A Study on (Q,L)-Fuzzy Normal Subsemiring of a Semiring

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Abstract: In this paper, we introduce the concept of (Q,L)-fuzzy normal subsemirings of a semiring and establish some results on these. We also made an attempt to study the properties of (Q,L)-fuzzy normal subsemirings of semiring under homomorphism and anti-homomorphism, and study the main theorem for this. We shall also give new results on this subject.

Keywords: (Q,L)-Fuzzy Subset, (Q,L)-Fuzzy Subsemiring, (Q,L)-Fuzzy Normal Subsemiring, Product Of (Q,L)-Fuzzy Subsets, Strongest (Q, L)-Fuzzy Relation, Pseudo (Q, L)-Fuzzy Coset

1. Introduction

There are many concepts of universal algebras generalizing an associative ring $(R; +, \cdot)$. Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra $(R; +, \cdot)$ is said to be a semiring if $(R; +)$ and $(R; \cdot)$ are semigroups satisfying $a(b+c) = a.b + a.c$ and $(b+c).a = b.a + c.a$ for all a, b and c in R . A semiring R is said to be additively commutative if $a+b = b+a$ for all a, b in R . A semiring R may have an identity 1, defined by $1.a = a = a.1$ and a zero 0, defined by $0+a = a = a+0$ and $a.0 = 0 = 0.a$ for all a in R . After the introduction of fuzzy sets by L.A. Zadeh [15], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S. Abou Zaid [10]. A. Solairaju and R. Nagarajan [12] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the concept of (Q,L)-fuzzy normal subsemiring of a semiring and established some results.

2. Preliminaries

2.1. Definition 1

Let X be a non-empty set and $L = (L, \leq)$ be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function $A: X \times Q \rightarrow L$.

2.2. Definition 2

Let $(R, +, \cdot)$ be a semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-fuzzy subsemiring (QLFSSR) of R if the following conditions are satisfied:

- (i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,
- (ii) $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q .

2.3. Definition 3

Let R be a semiring and Q be a non-empty set. An (Q, L)-fuzzy subsemiring A of R is said to be an (Q, L)-fuzzy normal subsemiring (QLFNSSR) of R if it satisfies the following conditions:

- (i) $A(x+y, q) = A(y+x, q)$,
- (ii) $A(xy, q) = A(yx, q)$, for all x and y in R and q in Q .

2.4. Definition 4

Let A and B be any two (Q,L)-fuzzy subsets of sets G and H , respectively. The product of A and B , denoted by $A \times B$, is defined as $A \times B = \{ \langle (x, y), q \rangle, A \times B((x, y), q) \mid \text{for all } x \text{ in } R \text{ and } y \text{ in } H \text{ and } q \text{ in } Q \}$, where $A \times B((x, y), q) = A(x, q) \wedge B(y, q)$.

2.5. Definition 5

Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings and Q be a non empty set. Let $f: R \rightarrow R'$ be any function and A be a (Q,L)-fuzzy subsemiring in R , V be a (Q,L)-fuzzy subsemiring in $f(R) = R'$, defined by $V(y, q) = \sup_{x \in f^{-1}(y)} A(x, q)$, for

all x in R and y in R^1 and q in Q . Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

2.6. Definition 6

Let A be a (Q,L) -fuzzy subset in a set S , the strongest (Q,L) -fuzzy relation on S , that is a (Q,L) -fuzzy relation V with respect to A given by $V((x,y),q) = A(x,q) \wedge A(y,q)$, for all x and y in S and q in Q .

2.7. Definition 7

A (Q,L) -fuzzy subset A of a set X is said to be normalized if there exists an element x in X such that $A(x,q)=1$.

2.8. Definition 8

Let A be an (Q,L) -fuzzy subsemiring of a semiring $(R, +, \cdot)$ and a in R . Then the pseudo (Q,L) -fuzzy coset $(aA)^p$ is defined by $((aA)^p)(x,q) = p(a)A(x,q)$, for every x in R and for some p in P and q in Q .

2.9. Definition 9

Let A be a (Q,L) -fuzzy subset of X . For α in L , a Q -level subset of A is the set $A_\alpha = \{x \in X : A(x,q) \geq \alpha\}$.

3. Properties of (Q,L)-Fuzzy Normal Subsemiring of a Semiring

3.1. Theorem 1

Let $(R, +, \cdot)$ be a semiring and Q be a non-empty set. If A and B are two (Q,L) -fuzzy normal subsemirings of R , then their intersection $A \cap B$ is an (Q,L) -fuzzy normal subsemiring of R .

Proof: Let x and $y \in R$. Let $A = \{(x,q), A(x,q) \mid x \in R \text{ and } q \in Q\}$ and $B = \{(x,q), B(x,q) \mid x \in R \text{ and } q \in Q\}$ be (Q,L) -fuzzy normal subsemirings of a semiring R . Let $C = A \cap B$ and $C = \{(x,q), C(x,q) \mid x \in R \text{ and } q \in Q\}$. Then, Clearly C is an (Q,L) -fuzzy subsemiring of a semiring R , since A and B are two (Q,L) -fuzzy subsemirings of a semiring R . And (i) $C(x+y,q) = A(x+y,q) \wedge B(x+y,q) = A(y+x,q) \wedge B(y+x,q) = C(y+x,q)$, for all x and y in R and q in Q . Therefore, $C(x+y,q) = C(y+x,q)$, for all x and y in R and q in Q . (ii) $C(xy,q) = A(xy,q) \wedge B(xy,q) = A(yx,q) \wedge B(yx,q) = C(yx,q)$, for all x and y in R and q in Q . Therefore, $C(xy,q) = C(yx,q)$, for all x and y in R and q in Q . Hence $A \cap B$ is an (Q,L) -fuzzy normal subsemiring of a semiring R .

3.2. Theorem 2

Let R be a semiring and Q be a non-empty set. The intersection of a family of (Q,L) -fuzzy normal subsemirings of R is an (Q,L) -fuzzy normal subsemiring of R .

Proof: Let $\{A_i\}_{i \in I}$ be a family of (Q,L) -fuzzy normal subsemirings of a semiring R and let $A = \bigcap_{i \in I} A_i$. Then for x and y in R and q in Q . Clearly the intersection of a family of (Q,L) -fuzzy subsemirings of a semiring R is an (Q,L) -fuzzy subsemiring of a semiring R . (i) $A(x+y,q) = \inf_{x \in f^{-1}(y)} A_i(x+y,q) =$

$\inf_{x \in f^{-1}(y)} A_i(y+x,q) = A(y+x,q)$. Therefore, $A(x+y,q) = A(y+x,q)$,

for all x and y in R and q in Q . (ii) $A(xy,q) = \inf_{x \in f^{-1}(y)} A_i(xy,q) =$

$\inf_{x \in f^{-1}(y)} A_i(yx,q) = A(yx,q)$. Therefore, $A(xy,q) = A(yx,q)$, for all

x and y in R and q in Q . Hence the intersection of a family of (Q,L) -fuzzy normal subsemirings of a semiring R is an (Q,L) -fuzzy normal subsemiring of a semiring R .

3.3. Theorem 3

Let A and B be (Q,L) -fuzzy subsemiring of the semirings G and H , respectively. If A and B are (Q,L) -fuzzy normal subsemirings, then $A \times B$ is an (Q,L) -fuzzy normal subsemiring of $G \times H$.

Proof: Let A and B be (Q,L) -fuzzy normal subsemirings of the semirings G and H respectively. Clearly $A \times B$ is an (Q,L) -fuzzy subsemiring of $G \times H$. Let x_1 and x_2 be in G , y_1 and y_2 be in H and q in Q . Then (x_1, y_1) and (x_2, y_2) are in $G \times H$. Now, $A \times B[(x_1, y_1) + (x_2, y_2), q] = A \times B((x_1 + x_2, y_1 + y_2), q) = A(x_1 + x_2, q) \wedge B(y_1 + y_2, q) = A(x_2 + x_1, q) \wedge B(y_2 + y_1, q) = A \times B((x_2 + x_1, y_2 + y_1), q) = A \times B[(x_2, y_2) + (x_1, y_1), q]$. Therefore, $A \times B[(x_1, y_1) + (x_2, y_2), q] = A \times B[(x_2, y_2) + (x_1, y_1), q]$. And, $A \times B[(x_1, y_1)(x_2, y_2), q] = A \times B((x_1 x_2, y_1 y_2), q) = A(x_1 x_2, q) \wedge B(y_1 y_2, q) = A(x_2 x_1, q) \wedge B(y_2 y_1, q) = A \times B((x_2 x_1, y_2 y_1), q) = A \times B[(x_2, y_2)(x_1, y_1), q]$. Therefore, $A \times B[(x_1, y_1)(x_2, y_2), q] = A \times B[(x_2, y_2)(x_1, y_1), q]$. Hence $A \times B$ is an (Q,L) -fuzzy normal subsemiring of $G \times H$.

3.4. Theorem 4

Let A be a fuzzy subset in a semiring R and V be the strongest (Q,L) -fuzzy relation on R . Then A is an (Q,L) -fuzzy normal subsemiring of R if and only if V is an (Q,L) -fuzzy normal subsemiring of $R \times R$.

Proof: Suppose that A is a (Q,L) -fuzzy normal subsemiring of R . Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$ and q in Q . Clearly V is a (Q,L) -fuzzy subsemiring of $R \times R$. We have, $V(x+y,q) = V[(x_1, x_2) + (y_1, y_2), q] = V((x_1 + y_1, x_2 + y_2), q) = A((x_1 + y_1), q) \wedge A((x_2 + y_2), q) = A((y_1 + x_1), q) \wedge A((y_2 + x_2), q) = V((y_1 + x_1, y_2 + x_2), q) = V[(y_1, y_2) + (x_1, x_2), q] = V(y+x,q)$. Therefore, $V(x+y,q) = V(y+x,q)$, for all x and y in $R \times R$ and q in Q . We have, $V(xy,q) = V[(x_1, x_2)(y_1, y_2), q] = V((x_1 y_1, x_2 y_2), q) = A((x_1 y_1), q) \wedge A((x_2 y_2), q) = A((y_1 x_1), q) \wedge A((y_2 x_2), q) = V((y_1 x_1, y_2 x_2), q) = V[(y_1, y_2)(x_1, x_2), q] = V(yx,q)$. Therefore, $V(xy,q) = V(yx,q)$, for all x and y in $R \times R$ and q in Q . This proves that V is a (Q,L) -fuzzy normal subsemiring of $R \times R$. Conversely, assume that V is a (Q,L) -fuzzy normal subsemiring of $R \times R$, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$, we have

$A(x_1 + y_1, q) \wedge A(x_2 + y_2, q) = V((x_1 + y_1, x_2 + y_2), q) = V[(x_1, x_2) + (y_1, y_2), q] = V(x+y,q) = V(y+x,q) = V[(y_1, y_2) + (x_1, x_2), q] = V((y_1 + x_1, y_2 + x_2), q) = A(y_1 + x_1, q) \wedge A(y_2 + x_2, q)$. We get, $A((x_1 + y_1), q) = A((y_1 + x_1), q)$, for all x_1 and y_1 in R and q in Q . And $A(x_1 y_1, q) \wedge A(x_2 y_2, q) = V((x_1 y_1, x_2 y_2), q) = V[(x_1, x_2)(y_1, y_2), q] = V(xy,q) = V(yx,q) = V[(y_1, y_2)(x_1, x_2), q] = V((y_1 x_1, y_2 x_2), q) = A(y_1 x_1, q) \wedge A(y_2 x_2, q)$. We get, $A((x_1 y_1), q) = A((y_1 x_1), q)$, for all x_1 and y_1 in R and q in Q . Hence A is a (Q,L) -fuzzy normal

subsemiring of R .

3.5. Theorem 5

Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set. The homomorphic image of an (Q, L) -fuzzy normal subsemiring of R is an (Q, L) -fuzzy normal subsemiring of R^1 .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings Q be a non-empty set and $f: R \rightarrow R^1$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R . Let $V = f(A)$, where A is an (Q, L) -fuzzy normal subsemiring of a semiring R . We have to prove that V is an (Q, L) -fuzzy normal subsemiring of a semiring R^1 . Now, for $f(x)$, $f(y)$ in R^1 , clearly V is an (Q, L) -fuzzy subsemiring of a semiring R^1 , since A is an (Q, L) -fuzzy subsemiring of a semiring R . Now, $V(f(x) + f(y), q) = V(f(x+y), q) \geq A(x+y, q) = A(y+x, q) \leq V(f(y+x), q) = V(f(y) + f(x), q)$, which implies that $V(f(x) + f(y), q) = V(f(y) + f(x), q)$, for all $f(x)$ and $f(y)$ in R^1 . Again, $V(f(x)f(y), q) = V(f(xy), q) \geq A(xy, q) = A(yx, q) \leq V(f(yx), q) = V(f(y)f(x), q)$, which implies that $V(f(x)f(y), q) = V(f(y)f(x), q)$, for all $f(x)$ and $f(y)$ in R^1 . Hence V is an (Q, L) -fuzzy normal subsemiring of a semiring R^1 .

3.6. Theorem 6

Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set. The homomorphic preimage of an (Q, L) -fuzzy normal subsemiring of R^1 is an (Q, L) -fuzzy normal subsemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set and $f: R \rightarrow R^1$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R . Let $V = f(A)$, where V is an (Q, L) -fuzzy normal subsemiring of a semiring R^1 . We have to prove that A is an (Q, L) -fuzzy normal subsemiring of a semiring R . Let x and y in R . Then, clearly A is an (Q, L) -fuzzy subsemiring of a semiring R , since V is an (Q, L) -fuzzy subsemiring of a semiring R^1 . Now, $A(x+y, q) = V(f(x+y), q) = V(f(x) + f(y), q) = V(f(y) + f(x), q) = V(f(y+x), q) = A(y+x, q)$, which implies that $A(x+y, q) = A(y+x, q)$, for all x and y in R and q in Q . Again, $A(xy, q) = V(f(xy), q) = V(f(x)f(y), q) = V(f(y)f(x), q) = V(f(yx), q) = A(yx, q)$, which implies that $A(xy, q) = A(yx, q)$, for all x and y in R and q in Q . Hence A is an (Q, L) -fuzzy normal subsemiring of a semiring R .

3.7. Theorem 7

Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set. The anti-homomorphic image of an (Q, L) -fuzzy normal subsemiring of R is an (Q, L) -fuzzy normal subsemiring of R^1 .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set and $f: R \rightarrow R^1$ be an anti-homomorphism. Then, $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R . Let $V = f(A)$, where A is an (Q, L) -fuzzy normal subsemiring of a semiring R . We have to prove that V is an (Q, L) -fuzzy normal subsemiring of a semiring R^1 . Now, for $f(x)$ and $f(y)$ in R^1 , clearly V is an (Q, L) -fuzzy subsemiring of a semiring R^1 , since A is an (Q, L) -fuzzy subsemiring of a semiring R . Now, $V(f(x) + f(y), q) = V(f(y+x), q) \geq A(y+x, q) = A(x+y, q) \leq V(f(x+y), q)$

$= V(f(y) + f(x), q)$, which implies that $V(f(x) + f(y), q) = V(f(y) + f(x), q)$, for all $f(x)$ and $f(y)$ in R^1 . Again, $V(f(x)f(y), q) = V(f(yx), q) \geq A(yx, q) = A(xy, q) \leq V(f(xy), q) = V(f(y)f(x), q)$, which implies that $V(f(x)f(y), q) = V(f(y)f(x), q)$, for all $f(x)$ and $f(y)$ in R^1 . Hence V is an (Q, L) -fuzzy normal subsemiring of a semiring R^1 .

3.8. Theorem 8

Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set. The anti-homomorphic preimage of an (Q, L) -fuzzy normal subsemiring of R^1 is an (Q, L) -fuzzy normal subsemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two semirings and Q be a non-empty set and $f: R \rightarrow R^1$ be an anti-homomorphism. Then, $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R . Let $V = f(A)$, where V is an (Q, L) -fuzzy normal subsemiring of a semiring R^1 . We have to prove that A is an (Q, L) -fuzzy normal subsemiring of a semiring R . Let x and y in R , then clearly A is an (Q, L) -fuzzy subsemiring of a semiring R , since V is an (Q, L) -fuzzy subsemiring of a semiring R^1 . Now, $A(x+y, q) = V(f(x+y), q) = V(f(y) + f(x), q) = V(f(x) + f(y), q) = V(f(y+x), q) = A(y+x, q)$, which implies that $A(x+y, q) = A(y+x, q)$, for all x and y in R and q in Q . Again, $A(xy, q) = V(f(xy), q) = V(f(y)f(x), q) = V(f(x)f(y), q) = V(f(yx), q) = A(yx, q)$, which implies that $A(xy, q) = A(yx, q)$, for all x and y in R and q in Q . Hence A is an (Q, L) -fuzzy normal subsemiring of a semiring R .

3.9. Theorem 9

Let A be an (Q, L) -fuzzy normal subsemiring of a semiring $(R, +, \cdot)$, then the pseudo (Q, L) -fuzzy coset $(aA)^p$ is an (Q, L) -fuzzy normal subsemiring of a semiring R , for a in R and q in Q .

Proof: Let A be an (Q, L) -fuzzy normal subsemiring of a semiring R . For every x and y in R and q in Q , we have, $((aA)^p)(x+y) = p(a)A(x+y) \geq p(a)\{A(x) \wedge A(y)\} = \{p(a)A(x) \wedge p(a)A(y)\} = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Therefore, $((aA)^p)(x+y) = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Now, $((aA)^p)(xy) = p(a)A(xy) \geq p(a)\{A(x) \wedge A(y)\} = \{p(a)A(x) \wedge p(a)A(y)\} = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Therefore, $((aA)^p)(xy) = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Hence $(aA)^p$ is an (Q, L) -fuzzy normal subsemiring of a semiring R .

3.10. Theorem 10

Let A and B be (Q, L) -fuzzy subsets of the sets R and H respectively, and let α in L . Then $(A \times B)_\alpha = A_\alpha \times B_\alpha$.

Proof: Let α in L . Let (x, y) be in $(A \times B)_\alpha$ if and only if $A \times B((x, y), q) \geq \alpha$
if and only if $\{A(x, q) \wedge B(y, q)\} \geq \alpha$
if and only if $A(x, q) \geq \alpha$ and $B(y, q) \geq \alpha$
if and only if $x \in A_\alpha$ and $y \in B_\alpha$
if and only if $(x, y) \in A_\alpha \times B_\alpha$
Therefore, $(A \times B)_\alpha = A_\alpha \times B_\alpha$.

3.11. Theorem 11

Let A be a (Q, L) -fuzzy normal subsemiring of a semiring R . If $A(x, q) < A(y, q)$, for some x and y in R and q in Q , then $A(x+y, q) = A(x, q) = A(y+x, q)$, for some x and y in R and q in Q .

Proof: It is trivial.

3.12. Theorem 12

Let A be a (Q,L)-fuzzy normal subsemiring of a semiring R . If $A(x,q) > A(y,q)$, for some x and y in R and q in Q , then $A(x+y,q)=A(y,q)=A(y+x,q)$, for some x and y in R and q in Q .

Proof: It is trivial.

3.13. Theorem 13

Let A be a (Q,L)-fuzzy normal subsemiring of a semiring R such that $\text{Im } A = \{\alpha\}$, where α in L . If $A = B \bigcup C$, where B and C are (Q,L)-fuzzy normal subsemiring of a semiring R , then either $B \subseteq C$ or $C \subseteq B$.

Proof: It is trivial.

4. In the Following Theorem is the Composition Operation of Functions

4.1. Theorem 1

Let A be an (Q, L)-fuzzy normal subsemiring of a semiring H and f is an isomorphism from a semiring R onto H . Then $A \circ f$ is an (Q,L)-fuzzy normal subsemiring of the semiring R .

Proof: Let x and y in R and A be an (Q,L)-fuzzy normal subsemiring of a semiring H . Then clearly $A \circ f$ is an (Q,L)-fuzzy subsemiring of a semiring R . Now, $(A \circ f)(x+y, q) = A(f(x+y), q) = A(f(x)+f(y), q) = A(f(y)+f(x), q) = A(f(y+x), q) = (A \circ f)(y+x, q)$, which implies that $(A \circ f)(x+y, q) = (A \circ f)(y+x, q)$, for all x and y in R and q in Q . And, $(A \circ f)(xy, q) = A(f(xy), q) = A(f(x)f(y), q) = A(f(y)f(x), q) = A(f(yx), q) = (A \circ f)(yx, q)$, which implies that $(A \circ f)(xy, q) = (A \circ f)(yx, q)$, for all x and y in R and q in Q . Hence $A \circ f$ is an (Q,L)-fuzzy normal subsemiring of a semiring R .

4.2. Theorem 2

Let A be an (Q,L)-fuzzy normal subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H . Then $A \circ f$ is an (Q,L)-fuzzy normal subsemiring of the semiring R .

Proof: Let x and y in R and A be an (Q,L)-fuzzy normal subsemiring of a semiring H . Then clearly $A \circ f$ is an (Q,L)-fuzzy subsemiring of a semiring R . Now, $(A \circ f)(x+y, q) = A(f(x+y), q) = A(f(y)+f(x), q) = A(f(x)+f(y), q) = A(f(y+x), q) = (A \circ f)(y+x, q)$, which implies that $(A \circ f)(x+y, q) = (A \circ f)(y+x, q)$, for all x and y in R and q in Q . And, $(A \circ f)(xy, q) = A(f(xy), q) = A(f(y)f(x), q) = A(f(x)f(y), q) = A(f(yx), q) = (A \circ f)(yx, q)$, which implies that $(A \circ f)(xy, q) = (A \circ f)(yx, q)$, for all x and y in R and q in Q . Hence $A \circ f$ is an (Q,L)-fuzzy

normal subsemiring of a semiring R .

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