots of money on small improvements. Reliability must also measure by money made or lost for the business. Reliability specialists often describe the lifetime of a population of products using a graphical representation. The shape parameter,  $\beta$ , is used to determine early failure period with a decreasing failure rate followed by a normal life period with a low, relatively constant failure rate and concluding with a wear-out failure period that exhibits an increasing failure rate.

## 2. Research Methodology

The data gave a set of observations collected during the fineness experiment that is conducted in the optimization of the effect of grinding machines on calcite and barite grinding industries. The various times to filths formation were studied and the effects on the superiority and quantity of calcite and barite powder when grinded with Zenith powder grinding machine were recorded daily. This real life experiment allowed us to examine what happens on the design and analysis of Weibull scale and shape parameters in line with the reliability level of the grinding process. The research enables us to determine the goodness of fit and estimate the parameters of the Weibull distribution. The parameters informed us directly about the shape parameters  $\beta$  and the scale parameter  $\alpha$ . The data was fit to visually determine the scale and shape parameters using linear transformation of the Weibull cumulative distribution function

$$f(x; \beta, \alpha) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta} \quad (1)$$

for  $x \geq 0$  and  $f(x; \beta, \alpha) = 0$  for  $x < 0$ . The variables were transformed to formulate a linear regression model for the Weibull cumulative distribution function and the axes of  $\ln(-\ln(1-f(x)))$  and  $\ln(x)$  were plotted in a standard form of a straight line. After obtaining the observed distribution function from the vertical coordinate for each point using the median rank formula where  $y_i$  and  $N$  are the rank of the data point and the number of the data points respectively (Nelson 2004).

$$\frac{y_i - 0.3}{N + 0.4} \quad (2)$$

The basic Weibull distribution has two parameters, a shape parameter, often termed beta ( $\beta$ ), and a scale parameter, often

termed alpha ( $\alpha$ ). The scale parameter,  $\alpha$ , determines when, in time, a given portion of the population will fail [4]. The shape parameter, beta, is the key feature of the Weibull distribution that enables it to be applied to any phase of the curve. A beta less than 1 models a failure rate that decreases with time. A beta equal to 1 models a constant failure rate. And a beta greater than 1 models an increasing failure rate. The Weibull distribution is the one of the most commonly used analysis methods for lifetime distributions, and is widely applied in non-repairable systems analysis. The Weibull distribution is directly related to the Power Law process. And the two and three parameter Weibull distributions are amongst the most common distributions used. They can be manipulated to support accurate representations using their shape ( $\beta$ ) and scale ( $\alpha$ ) parameters and can thus model a wide variety of data and life characteristics. Since the form of a life distribution is often composed of more than one shape the application of a mixed distribution pattern becomes a natural alternative. According to [5], the application of a mixed distribution Weibull methodology is always possible for the reliability approximation of any arbitrary system.

At this point, the Weibull cumulative distribution function  $F(x)$ , was transformed to appear in form of a simple regression,  $Y = \alpha_1 + \beta_1 x$  by using the (1) and comparing the parameters.

$$1 - F(x) = e^{-\left[\frac{x}{\alpha}\right]^\beta} \tag{3}$$

$$\ln(1 - F(x)) = -\left[\frac{x}{\alpha}\right]^\beta \tag{4}$$

$$\ln\left(\frac{1}{1-F(x)}\right) = \left[\frac{x}{\alpha}\right]^\beta \tag{5}$$

$$\ln\left[\ln\left(\frac{1}{1-F(x)}\right)\right] = \beta \ln\left[\frac{x}{\alpha}\right] \tag{6}$$

$$\ln\left[\ln\left(\frac{1}{1-F(x)}\right)\right] = \beta \ln x - \beta \ln \alpha \tag{7}$$

Comparing (7) with the linear regression model, we see that the left side of the equation corresponds to  $Y$ ,  $\ln x$  corresponds to  $x$ ,  $\beta$  corresponds  $\beta_1$  and  $-\beta \ln \alpha$  corresponds to  $\alpha_1$ . Thus, when we perform the linear regression, the estimate for the Weibull parameter comes directly from the slope of the line  $\beta_1$  since it is equal to  $\beta$ . It is only left for the value of  $\alpha$  to be computed where  $\alpha$  is the estimated value from the linear transformation of the regression coefficients. It is the intercept of the model. In (10),  $\beta$  is the coefficient of the linear model. The equation is used to generate  $\alpha$  which is a real estimated value of the parameter.

$$-\beta \ln \alpha = \alpha_1 \tag{8}$$

$$\ln \alpha = -\frac{\alpha_1}{\beta} \tag{9}$$

$$\alpha = e^{-\left[\frac{\alpha_1}{\beta}\right]} \tag{10}$$

Generally, there is a set goal on every project. In this case, it is desirable to achieve 500 cycles (in minutes) of operation at 0.90. This implies that the grinding machines are expected to operate for at least 500 cycles each day without felt formation each at 90 percent of the time. So, we can expressed the reliability goal mathematically as  $R(500) \geq 0.90$ . In this research, sixty samples of data were gathered over a period of three months (last business quarter of year 2016) namely, October, 2016 to December, 2016. The mill was monitored for twenty days per month for the period of three months. The grinding process was observed each day from the start time until the time felt started forming. The collected data are shown in Table 1.

Table 1. Failure Data from October 2016 to December 2016.

SN	OCT			NOV			DEC		
	Start Time	End Time	Cycles (hrs)	Start Time	End Time	Cycles (hrs)	Start Time	End Time	Cycles (hrs)
1	8:10AM	3:00PM	6:50	8:15AM	12:34PM	4:19	8:30AM	1:45PM	5:05
2	8:00AM	1:55PM	5:55	8:00AM	3:03PM	7:03	8:10AM	3:30PM	7:20
3	8:10AM	3:20PM	7:10	9:10AM	3:26PM	6:16	8:20AM	3:30PM	7:10
4	10:10AM	3:00PM	4:50	8:02AM	4:00PM	7:56	8:25AM	2:55PM	6:30
5	9:20AM	2:30PM	5:10	9:30AM	3:02PM	5:32	8:30AM	1:50PM	5:20
6	8:10AM	3:40PM	7:30	8:00AM	3:51PM	7:51	8:10AM	3:50PM	7:40
7	8:10AM	12:05PM	3:55	8:05AM	3:26PM	7:21	10:30AM	2:55PM	4:25
8	8:30AM	3:00PM	6:30	9:21AM	3:54PM	6:33	8:30AM	1:00PM	4:30
9	8:30AM	11:45AM	3:15	10:01AM	3:25PM	5:24	12:10PM	3:55PM	3:45
10	10:30AM	1:20PM	2:50	8:10AM	2:55PM	6:45	12:30PM	3:55PM	3:25
11	9:01AM	1:20PM	6:10	8:10AM	2:35PM	6:25	8:10AM	3:05PM	6:55
12	8:05AM	3:11PM	6:40	12:30PM	3:45PM	3:15	8:30AM	1:49PM	5:19
13	9:00AM	3:45PM	5:18	12:20PM	3:45PM	3:30	9:10AM	3:47PM	6:37
14	8:05AM	2:18PM	6:31	10:30AM	4:00PM	5:30	8:20AM	3:31PM	7:11
15	8:06AM	2:36PM	7:19	9:15AM	3:25PM	4:10	8:30AM	3:51PM	7:21
16	8:10AM	3:25PM	7:49	8:05AM	3:25PM	7:35	9:30AM	3:49PM	6:19
17	9:10AM	3:59PM	4:13	9:10AM	2:25PM	5:15	8:00AM	3:59PM	7:59
18	8:20AM	3:21PM	7:01	10:30AM	3:25PM	4:55	8:10AM	3:15PM	7:05
19	8:15AM	3:29PM	7:14	8:08AM	3:23PM	7:15	8:30AM	3:59PM	7:29
20	9:25AM	3:01PM	5:26	8:10AM	3:40PM	7:30	8:30AM	2:43PM	6:13

In Weibull analysis, one of the important steps to take in data analysis is preparation [6]. This preparation is the rigorous transformations from exponential to linear model. The essence of transformation is for easy computation and comparison with the linear regression model. The data was prepared by taking the preliminary transformations and computations as shown in Tables 1, 2 and 3. After arranging the data in terms of number of cycles, in minutes, before felt formation, the data were entered and sorted in ascending order from lowers to the highest which form the ranks. The ranks represent the position of each cycle when displayed in ascending order of magnitude. After that, the column median ranks were estimated. This gives the proportion of samples

that will fall by the number of cycles listed column-wise.

In the computation of the column median ranks, we estimated the proportion of the population that will fall within the number of cycles in the Column Cycles (column-wise). There are different approaches of computing the median ranks [7]. The most common formula was used for the purpose of this research. The formula is presented mathematically in equation (1). The  $y_i$  is the column rank which run from 1 to 20 per month and 1 to 60 for the entire quarter. In other words, N is the total number of cycles observed which is equal to the highest rank at each level under consideration (monthly or quarterly).

**Table 2.** Preparation Design for Weibull Analysis for October, 2016.

Cycles(mins)	Ranks	Median Ranks	1/(1-Median Rank)	ln(ln(1/(1-Median Rank)))	ln(Cycles)
170	1	0.0343137	1.035533	-3.3548	5.135798
195	2	0.0833333	1.090909	-2.44172	5.273
235	3	0.1323529	1.152542	-1.95214	5.459586
253	4	0.1813725	1.221557	-1.60881	5.533389
290	5	0.2303922	1.299363	-1.33989	5.669881
310	6	0.2794118	1.387755	-1.1157	5.736572
318	7	0.3284314	1.489051	-0.92095	5.762051
326	8	0.377451	1.606299	-0.74669	5.786897
355	9	0.4264706	1.74359	-0.58708	5.872118
370	10	0.4754902	1.906542	-0.43805	5.913503
390	11	0.5245098	2.103093	-0.29651	5.966147
400	12	0.5735294	2.344828	-0.15992	5.991465
410	13	0.622549	2.649351	-0.02602	6.016157
421	14	0.6715686	3.044776	0.107443	6.042633
430	15	0.7205882	3.578947	0.243	6.063785
434	16	0.7696078	4.340426	0.383882	6.073045
439	17	0.8186275	5.513514	0.534856	6.084499
450	18	0.8676471	7.555556	0.704227	6.109248
451	19	0.9166667	12	0.910235	6.111467
469	20	0.9656863	29.14286	1.215568	6.150603

**Table 3.** Preparation Design for Weibull Analysis for November, 2016.

Cycles(mins)	Ranks	Median Ranks	1/(1-Median Rank)	ln(ln(1/(1-Median Rank)))	ln(Cycles)
195	1	0.0343137	1.035533	-3.3548	5.273
210	2	0.0833333	1.090909	-2.44172	5.347108
250	3	0.1323529	1.152542	-1.95214	5.521461
295	4	0.1813725	1.221557	-1.60881	5.686975
315	5	0.2303922	1.299363	-1.33989	5.752573
324	6	0.2794118	1.387755	-1.1157	5.780744
330	7	0.3284314	1.489051	-0.92095	5.799093
332	8	0.377451	1.606299	-0.74669	5.805135
376	9	0.4264706	1.74359	-0.58708	5.929589
385	10	0.4754902	1.906542	-0.43805	5.953243
393	11	0.5245098	2.103093	-0.29651	5.97381
405	12	0.5735294	2.344828	-0.15992	6.003887
423	13	0.622549	2.649351	-0.02602	6.047372
435	14	0.6715686	3.044776	0.107443	6.075346
439	15	0.7205882	3.578947	0.243	6.084499
441	16	0.7696078	4.340426	0.383882	6.089045
450	17	0.8186275	5.513514	0.534856	6.109248
455	18	0.8676471	7.555556	0.704227	6.120297
471	19	0.9166667	12	0.910235	6.154858
476	20	0.9656863	29.14286	1.215568	6.165418

**Table 4.** Preparation Design for Weibull Analysis for December, 2016.

Cycles(mins)	Ranks	Median Ranks	1/(1-Median Rank)	ln(ln(1/(1-Median Rank)))	ln(Cycles)
205	1	0.0343137	1.035533	-3.3548	5.32301
225	2	0.0833333	1.090909	-2.44172	5.4161
265	3	0.1323529	1.152542	-1.95214	5.57973
270	4	0.1813725	1.221557	-1.60881	5.598422
305	5	0.2303922	1.299363	-1.33989	5.720312
319	6	0.2794118	1.387755	-1.1157	5.765191
320	7	0.3284314	1.489051	-0.92095	5.768321
373	8	0.377451	1.606299	-0.74669	5.921578
379	9	0.4264706	1.74359	-0.58708	5.937536
390	10	0.4754902	1.906542	-0.43805	5.966147
397	11	0.5245098	2.103093	-0.29651	5.983936
415	12	0.5735294	2.344828	-0.15992	6.028279
425	13	0.622549	2.649351	-0.02602	6.052089
430	14	0.6715686	3.044776	0.107443	6.063785
431	15	0.7205882	3.578947	0.243	6.066108
440	16	0.7696078	4.340426	0.383882	6.086775
441	17	0.8186275	5.513514	0.534856	6.089045
449	18	0.8676471	7.555556	0.704227	6.107023
460	19	0.9166667	12	0.910235	6.131226
479	20	0.9656863	29.14286	1.215568	6.171701

In this section, the model adequacy was analyzed. It is important to examine if linear regression model was a suitable approximation of the Weibull model. Model adequacy is very necessary in regression analysis [8]. It is not good to use insufficient experimental design to represent a Weibull model. We tested if the model and experiment followed Weibull distribution. Also, we examined the

distribution of the error terms. When the errors are normally distributed, the design is said to be unbiased. When the model is not fit, the analysis will show non-significant lack of fit and that means the model is not adequate enough for further decisions. The monthly estimation of the shape and scale parameters are represented in Table 5.

**Table 5.** Model Summary for October, November and December, 2016.

		R Square	F	df <sub>1</sub>	df <sub>2</sub>	Sig.	Constant	b <sub>1</sub>	α <sub>1</sub>
OCT	Linear	0.965342	501.3614	1	18	1.36E-14	-23.4998	3.932326	393.884
NOV	Linear	0.961445	448.8685	1	18	3.55E-14	-26.3181	4.380561	406.64
DEC	Linear	0.9588	418.8961	1	18	6.46E-14	-27.7276	4.616065	406.168
		The independent variable is X. Dependent Variable: Y							

The following equations are generated for the model.

$$\alpha = -e^{\left[\frac{23.4998}{3.932326}\right]} = -e^{5.97606} = 393.884$$

$$\hat{Y} = -23.4998 + 3.932326x$$

$$\ln\hat{Y} = 3.932326\ln x - \ln 393.884$$

November:

$$\alpha = -e^{\left[\frac{26.3181}{4.380561}\right]} = -e^{6.00793} = 406.64$$

$$\hat{Y} = -26.3181 + 4.380561x$$

$$\ln\hat{Y} = 4.380561\ln x - \ln 406.64$$

December:

$$\alpha = -e^{\left[\frac{27.7276}{4.61606}\right]} = -e^{6.00677} = 406.168$$

$$\hat{Y} = -27.7276 + 4.61606x$$

$$\ln\hat{Y} = 4.61606\ln x - \ln 406.168$$

Table 5 describes the presence of one independent effect for the respective monthly analysis where α represents the intercept scale parameter. In the linear regression model, there was only one independent variable which was represented as the grinding time to felts formation in terms of the mesh value of the grinded calcite and barite of the paper producing industries. The coefficients of the respective model parameters were further analysed and presented in terms of Weibull transformation.

**Table 6.** Coefficients of α and β Quarterly Analysis.

Model		Unstandardized Coefficients		Standardized Coefficients		t	Sig.
		B	Std. Error	Beta			
1	Constant	-27.201	0.115			39.622	0.000
	X	4.538	0.673	0.982		-40.419	0.000

a. Dependent Variable: Y

From Table 6, the estimated value of  $Y = \alpha_1 + \beta_1x$  obtained as

$$\hat{Y} = -7.62 + 1.104x \tag{11}$$

The value of  $\alpha$  is calculated using (10) as

$$\alpha = -e^{\left[\frac{27.201}{4.538}\right]} = -e^{\left[\frac{27.201}{4.538}\right]}$$

$$\alpha = -e^{5.99405} = 401.0356$$

The analysis of the regression model for the quarterly data gave the estimated values for  $\alpha$  and  $B$  as 401.0356 and 4.538 respectively. The simple regression model when compared to the Weibull parameter after the transformation is presented in equation (12).

$$\hat{Y} = 4.538\ln x - 4.538\ln 401.0356 \tag{12}$$

When we critically examined the model, we noticed the f-value of 0.00417 which indicated significant model parameters. The model error of 0.00% is the probability of the model F-value which occurred as a result of obstructions in the experiment. The amount of "Prob > F < 0.0500" indicates significant model terms as seen in Table 6. The linear analysis done in Table 6 shows that model parameters for time to failure of the machines are significant in the approximated model. When the (Prob>F) is greater than 0.1000, there is an evidence that the model terms are not present.

**Table 7. Linear Model Regression Analysis of Variance.**

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	85.292	1	85.292	0.00417	.000
	Residual	3.151	58	0.054		
	Total	88.443	59	0.054		
a. Predictors (Constant): X						
b. Dependent Variable: Y						

These model terms that are significant add to improve the power of the design are considered in a regression model [9]. The dependent variable and the independent variables in terms of Weibull cumulative functions are true representative

of the regression model. The residual squared has a value of 0.9638 which is in realistic conformity with the adjusted residual squared of value 0.9820.

**Table 8. Testing the Precision Level.**

Model	Observation	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	30	0.982024	0.963757	0.982024	0.964371

The linear model is considered to be relevant to take the helm of the experimental space and it is adequate to be adopted as a true representation of the Weibull function and used to determine the grinding fineness in paper producing

industries. The residuals explain how sufficient, efficient and unbiased the estimated model could explain and truly represent the design and true nature of the experimental region [10].

### 3. Results and Discussion

Table 9 shows the residuals for the quarterly observation from 1 to 120. The estimate of Y is denoted by Y Est. The residual is calculated using  $R = Y_i - \hat{Y}$ :

$$R = \ln\left(\ln\left(\frac{1}{1 - \text{Median Rank}}\right)\right) - 4.538\ln x - 4.538\ln 401.0356 \tag{13}$$

W were  $x$  = the cycles in minutes.

**Table 9. Residual Outputs.**

SN	Y	Y Est.	Residuals	SN	Y	Y Est.	Residuals
1	-4.45184	-3.89475	-0.55709	31	-0.36651	-0.12663	-0.23988
2	-3.29404	-3.27213	-0.02191	32	-0.29559	-0.09185	-0.20374
3	-3.29404	-3.27213	-0.02191	33	-0.24903	-0.0459	-0.20313
4	-2.76122	-3.04518	0.283961	34	-0.20294	-0.01173	-0.19121
5	-2.5132	-2.93583	0.422626	35	-0.15724	0.04464	-0.20188
6	-2.31137	-2.62274	0.311366	36	-0.11184	0.100321	-0.21216
7	-2.14067	-2.4254	0.284731	37	-0.06666	0.155328	-0.22199
8	-1.99236	-2.14461	0.15225	38	-0.02161	0.220468	-0.24208
9	-1.86091	-2.09048	0.229568	39	0.023401	0.241975	-0.21857
10	-1.74261	-1.88019	0.137576	40	0.068458	0.263381	-0.19492
11	-1.63484	-1.79536	0.160521	41	0.136346	0.316457	-0.18011

SN	Y	Y Est.	Residuals	SN	Y	Y Est.	Residuals
12	-1.53568	-1.47108	-0.0646	42	0.136346	0.316457	-0.18011
13	-1.44368	-1.39351	-0.05017	43	0.204918	0.326999	-0.12208
14	-1.35773	-1.24222	-0.11551	44	0.25121	0.358476	-0.10727
15	-1.27693	-1.16843	-0.1085	45	0.298124	0.36892	-0.0708
16	-1.20059	-1.09583	-0.10476	46	0.370009	0.410459	-0.04045
17	-1.12812	-1.05281	-0.07531	47	0.370009	0.410459	-0.04045
18	-1.05904	-1.03856	-0.02048	48	0.444293	0.420784	0.023509
19	-0.99293	-1.02436	0.031429	49	0.52181	0.431086	0.090724
20	-0.92947	-0.96799	0.038516	50	0.52181	0.431086	0.090724
21	-0.86836	-0.94006	0.071699	51	0.603695	0.51267	0.091025
22	-0.80935	-0.88472	0.075367	52	0.691573	0.522766	0.168807
23	-0.7522	-0.8573	0.105097	53	0.691573	0.522766	0.168807
24	-0.69673	-0.55333	-0.1434	54	0.787949	0.532839	0.25511
25	-0.64276	-0.36552	-0.27724	55	0.858883	0.57291	0.285973
26	-0.59014	-0.32888	-0.26126	56	0.937436	0.622506	0.31493
27	-0.53873	-0.29252	-0.24621	57	1.026993	0.710436	0.316557
28	-0.48839	-0.25646	-0.23193	58	1.133895	0.729746	0.404149
29	-0.43901	-0.18518	-0.25383	59	1.365752	0.777666	0.588086
30	-0.36651	-0.12663	-0.23988	60	1.365752	0.806177	0.559575

Generally, it is expected that a fit model should be able to adequately approximate the true situation in the model form. Any regression model that explains the standard errors of the dependent and independent variables in a model is considered to be normal. There are assumptions which every researcher should have in mind while formulating, analyzing and interpreting any estimated regression model. The most common but highly effective one is that the standard error terms  $e_i$ 's should be independently, identically and normally distributed [11]. The next is the mean and variance of the  $e_i$ 's must be zero and  $s^2$  respectively. To examine if the model considers the assumptions mentioned earlier, we plot the graph of the observed and predicted residuals as shown in Figure 1.

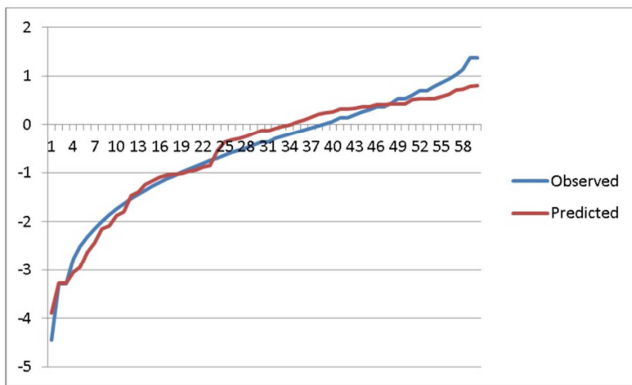


Figure 1. Graph of observed and predicted residuals.

In the Weibull reliability analysis shown in the paper producing industries, the shape parameter,  $\beta$ , indicates whether the rate of felt formation is increasing, constant or decreasing. If  $\beta$  is less than 1.0, it means that the grinding process has a decreasing felt formation rate [12]. This situation is typical of creeping survival and indicates that the machines are failing during its depreciation period. In a case where  $\beta$  is equal to 1.0, there exists a constant felt formation rate in the grinding process. Frequently, grinding components that have reached depreciation period will subsequently exhibit a constant failure rate. A  $\beta$  greater than 1.0 indicates

an increasing felt formation rate. This is typical of systems with components that are wearing out. Such is the case in this study where we have a  $\beta$  value of 4.538 which is much higher than 1.0. This indicates that felt formation could also be occurring due to fatigue, i.e., sub assemblies wearing out.

The Weibull characteristic life, called  $\alpha$ , is a measure of the scale, or spread, in the distribution of data [13]. It so happens that  $\alpha$  equals the number of cycles at which 63.2 percent of the product has failed. In this case, 401.0356 equals 63.2 percent felt formation. In other words, for a Weibull distribution  $R(\alpha)$  equals 0.368, regardless of the value of  $\beta$ . While this is interesting, it still doesn't reveal whether the grinding process meets the reliability goal of  $R(500)$  of 0.90. The formula for reliability assuming a Weibull distribution can be applied using the goal analysis.

$$R(t) = e^{-\left[\frac{x}{\alpha}\right]^\beta} \tag{14}$$

where  $x$  is the time (or number of cycles in minutes) until felt formation

$$R(500) = e^{-\left[\frac{500}{401.0356}\right]^{4.538}} = 0.065828$$

The value is far compared to the goal of obtaining 90 percent reliability at 500 minutes. Table 10 shows the reliability tables results based on cycle times in intervals of 100 minutes in a day.

Table 10. Survival Probability and Reliability for the Paper Producing Industries.

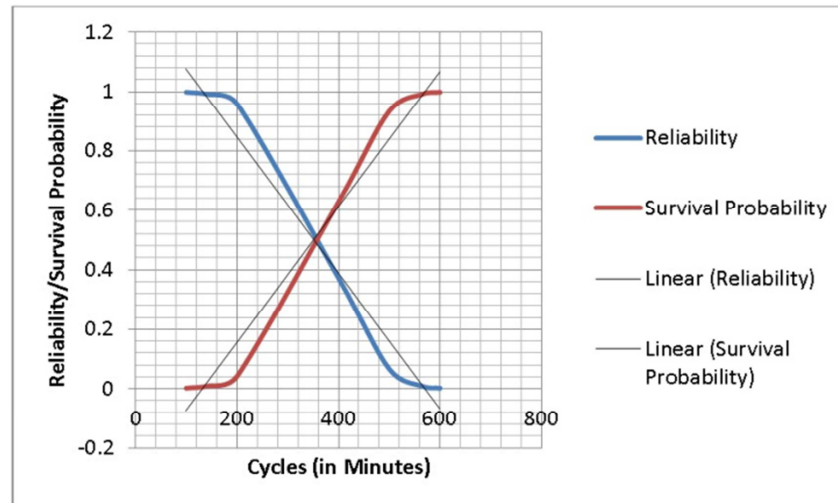
Cycles	Reliability	Survival Probability
100	0.99817	0.00183
145	0.990162	0.009838
200	0.958349	0.041651
400	0.372196	0.627804
500	0.065828	0.934172
564	0.009099	0.990901
600	0.001983	0.998017

In an attempt to compute the number of cycles (or time to

failure) corresponding to a certain reliability level, we did some system iterations to arrive at the results in Table 10. The analysis demonstrates 99 percent chances of felt formation. Looking at the result, we find that for paper producing industries,  $R(564)$  gives 0.01, or 99 percent chance of failure is expected by the 564th cycle in minutes. This

implies that when the grinding process would experience 99 percent level of felt formation for every 9 hours 24 minutes grinding (equivalent to daily production).

An easy way to depict the reliability of grinding process of the paper producing industries based on the model carried out is by using a survival graph which is shown in Figure 2.



**Figure 2.** Survival Graph For The Steel Rolling Mill.

Using the graph, it is expected that 99 percent of the grinding process should survive at least 145 cycles (mins) before felt formation could possibly occur. This implies that the establishment can only boost the grinding process and achieve a competitive grinding result, if and if only 99 percent success for every 2 hours 25 minutes production is achieved. The graph of Figure 2 also indicates that  $R(370 \text{ mins})$  is a point where the reliability and survival probability is at equilibrium. At this point, there exist 50 percent chances of survival and 50 percent chances of failure.

## 4. Conclusion

The analysis of the case study using a transformed Weibull model shows that there were several cases of felt formation in the grinding process of calcite and barite. This is a serious problem in paper producing industries. It is one of the utmost aim of every paper producing industry to maintain at least a 90 percent grinding excellence. There is every indication that the machine voltage regulator, maximum grinding efficiency, adequate machine maintenance, operators training, average temperature can be collectively checked and monitored. Any obstruction in the systems mentioned results to a very difficult and terrible grinding and production process in paper producing industries.

In the study, the extent of reliability of the paper producing industries have been analyzed with the recommended extend of production that the goal of achieving optimum reliability. The grinded calcite are used frequently in many industries for the production of toiletries, coded drugs, pomades, tissues, exercise books and many more. Similarly, grinded and processed barites are progressively used more than before in

the manufacturing sectors and even in individual houses. The two are used for as productive materials because they are more economical than other resources. Their excellence varies between 1250 and 1400 mesh. It is very necessary to maintain optimum grinding that has less or no felts to suit the varied demands of the use of these grinded materials. According to [14], Zenith grinding mall offers unique barite and ultimate calcite powder ranging up to 1399.36 at 50 minutes without felts when the process is fully optimized.

It is recommended from the research that having determined the reliability of the existing system using the weibull methodology, it is recommended that the grinding machines should be regularly serviced and monitored to ensure that there has been no dislodgment since this is the root cause of the felt formation occurrence as observed. It may be necessary to suspend operations for a few minutes in order to achieve this, but it will save time and money in the long run by effectively ensuring that the assembly maintains 0.99 or 99% reliability. Based on the research findings, the grinding industries should apply reliability assessment on the chemical properties of the grinding machines as this was also found to be another cause of random felt formation.

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