On chemically reacting hydromagnetic flow over a flat surface in the presence of radiation with viscous dissipation and convective boundary conditions

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Abstract: This paper presents an investigation of the hydromagnetic flow over a flat surface with convective boundary conditions and internal heat generation in the presence of chemical reaction. The Newton-Raphson shooting method along with the fourth-order Runge-Kutta integration algorithm has been employed to tackle the third order, nonlinear boundary layer equation governing the problem. Results have been graphically displayed and discussed quantitatively to show some interesting aspects of the controlling parameters on the dimensionless axial velocity, temperature and the concentration profiles, local skin friction, and the rate of heat and mass transfer. Comparison of the numerical results of the present paper with earlier published works under some special cases showed consistency.

Keywords: Convective Boundary Condition, Hydrodynamics, Brinkmann Number, Viscous Dissipation, Heat Generation

1. Introduction

The study of hydrodynamic flow with heat and mass transfer over a flat plate may find its application to sheet extrusion in order to make flat plastic sheets. In doing so, it is important to investigate cooling and heat transfer for the improvement of the final products. The conventional fluids such as water and air are amongst the most widely used fluids as the cooling medium.

However, the rate of heat exchange achievable by the above fluids is realized to be unsuitable for certain sheet materials. Thus, in recent years, it has been proposed to alter flow kinematics so that, it will leads to a slower rate of solidification, as compared with water. Among the techniques to control flow kinematics, the idea of using magnetic fields appears to be the most attractive one, both because of its ease of implementation and also because of its non-intrusive nature. This has diverted the attention of researchers to the mathematical investigations of hydrodynamic flow of electrically conducting fluids with heat and mass transfer over surfaces with different orientations.


Again, Ibrahim and Makinde [7] investigated into radiation effect on chemically reacting Magnetohydrodynamics (MHD) boundary layer flow of heat and mass transfer through a porous vertical plate. In that same year, Ibrahim and Makinde [8] presented a numerical analysis of chemically reacting Magneetohydrodynamics (MHD) boundary layer flow of heat and mass transfer past a low-heat-sheet moving vertically downwards. Heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation was studied by Makinde [18]. Singh

Moreover, Uddin et al. [17] studied MHD forced convective laminar boundary layer flow from a convectively heated moving vertical plate with radiation and transpiration effect and recently, Arthur and Seini [4] studied MHD thermal stagnation point flow towards a stretching porous surface. This fact motivated the present communication to consider chemically reacting MHD flow over a flat surface in the presence of internal heat generation with convective boundary conditions.

2. Mathematical Model

The steady laminar two-dimensional hydrodynamic boundary layer flow with heat and mass transfer over a flat plate in a stream of cold fluid at temperature $T_s$ in the presence of a volumetric rate of heat generation and magnetic field is considered.

It is assumed that the fluid property variations due to temperature and chemical species concentration are limited to fluid density.

It is also assumed that the lower surface of the plate is heated by convection from a hot fluid at temperature $T_s$ which provides the heat transfer coefficient $h_s$. The cold fluid on the upper side of the plate is assumed to be Newtonian and electrically conducting with constant fluid property. A uniform transverse magnetic field $B_0$ is imposed normal to the $x$-axis, as shown in Figure 1. Both the induced magnetic field due to the motion of the electrically conducting fluid and the electric field due to the polarisation of charges are assumed to be negligible.

Let the $x$-axis be taken along the direction of plate and $y$-axis, normal to it. If $u$, $v$, $T$ and $C$ are the fluid $x$-component of velocity, $y$-component of velocity, temperature and concentration respectively, then under the Boussinesq and boundary-layer approximations, and based on the above assumptions the continuity, momentum, energy and mass transfer (concentration) equations for the problem under consideration can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - U_s)$$  \hspace{1cm} (2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \left( \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \frac{\partial \psi}{\partial y} \right) + \frac{\sigma B_0^2 (u - U_s)^2}{\rho c_p}$$  \hspace{1cm} (3)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \gamma (C - C_s)$$  \hspace{1cm} (4)

Boundary conditions:

$$y = 0: u = v = 0, -\kappa \frac{\partial T}{\partial y} = h_s (T_s - T), \hspace{1cm} C = C_s$$

$$y \rightarrow \infty: u(x, y) = 0, T = T_s, \hspace{1cm} C = C_\infty$$  \hspace{1cm} (5)

where $\nu$ is the kinematic viscosity, $\sigma$ is the electrical conductivity, $T_s$ is the free stream temperature, $C_s$ is the free stream concentration, $U_s$ is the free stream velocity, $\alpha$ is the thermal diffusivity, $D$ is the mass diffusivity, $\kappa$ is the thermal conductivity, $B_0$ is the magnetic field of constant strength, $\rho$ is the fluid density, $c_p$ is the specific heat capacity at constant pressure, $\gamma$ is the reaction rate and $q_s$ is the radiative heat flux.

3. Method of Solution

The velocity components $u$ and $v$ can be expressed in terms of the stream function, $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \hspace{1cm} v = -\frac{\partial \psi}{\partial x}$$  \hspace{1cm} (6)

We seek for the stream function and the similarity variable respectively such that

$$\psi(x, y) = x^{\frac{\nu}{U_s}} \sqrt{2 U_s} f(\eta), \hspace{1cm} \eta = x^{\frac{\nu}{U_s}} \sqrt{\frac{U_s}{v}}$$  \hspace{1cm} (7)
It may be verified that the continuity equation in (1), is
identically satisfied.

Using the Rosseland approximation for radiation, Ibrahim
and Makinde [7] simplified the heat flux as

\[ q_r = -\frac{4\sigma^* T^4}{3 K'} \frac{\partial T^4}{\partial y} \]  

(8)

where \( \sigma \) and \( K' \) are the Stefan-Boltzmann constant and the
mean absorption coefficient respectively. We assume that the
temperature differences within the flow such as the term \( T^4 \)
may be expressed as a linear function of temperature. Hence,
expanding \( T^4 \) in a Taylor series about \( T_\infty \), and neglecting higher
order terms, we get;

\[ T^4 \approx 4T^3 - 3T^2 \]  

(9)

Also, the following non-dimensional temperature and
concentration can be introduced as

\[ \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_\infty - T_\infty} \]  

(10)

Considering (6)-(10), we can transform (2)-(4) into the
following ordinary nonlinear system of differential equations:

\[ f''''(\eta) + \frac{1}{3} f'(\eta) f''(\eta) - \text{Ha}_s (f'(\eta) - 1) = 0 \]  

(11)

\[ \left( 1 + \frac{4}{3} \text{Ra} \right) \theta''(\eta) + \frac{1}{2} \text{Pr} f'(\eta) \theta'(\eta) + \text{Br} f''(\eta) + \text{Br} \text{Ha}_s (f'(\eta) - 1) = 0 \]  

(12)

\[ \phi''(\eta) + \frac{1}{2} Scf(\eta) \phi'(\eta) - Sc \beta_s \phi(\eta) = 0 \]  

(13)

The corresponding boundary conditions in (3) now become

\[ f(0) = 0, f'(0) = 0, \theta'(0) = \text{Bi}_s [\theta(0) - 1], \phi(0) = 1, \]  

\[ f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0. \]  

(14)

In the above equations, primes denote the order of
differentiation with respect to the similarity variable \( \eta \), where

\[ \text{Pr} = \frac{\nu}{\alpha} \] (Prandtl number), \( \text{Ra} = \frac{4 \sigma^* T^3}{K'\nu} \) (Thermal radiation
parameter), \( \text{Ha}_s = \frac{\sigma \text{Bi}_s x}{\kappa} \) (Local magnetic field parameter),

\[ \text{Br} = \frac{\mu U^2}{\kappa (T_\infty - T_\infty)} \] (Brinkmann number), \( \text{Bi}_s = \frac{h_x}{\kappa} \sqrt{\frac{\nu}{U^2}} \) (Local
Biot number), \( Sc = \frac{\nu}{D} \) (Schmidt number) and \( \beta_s = \frac{\gamma x}{U^2} \)
( Local reaction rate parameter).

Obviously the local parameters \( \text{Ha}_s, \text{Bi}_s, \) and \( \beta_s \) in (11)-(14)
are functions of \( x \). In order to have a similarity solution, all the
parameters must be constant and we therefore assume that

\[ h_x = ax^{-3/2}, \sigma = bx^{-1} \] and \( \gamma = cx^{-1} \)

where \( a, b \) and \( c \) are constants.

4. Numerical Procedure

The governing equations in (11)-(13) with the boundary
conditions in (14) have been solved numerically by the use of
Runge–Kutta integration along with the Newton Raphson
algorithm to obtain approximate solutions. From the
computations, the local skin friction coefficient, the local
Nusselt number and the local Sherwood number at the plate
surface which are respectively proportional to \( -f'(0), -\theta'(0) \) and
\( -\phi'(0) \) are also worked out and their numerical values are
presented in tabular form.

5. Numerical Results

The numerical results are compared with Aziz [19] and
Makinde [11] in Table 1 to justify the accuracy of the method
used. From the comparison, the present results are consistent
with theirs.

In Table 2, it is observed that the rate of heat transfer, \( -\theta'(0) \),
increases with increasing the Prandtl number and the local Biot
due to convective heat exchange at the surface of the
plate. At the same time, the Lorenz force induced by increasing
the magnetic field intensity increased the skin friction at the
surface. Though this force is a retarding one, it enhanced the
rate at which mass is transferred, \( -\phi'(0) \). Meanwhile this
retarding force was distractive to the rate of heat transfer. Just as
the magnetic field parameter, increasing the thermal radiation
parameter and the Brinkmann number reduced the rate of heat
transfer due to viscous dissipation. Furthermore, increasing the
Schmidt number and the reaction rate parameter increases the
rate of mass transfer for obvious reasons.

5.1. Effects of Parameter Variation on the Velocity Profiles

The effect of varying the controlling parameters on the
velocity boundary layer is depicted in Figure 2. Generally, the
fluid velocity is lowest at the plate surface and increases to the
free stream value satisfying the far field boundary condition. A

\[ \text{Figures} \]

consistent decrease in the longitudinal velocity, informed by increasing magnetic field intensity, with all profiles tending asymptotically to the free stream value away from the plate, is observed. In practice, this phenomenon is due to the fact that increasing the magnetic field strength increases the Lorenz force which causes a greater opposition to fluid transport.

5.2. Effects of Parameter Variation on the Temperature Profiles

Figures 3 – 7 show the effects of the magnetic field parameter, Biot number, radiation parameter, Brinkmann number and Prandtl number respectively on the temperature profiles. It is observed that increasing the magnetic field intensity increases the fluid temperature which in turn, increases the thermal boundary layer. This can be attributed to the effect of ohmic heating on the flow system. An increase in the Biot number is observed to increase the temperature of the fluid due to the convective heat exchange between the hot fluid at the lower surface of the plate and the cold fluid at the upper surface of the plate. Increasing the thermal radiation causes an increase in the fluid temperature within the boundary layer. We observe the same trend for the Brinkmann number.

Meanwhile, increasing the Prandtl number decreases the fluid temperature within the boundary layer. When the Prandtl number is high, the fluid velocity decreases, which implies lower thermal diffusivity and hence, decrease in fluid temperature.
5.3. Effects of Parameter Variation of Concentration Profiles

![Figure 8. Concentration Profiles for increasing magnetic field parameter.](image)

![Figure 9. Concentration Profiles for increasing local reaction rate parameter.](image)

![Figure 10. Concentration Profiles for increasing Schmidt number.](image)

Figures 8 – 10 show the effects of the Magnetic field parameter, reaction rate parameter and the Schmidt number respectively on the concentration boundary layer. It is observed that increasing the magnetic parameter reduces the species concentration boundary layer thickness. The same is true for increasing reaction rate parameter and the Schmidt number for obvious reasons.

### Table 1. Comparison of results for $H_a = 0$, $Br = 0$ and $Pr = 0.72$

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### Table 2. Results of skin friction coefficient, Nusselt number and Sherwood number for various values of controlling parameters

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<th>$Sc$</th>
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<th>$Ra$</th>
<th>$Br$</th>
<th>$Bx$</th>
<th>$Bix$</th>
<th>$f''(0)$</th>
<th>$-\phi'(0)$</th>
<th>$-\phi'(0)$</th>
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<td>0.451835</td>
<td>0.068283</td>
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6. Conclusion

Chemically reacting MHD flow over a flat surface in the presence of internal heat generation with convective boundary conditions has been studied. Numerical results have been compared to earlier results published in the literature and consistency is shown. Our results revealed that:

i. A consistent decrease in the longitudinal velocity within the boundary layer accompanies a rise in the magnetic field intensity with all profiles tending asymptotically to the free stream value away from the plate.

ii. The thermal boundary layer increases with increasing values of the magnetic field parameter, Biot number, radiation parameter and Brinkmann number. Meanwhile, increasing the Prandtl number reduces it.

iii. The concentration boundary layer decreases with increasing Magnetic field parameter, reaction parameter and the Schmidt number.

iv. The skin friction at the surface increases for the increase in the magnetic field parameter.

v. The rate of heat transfer at the surface increases with increasing values of the Prandtl number and the Biot number; whereas a decrease is observed for increasing the magnetic field parameter, the radiation parameter and the Brinkmann number.

vi. The rate of mass transfer at the surface increases with increasing values of the magnetic field parameter, the reaction rate parameter and the Schmidt number.

References


