Modeling the spread of malaria in North-Eastern Nigeria: a case study of Adamawa state

Adamu Abdul Kareem\textsuperscript{1}, Moses Vandi Tumba\textsuperscript{1}, Dang Bwebum Cleofas\textsuperscript{2}

\textsuperscript{1}Mathematical Sciences Department, School of Pure & Applied Sciences, Modibbo Adama University of Technology Yola, Adamawa State, Nigeria
\textsuperscript{2}Mathematical Sciences Department, University of Jos, Plateau State, Nigeria

Email address: mathsmanadams@yahoo.com (A. Abdul Kareem), vandimoses@yahoo.com (M. Vandi Tumba), bwebumcd@yahoo.com (D. Bwebum Cleofas),

To cite this article:

Abstract: This paper deals with modeling the spread of Malaria in Adamawa State of Nigeria. The simple SIR model and data obtained from the World Health Organization WHO, Adamawa was used to analyze the rate of infection of malaria in the state. It was discovered that $R_0 > 0$. Based on the principle of $R_0$ which states that when $R_0 < 0$, the infection will die out with a certainty, but if $R_0 > 0$, there will be a proper malaria outbreak. It was observed that the force of infection of malaria in the Adamawa state is high. Some necessary recommendations for the eradication of malaria were made.

Keywords: Disease-Free Equilibrium, Force Of Infection, Infectious disease, Jacobian Matrix, Reproduction Ratio

1. Introduction

Malaria, derived from male aria (Italian for “bad air”) and formerly called ague or marsh fever in English, is an infectious disease, which cause at least 300 million infections in humans and approximately 1.2-2 million deaths annually, mainly in the Tropics, Sub-Saharan Africa (Engers and Godal, 1998). The exact statistics are unknown because many cases occur in the rural areas where people do not have access to hospital and/or the means to afford health care. Consequently, many cases are treated at home and are not documented. Malaria is the common name for diseases caused by single-celled parasites of the genus Plasmodium. Among the parasites of the genus Plasmodium, four species have been identified which can cause disease in humans. These include: Plasmodium falciparum, Plasmodium vivax, Plasmodium malaria and Plasmodium ovale. Of these, Plasmodium falciparum is of greatest risk to non-immune humans. The Plasmodium falciparum variety of parasites account for 80% of cases and 90% off deaths Malaria is a major cause of morbidity and mortality. It ranks alongside acute respiratory infections, measles and diarrhea diseases as a major cause of mortality worldwide. Malaria is spread by the bite of an infected female mosquito, of the genus anophelus each time the infected insect takes a blood meal. The symptoms in an infected human include bouts of fever, headache, vomiting flu-like, anemia (destroying red blood cell) and malaria can kill by clogging the capillaries that carry blood to the brain (cerebral malaria) or other vital organs. On the average the incubation period of Plasmodium falciparum is about 12 days in humans. Malaria is endemic in tropical areas where the climatic and weather conditions allow continuous breeding of the mosquito. Malaria is one of the most important parasitic and infectious diseases especially in tropical and sub-tropical areas caused by protozoan parasites of the genus plasmodium. Malaria, affecting mainly children and pregnant women is one of the greatest menace of our society in terms of morbidity and mortality and the occurrence of malaria in our part of the world correlates with poverty and ignorance (Perandin, 2003). Malaria is a major public health problem in the world. The WHO estimates that in tropical countries among the 500 million cases of malaria infection, one million deaths occur annually. Malaria affects the health and wealth of nations and individuals alike. In Africa today, malaria is understood to be both disease of poverty and causing poverty. Malaria has significantly measurable direct and indirect cost and has recently been shown to be a major constraint to economic development. The direct cost of malaria include a combination of personal and public expenditure, include individual and family spending on Insecticide, Treated Mosquito Nets (ITNS), doctor fees, anti malaria drugs, transport to health facilities,
support for patient and sometimes an accompanying family members during hospital stays. The indirect cost of malaria includes lost productivity or low income associated with illness or death. Another indirect cost of malaria is the human pain and suffering caused by the disease. It also hampers children schooling and social development through both absenteeism and permanent neurological. The simple presence of malaria in a community also hampers individual and national prosperity due to its influence on social and economic decisions. The risk of contracting malaria in endemic area can deter investment both internally and externally and affect individual and household decision making in many ways that have negative impact on economic productivity and growth.

Several researches have been carried out on malaria and studies have been reviewed, relevant publications are the work done by Aron and May (1982), Bailey (1982), MacDonald (1957), Nasell (1987), Day et al. (1998), Anderson and May, (1989), and Carter et al. (2000). However, the increase in this epidemic in the sub-African regions has created much vacuum and left much to be desired. It is in the light of the above that this study is carried out to look at modeling the epidemiology of malaria: a case study of Adamawa State in the North Eastern Region of Nigeria. This study is of great importance to Adamawa State because a slightly high rainfall pattern is recorded in the northern part of the nation of which the state is located.

1.1. Mathematical Model

Variables and parameters: We use the following notations and/or definitions for the human populations:

Susceptibles (S): The number of individuals who can be infected but have not yet contracted the malaria fever but may contract it if exposed to any mode of its transmission.

Infectives I: The number of individuals who have been infected of malaria fever.

Recoveries R: The number of individuals who are recovered after treatment and are immune to the disease.

Assuming in the give population at time say t, S(t), I(t) and R(t) denote susceptible, infective carriers and recoveries respectively.

We define the following parameters as follows

\[ N(t) \] The total human population size at time t.

\[ \beta \] The contact rate, which takes into account the probability of getting the disease in contact between a susceptible and an infectious object.

\[ V \] The rate of recovery from the disease by an individual.

\[ F \] The force of infection (coefficient of interaction between the susceptibles and the infectious).

Once a human being recovers from the disease, the person acquires immunity against the disease. The immunity may be temporal or permanent. The natural process of reproduction also occurs in each of the compartments. Natural death (due to other diseases) also occurs in each of the compartments.

1.2. Assumptions of the Model

The disease is short lived and rarely fatal.
The disease is spread by contact between parasites and the individual host.
Individuals who recover develop immunity.

2. Equations of the Model

In view of the above assumptions and inter-relations between the variables and parameters as described in the compartmentalized model in Fig. 1, we have the following system of differential equation.

\[ \frac{dS}{dt} = - \beta IS \] (2.1)

\[ \frac{dI}{dt} = \beta IS - VI \] (2.2)

\[ \frac{dR}{dt} = VI \] (2.3)

Analysis of the Model

Here present the results of the existence and stability analysis of the likely steady states in a human individual as far as the parasites at different stages are concerned and the state of immunity of the individual likewise the equilibrium point.

The force of infection (F)

In the above model, the function F = \( \frac{\beta I}{N} \) models the transition rate from the compartment of susceptible individuals to the compartment of infectious individuals. This is the force of infection of the malaria parasites in the population. However since the population under consideration is large, it is more realistic to consider a force of infection that does not depend on the absolute number of infectious subjects, but rather on their fraction (with respect to the total population size N)

Thus

\[ F = \beta \frac{1}{N} \]

The basic reproduction ratio (\( R_0 \))

The basic reproduction ratio is given by:

\[ R_0 = \frac{\beta}{V} \]

This ratio is derived as the number of new infections from a single infection in a population where all subjects are susceptible. The role of the basic reproduction number in determining the rate of infection is extremely important. In fact upon rewriting the equation of the rate of infection as follows:

\[ \frac{dI}{dt} = \beta \frac{1}{V} VIS - VI \]
\[ R_o V S - V I = (R_o S - I) V I \]  
\hfill (2.4)

It yields that for \( R_o > 0 \) in Eq. (4) above:
\[
\frac{dI}{dt} > 0
\]

Hence there will be a proper malarial outbreak in the state with an increase in the number of the infected individuals in the population (which can reach a considerable fraction of the population). The disease free equilibrium is unstable and the host will not recover from the disease.

On the contrary, if \( R_o < 0 \),
\[
\frac{dI}{dt} > 0
\]

That is independently from the initial size of the susceptible population, the disease can never cause a proper epidemic outbreak. This is the case where the human host will always recover from the disease.

**Equilibrium analysis:**
At equilibrium:
\[
\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0
\]

i. \[ -\beta I S = -V I \]

ii. \[ \beta S = \frac{1}{2}(V) \]

iii. \[ \beta S = \frac{V}{2\beta} \]

\[
S = (\frac{1}{2}\left(\frac{1}{R_o}\right))
\]

\[
S \propto \frac{K}{R_o}
\]

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

**Existence of steady states:** Let E(S*, I*, R*) be steady states such that,
\[
\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0
\]

Then Equation (1), (2) and (3) becomes:
\[
\beta I S* = V I* = 0 \quad (2.4)
\]

\[
\beta I S* - V I* = 0 \quad (2.5)
\]

\[
V I* = 0 \quad (2.6)
\]

On existence of trivial equilibrium:
Solving Eq. (2.4), (2.5) and (2.6) at \((S*, I*, R*) = (0, 0, 0)\) and with all parameters greater than zero, \(\beta, v > 0\) then \(\beta_{,v} \neq 0\). Moreover when the human host is not infected with the malaria, the individual is susceptible to the disease. This implies that there is no trivial equation and thus we cannot have \((S*, I*, R*) = (0, 0, 0)\).

**Disease free equilibrium:**
In this state the individual has no malaria parasites in the body. Thus we take \(I* = 0\). Hence from Eq. (2.5) we have:
\[
\beta I S* - V I* = 0
\]

\[
S* = \frac{V}{\beta}
\]
Thus the disease free equilibrium is given as \( E^*(\frac{v}{\beta}, 0, 0) \).

**Stability of steady states:** We establish the stability of equilibrium points from the Jacobian matrix of Eq. (2.1) to (2.3) which is given as:

\[
J = \begin{bmatrix}
-\beta I & 0 & 0 \\
\beta I & -v & 0 \\
0 & v & 0
\end{bmatrix}
\]

where \( \lambda \) are the eigenvalues of \( J \)

\[
\text{Det}(J) = (-\beta I - \lambda)(-v - \lambda) = 0
\]

\[
\lambda_1 = -\beta, \lambda_2 = -v, \lambda_3 = 0
\]

It is clear that \( \text{Det}(J) < 0 \), which implies that the system is unstable. This contradicts with the initial assumption that individuals are permanently immune, hence there is no permanent immunity in the state.

### 3. Results and Discussion

#### 3.1. Data collected from the World Health Organization, Adamawa State

Table 2.1 shows the statistical analysis of the state population with the target groups 2011 and Table 2.2 shows Malaria cases from 2006 to 2011 in the state. Moreover, Fig. 3.1 to 3.3 is a narrative to the basic reproductive ratio.

#### Table 2.1. Adamawa state population with target groups: 2011

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Demsa</td>
<td>95,149</td>
<td>180,251</td>
<td>210,722</td>
</tr>
<tr>
<td>Fufure</td>
<td>158,137</td>
<td>207,287</td>
<td>242,329</td>
</tr>
<tr>
<td>Ganye</td>
<td>146,835</td>
<td>164,087</td>
<td>191,826</td>
</tr>
<tr>
<td>Gombi</td>
<td>88,635</td>
<td>146,429</td>
<td>171,183</td>
</tr>
<tr>
<td>Guyuk</td>
<td>70,528</td>
<td>177,785</td>
<td>207,839</td>
</tr>
<tr>
<td>Hong</td>
<td>117,240</td>
<td>169,126</td>
<td>197,717</td>
</tr>
<tr>
<td>Maima</td>
<td>83,192</td>
<td>111,215</td>
<td>130,016</td>
</tr>
<tr>
<td>Mayo-belwa</td>
<td>125,073</td>
<td>153,129</td>
<td>179,015</td>
</tr>
<tr>
<td>Michika</td>
<td>117,684</td>
<td>155,302</td>
<td>181,556</td>
</tr>
<tr>
<td>Numan</td>
<td>130,450</td>
<td>90,723</td>
<td>106,060</td>
</tr>
<tr>
<td>Song</td>
<td>198,474</td>
<td>192,697</td>
<td>225,272</td>
</tr>
<tr>
<td>Jada</td>
<td>124,858</td>
<td>168,473</td>
<td>196,953</td>
</tr>
<tr>
<td>Madagali</td>
<td>90,159</td>
<td>134,827</td>
<td>157,619</td>
</tr>
<tr>
<td>Shelling</td>
<td>52,477</td>
<td>149,069</td>
<td>174,269</td>
</tr>
<tr>
<td>Girei</td>
<td>Not created</td>
<td>129,995</td>
<td>151,970</td>
</tr>
<tr>
<td>Lamurde</td>
<td>Not created</td>
<td>112,803</td>
<td>131,877</td>
</tr>
<tr>
<td>Toungo</td>
<td>Not created</td>
<td>052,040</td>
<td>060,837</td>
</tr>
<tr>
<td>Yola North</td>
<td>Not created</td>
<td>198,247</td>
<td>231,760</td>
</tr>
<tr>
<td>Yola South</td>
<td>Not created</td>
<td>194,607</td>
<td>227,505</td>
</tr>
<tr>
<td>Mubi North</td>
<td>Not created</td>
<td>129,937</td>
<td>150,734</td>
</tr>
<tr>
<td>Mubi South</td>
<td>Not created</td>
<td>151,072</td>
<td>176,611</td>
</tr>
<tr>
<td>Total</td>
<td>3,168,101</td>
<td>3,703,664</td>
<td>3,703,664</td>
</tr>
</tbody>
</table>

Source: National Population Commission, Abuja, Nigeria

#### Table 2.2. Malaria Cases in Adamawa state, North-Eastern Nigeria

<table>
<thead>
<tr>
<th>Years</th>
<th>cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>753,752</td>
</tr>
<tr>
<td>2007</td>
<td>881,895</td>
</tr>
<tr>
<td>2008</td>
<td>1,083,109</td>
</tr>
<tr>
<td>2009</td>
<td>1,476,383</td>
</tr>
<tr>
<td>2010</td>
<td>1,859,978</td>
</tr>
<tr>
<td>2011</td>
<td>2,200,270</td>
</tr>
</tbody>
</table>

![Fig. 3.1. Graph of the basic Reproduction Ratio \( R_0 \) in the state when \( K = \frac{v}{\beta} \).](image-url)
3.2. Calculation of the Basic Reproduction Ratio ($R_0$) with Target Groups: 2011

From the equilibrium analysis, it was deduced that $S \propto \frac{K}{R_0}$, thus $R_0 = \frac{K}{S(t)}$, for all $k > 0$ we have:

$R_0$ for Demsa
\[
R_0 = \frac{K}{210722} = 0.00000475K
\]

For all $K = \frac{1}{2}$, 1 and 2, $R_0 = 0.00000238, 0.00000475$ and $0.0000095$, respectively, $R_0 > 0$

$R_0$ for Fufore
\[
R_0 = \frac{K}{242329} = 0.00000413K
\]

For all $K = \frac{1}{2}$, 1 and 2, $R_0 = 0.00000207, 0.00000413$ and $0.00000826$, respectively, $R_0 > 0$

$R_0$ for Ganye
\[
R_0 = \frac{K}{191826} = 0.00000521K
\]

For all $K = \frac{1}{2}$, 1 and 2, $R_0 = 0.00000261, 0.00000521$ and $0.0000104$, respectively, $R_0 > 0$

$R_0$ for Gombi
\[
R_0 = \frac{K}{171183} = 0.00000584K
\]

For all $K = \frac{1}{2}$, 1 and 2, $R_0 = 0.00000292, 0.00000584$ and $0.0000117$, respectively, $R_0 > 0$

$R_0$ for Guyuk
\[
R_0 = \frac{K}{207839} = 0.00000481K
\]

For all $K = \frac{1}{2}$, 1 and 2, $R_0 = 0.00000241, 0.00000481$ and $0.00000962$, respectively, $R_0 > 0$
For all $K = \frac{1}{2}, 1$ and $2$, $R_o > 0$
\[ R_o = \frac{K}{150734} = 0.00000663K \]

For all \( K = \frac{1}{2}, 1 \) and 2, \( R_o = 0.00000332, 0.00000663 \) and 0.0000133, respectively, \( R_o > 0 \)

\( R_o \) for Mubi South

\[ R_o = \frac{K}{176611} = 0.00000566K \]

For all \( K = \frac{1}{2}, 1 \) and 2, \( R_o = 0.00000283, 0.00000566 \) and 0.0000113, respectively, \( R_o > 0 \)

Hence there is a proper malarial outbreak in the state with an increase in the number of the infected individuals in the population (which can reach a considerable fraction of the population). The disease free equilibrium is unstable and the host will not recover from the disease.

4. Interpretation of Result

The number of new infections increases over the years in this population of which every individual is susceptible and thus the basic reproduction ratio is positive (that is \( R_o > 0 \)). Hence there will be a proper malaria outbreak in the state with an increase in the number of the infected individuals in the population (Which can reach a considerable fraction of the population).

The force of infection increases over the years and thus the number of susceptible individuals in the population decreases making the relation \( S \propto \frac{K}{R_o} \) valid.

It is seen that from 2006 to 2011, the number of infected individuals keep increasing verifying the fact that as \( t \to \infty \), \( S(t) \to 0 \) and \( I(t) \to N \). Considering 2011, out of a total population of 3,703,664, 2,200,270 people contracted the disease which is about 68.3% of the population. This shows a high level of infection in the population and testifies the fact that every individual in the population is susceptible.

4.1. Conclusion

In conclusion, I will like to put forward the following to help check the rate of infection if not to eradicate the malarial fever from Adamawa state, the following recommendations should be taken into consideration:

- Malaria prevention must be a public health priority.
- The government should provide inhabitants of the state a clean environment and clean drainage system.
- Vaccines against malaria fever should be made available by the World Health Organization and should be used everywhere in the state.
- The battle against malaria can only be won through prevention.
- The public should undertake practices which can limit malaria transmissions by trying to reduce the number of mosquitoes and mosquito bites through the use of bed nets, spraying and wearing long sleeves shirts and trousers to reduce the amount of exposed skin.
- The public should remove potential mosquito breeding sites and so reduce the number of mosquitoes.

4.2. Recommendation

To eradicate malaria fever from Adamawa state, the following recommendations should be taken into consideration:

References