American Journal of Applied Mathematics
2020; 8(4): 230-235
http://www.sciencepublishinggroup.com/j/ajam
doi: 10.11648/j.ajam.20200804.18
ISSN: 2330-0043 (Print); ISSN: 2330-006X (Online)

Evacuation Contraflow Problems with Not Necessarily Equal Transit Time on Anti-parallel Arcs

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To cite this article:
Phanindra Prasad Bhandari, Shree Ram Khadka. Evacuation Contraflow Problems with Not Necessarily Equal Transit Time on Anti-parallel Arcs. *American Journal of Applied Mathematics*. Vol. 8, No. 4, 2020, pp. 230-235. doi: 10.11648/j.ajam.20200804.18

Received: July 27, 2020; **Accepted**: August 10, 2020; **Published**: August 17, 2020

Abstract: An evacuation planning problem provides a plan for existing road topology that sends maximum number of evacuees from risk zone to the safe destination in minimum time period during disasters. The problems with different road network attributes have been studied, and solutions have been proposed in literature. Evacuation planning problems with network contraflow approach, reversing the direction of traffic flow on lanes, with the same transit time on anti-parallel arcs have also been extensively studied. The approach, due to its lane-direction reversal property, can be taken as a potential remedy to mitigate congestion and reduce casualties during emergencies. In this paper, we propose a mathematical optimization contraflow model for the evacuation problem with the case where there may exist different transit time on anti-parallel arcs. We also propose analytical solutions to a few variants of problems, such as maximum dynamic contraflow problem and earliest arrival contraflow problem in which arc reversal capability is allowed only once at time zero. We extend the solution to solve the problems with continuous time settings by applying the natural relation between discrete time flows and continuous time flows. The solution procedures are based on application of temporally repeated flows (TRFs) on modified network, and they solve the problems optimally in strongly polynomial time.

Keywords: Network Flow, Contraflow, TTSP Network, Evacuation Planning Problem, Disaster Management

1. Introduction

An evacuation planning problem, important notion during the response phase of disaster management, attempts to find an optimal evacuation plan with a realistic flow model where each evacuee is supposed to be evacuated in a minimal time period from a risky site (source) to a safe site (sink). An efficient evacuation plan minimizes human casualties and their property during natural and human-created disasters, and also applicable in mitigation of rush-hour traffic in the crowded urban area.

The reversibility of direction of traffic flow in one or more lanes of roadways for fixed time period is termed as contraflow. The contraflow approach reconfigures the network identifying ideal direction and reallocating available capacity for each arc to improve the flow egress time and/or improve the number of flow units from source to destination. The approach, due to its lane-direction reversal property, can be taken as a potential remedy to mitigate congestion during emergencies by increasing outbound evacuation route capacity. It significantly reduces the total evacuation time and/or increase the number of evacuees sent from risk zone to safety. Studies show that reversing one lane of a four-lane dual highway increases the evacuation road capacity by approximately 30% and reversing all the inbound lanes, it increases by 67% [1].

Despite the long history of studies of evacuation problems with contraflow approach, there is limited implementation in real emergency evacuations due to difficulty in using commonly employed methods to duplicate traffic conditions of real contraflow lane during an emergency [1]. However, they have been adapted for evacuating some major metropolitan regions threatened by disasters. It was first applied during Hurricane Floyd in the United States in 1999 with mixed, though overall positive, results [2]. Contraflow was also implemented during hurricanes Katrina and Rita in the United States in 2005. However, it was criticized as unplanned contraflow orders and as failure to use contraflow

lanes [3]. Contraflow approach is primarily important for emergency evacuations, nonetheless, its applications are not limited to these. This is commonly used for accommodating directionally imbalanced traffic associated with daily commuter in big cities as well as consequences due to religious gathering, concerts, tournaments, etc.

Maximum dynamic flow problem that sends the maximum amount of flow from the source to the sink within the given time horizon was due to Ford and Fulkerson [4]. Various applications of this problem including evacuation planning problems are considered in the literature, e.g., [5-7]. Evacuation problems that allow evacuees to be held at temporary shelters at intermediate spots have also been studied in [8-10].

The first mathematical optimization model for contraflow problem is due to Rebennack et al. [11]. They have investigated analytical solution of maximum contraflow problem with polynomial time complexity for both static and dynamic networks. Their solution idea is based on transformation of given network into one at which the existing algorithms are applicable. There is extensive study of dynamic network flow problems with continuous time setting, e.g., [12-17]. The continuous time dynamic network contraflow problems have been considered in [18] and [19]. The earliest arrival flow (EAF) problem that ask to maximize flow into the sink at each time points within the time horizon have also been considered widely in the literature, e.g., [20-25]. The EAF problem for two terminal series parallel (TTSP) network without and with contraflow approach have been studied and proposed polynomial time solutions in [26] and [27], respectively. Contraflow approach has been incorporated in network flow model to study facility location problem in [28] and the notion of abstract flow has been applied to network contraflow problems in [29]. The partial contraflow approach over the abstract network setting has been introduced in [30].

Time parameter plays a vital role in designing evacuation planning models. Therefore, it is important to be careful about its nature: discrete or continuous, adapted in the model. Optimization contraflow models developed so far are based on equal transit time settings on anti-parallel arcs and these models do not allow multiple arcs of different transit time. We call the two directed arcs 'anti-parallel' if they join the same pair of nodes, but in opposite directions. It is crucial, in case of uneven road architecture, for example, to take contraflow models on networks with not necessarily equal transit time on anti-parallel arcs into account for preparing evacuation tasks.

In this paper, we propose mathematical optimization contraflow model with assumption where the transit time on anti-parallel arcs may have different values. Discrete as well as continuous aspect of transit time are considered in the model, whereas capacities and transit time on arc are time independent and behave symmetrically during the reversal of direction of arcs.

Remaining part of the paper is organized as follows. Mathematical formulation of the contraflow problems are given in Section 2. Section 3 contains solution of maximum dynamic contraflow problem with discrete time setting and with continuous time setting in Subsection 3.1 and 3.2, respectively, and that of earliest arrival contraflow problem in Subsection 3.3. Section 4 concludes the paper.

2. Problem Formulation

Consider an evacuation network $N = (V, E, \mathrm{c(e)}, \tau(\mathrm{e}), T)$ where V is the set of nodes v denoting the crossing of routes from dangerous place, the source s, to safer place, the sink d and E is the set of route segments, arc $e = (v, w)$ joining any two different nodes $v, w \in V$. Let $c: E \to Z^{\geq 0}$ be a capacity function denoting the upper bound for flow units to pass the arc at a time slot and $\tau: E \to R^{>0}$ be the transit time function denoting the time required for a flow unit to travel the arc. Moreover, we assume that the network N with not necessarily equal transit time on anti-parallel arcs. However, the transit time behave symmetrically during contraflow process. The total evacuation time period is denoted by T and we call it time horizon. An evacuation network has been depicted in Figure 1.

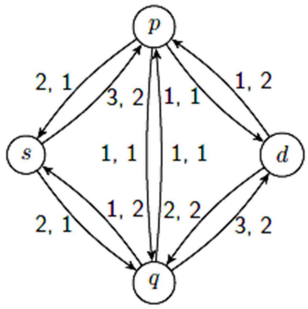

Figure 1. *An evacuation network N with source node s and sink node d. First and second numbers next to each arc are capacities and transit time, respectively.*

The flow of evacuees, say f, on the network N defined as $f: E \times [0, T) \to R^{\geq 0}$ satisfies the following conditions: Flow units travelling along arc e cannot exceed the arc capacity $c(e)$ for any time within given time horizon T. That is,

$$0 \leq f(e, \theta) \leq \mathrm{c(e)}\ \forall\, e \in E \text{ and } \forall\, \theta \in [0, T). \qquad (1)$$

Flow units that enter into node v for all $v \in V \backslash \{s, d\}$ must exit from it within given time horizon. That is,

$$\sum_{e \in \delta^-(v)} \int_0^{T-\tau(e)} f(e, \theta) d\theta = \sum_{e \in \delta^+(v)} \int_0^{T} f(e, \theta) d\theta\,,$$

$$\forall\, v \in V \backslash \{s, d\}. \qquad (2)$$

where $\delta^-(v)$ and $\delta^+(v)$ denote for the set of arcs entering into the node v and leaving from it, respectively.

A dynamic $s - d$ -flow on N that satisfies capacity constraints (1) is a feasible $s - d$ -flow. For a dynamic network $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c(e)}, \tau(\mathrm{e}), s, d, T)$, the objective of maximum dynamic $s - d$ -contraflow problem with continuous time setting is to maximize a net feasible

continuous dynamic flow, say f_c, from s to d within the given time horizon T, if the direction of the arcs on N can be reversed. The net flow f_c is given by

$$\boldsymbol{f_c} \coloneqq \sum_{e\in\delta^+(s)} \int_0^T f(e,\theta)d\theta - \sum_{e\in\delta^-(s)} \int_0^T f(e,\theta)d\theta$$

$$= \sum_{e\in\delta^-(d)} \int_{\theta=0}^{T} f(e,\theta)d\theta - \sum_{e\in+(d)} \int_{\theta=0}^{T} f(e,\theta)d\theta \quad (3)$$

If one wishes to send packets of flow units at discrete time points into the arcs instead of sending flow at continuous flow rates, the time horizon T is to be discretized into the time steps $\{0,1,\dots,T\}$. In discrete time flow model, the flow units sent into an arc $e=(v,w)$ at time θ totally reach the target node w at time $\theta+\tau(\mathrm{e})$, for $\tau: E \to Z^{>0}$. In discrete time setting, the flow function f defined as $f: E\times\{0,1,\dots,T\} \to Z^{\geq 0}$ satisfies the capacity constraint in the form:

$$0 \leq f(e,\theta) \leq \mathrm{c(e)}\ \forall\, e \in E\ \&\ \forall\, \theta \in \{0,1,\dots,T\} \quad (4)$$

and the flow conservation constraint in the form:

$$\sum_{e\in\delta^-(v)} \sum_{\theta=0}^{T-\tau(e)} f(e,\theta) = \sum_{e\in\delta^+(v)} \sum_{\theta=0}^{T} f(e,\theta) \quad (5)$$

A dynamic $s-d$-flow satisfying capacity constraints (4) is a feasible $s-d$-flow for discrete time setting. For a dynamic network N and T being discretized, the maximum dynamic $s-d$ contraflow problem maximizes the net feasible discrete dynamic flow, say $\boldsymbol{f}_d$, from s to d within the given time horizon T, if the direction of the arcs on N can be reversed. The net flow $\boldsymbol{f}_d$ is given by

$$\boldsymbol{f_d} \coloneqq \sum_{e\in\delta^+(s)} \sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in\delta^-(s)} \sum_{\theta=0}^{T} f(e,\theta)$$

$$= \sum_{e\in\delta^-(d)} \sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in\delta^+(d)} \sum_{\theta=0}^{T} f(e\,\theta). \quad (6)$$

A maximum dynamic $s-d$-contraflow problem is also known as a *maximum dynamic contraflow (MDCF) problem* for single-source-single-sink network. Obviously, the flow value before and after the time horizon T is zero and all flow units leave the network within it in both discrete and continuous cases.

For given network $N=(V,E,\mathrm{c(e)},\tau(\mathrm{e}),\mathrm{s},\mathrm{d},T)$, *earliest arrival contraflow (EACF) problem*, if the direction of the arcs on N are allowed to reverse, maximizes the feasible net flow from s to d at each time points $\theta\in[0,\mathrm{T})$.

3. Solution Discussion

Authors in [11] studied maximum dynamic contraflow problem with equal transit time on anti-parallel arcs. Their model is with discrete time setting and arc reversal capability has been allowed only once at time zero. In this section, we consider maximum dynamic contraflow problems on dynamic networks in which there may be unequal transit time on anti-parallel arcs with discrete as well as continuous time settings in which arc reversal capability is allowed only once at time zero. We also discuss the solution procedure for earliest arrival contraflow problem with these settings for two terminal series parallel (TTSP) network.

3.1. Maximum Dynamic Contraflow with Discrete Time Setting

Consider the discrete time maximum dynamic contraflow (DT-MDCF) problem on network $\mathrm{N}=(\mathrm{V},\mathrm{E},\mathrm{c(e)},\tau(\mathrm{e}),s,d,T)$ with not necessarily equal transit time on anti-parallel arcs and with integer inputs. In the following, we propose a solution procedure to this problem when the arc reversibility is allowed only once at time zero.

Algorithm 1: Algorithm DT-MDCF

1. Given network $\mathrm{N}=(\mathrm{V},\mathrm{E},\mathrm{c(e)},\tau(\mathrm{e}),s,d,T)$ with not necessarily equal transit time on anti-parallel arcs and with integer inputs.
2. Transform network N into $N^+=(V^+,E^+,\mathrm{c(e^+)},\tau(\mathrm{e^+}),s,d,T)$ where $V^+=V\cup V'$ such that $V'=\{\overrightarrow{vw}:(v,w)\in E\}$ and $E^+=\{(v,\overrightarrow{vw}),(\overrightarrow{vw},w):v,w\in V,\overrightarrow{vw}\in V'\ \&\ (v,w)\in E\}$ with capacities $c(v,\overrightarrow{vw})=c(v,w)$, $c(\overrightarrow{vw},w)=\infty$ and transit time $\tau(v,\overrightarrow{vw})=\tau(v,w)$, $\tau(\overrightarrow{vw},w)=0$.
3. Transform network N^+ into its auxiliary network $\widetilde{N^+}$ as in [11].
4. Compute the discrete dynamic, temporally repeated flow on network $\widetilde{\mathrm{N}}$ for time horizon T.
5. Perform the flow decomposition into chain and cycle flows of the maximum flow obtained from step-4 and remove all cycle flows.
6. Arc $\bar{\mathrm{e}}\in\mathrm{E}$ is reversed if and only if the flow along arc $\vec{\mathrm{e}}\in\mathrm{E}$ is greater than $\mathrm{c(e)}$ or if there is non-negative flow along arc $\mathrm{e}\notin\mathrm{E}$.
7. Get discrete time maximum dynamic contraflow on N for time horizon T.

The procedure (cf. Algorithm 1) is based on the network transformation. In contrast to the case of equal transit time on the arcs, addition of capacities of anti-parallel arcs, while constructing the auxiliary network, is no longer possible in the case of unequal transit time. We propose an alternative, a more general, way of constructing the auxiliary network for the latter case. Network N with unequal transit time and capacities on the arcs is transformed into new network by introducing an artificial node for each arc that separates it into two different arcs. Each arc on N is split into two arcs: real arc and artificial arc, in the auxiliary network N^+. Artificial arcs have infinite capacities and zero transit time, whereas the real arcs have the original arc capacities and transit time. We denote artificial node that splits arc $(v,w)\in E$ by $\overrightarrow{vw}$, see Figure 2. Then the solution procedures that solves the MDCF problem with equal transit time on anti-parallel arcs, given in [11], is applicable on transformed network N^+. Their algorithm is based on reduction of given network N into its auxiliary network $\widetilde{N}$. We denote $\vec{\mathrm{e}}\in E$ for an arc (v,w) in which the flow unit is sent from the node v to the node w and $\bar{\mathrm{e}}\in E$ for an arc (w,v) in which the flow unit is sent from the node w to the node v for all $\mathrm{v},\mathrm{w}\in\mathrm{V}$. Replacement of $\vec{\mathrm{e}}$ by $\bar{\mathrm{e}}$ is known as the arc reversal. For network N with equal transit time on anti-parallel arcs, its auxiliary network is $\widetilde{\mathrm{N}}=(\mathrm{V},\tilde{E},\mathrm{c}(\tilde{e}),\tau(\tilde{e}),\mathrm{s},\mathrm{d},\mathrm{T})$ where

$\widetilde{\mathrm{E}} = \{\tilde{\mathrm{e}} = \vec{\mathrm{e}} \text{ or } \overleftarrow{\mathrm{e}}\}$ with $\mathrm{c}(\tilde{e}) = c(\vec{\mathrm{e}}) + \mathrm{c}(\overleftarrow{\mathrm{e}})$, and $\tau(\tilde{e}) = \tau(e)$ if $\mathrm{e} \in \mathrm{E}$ and $\tau(\tilde{e}) = \tau(\overleftarrow{\mathrm{e}})$ otherwise. Any known technique can be applied to solve the maximum dynamic flow problem on $\widetilde{\mathrm{N}}$, which ultimately, solves the original maximum dynamic flow problem with contraflow approach.

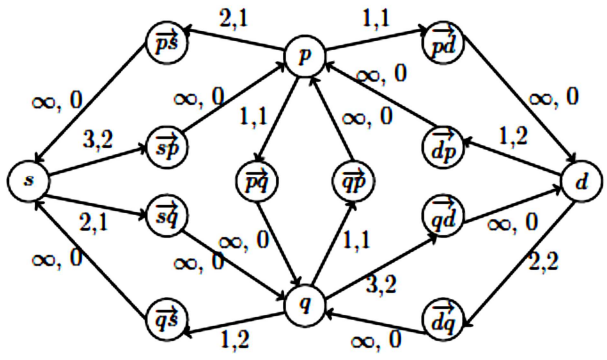

Figure 2. *Transformed network N^+ of the network N depicted in Figure 1.*

An optimal solution to minimum cost circulation (MCC) problem can be turned into a maximal discrete dynamic flow using the notion of temporally repeated flows (TRFs), [4]. The TRF is obtained by sending as much flow as possible along each path from source to sink at time zero and continue on them as long as there is enough time left within time horizon T to arrive at the sink d. To set the MCC problem on N, interpret the transit time $\tau(e)$ as cost coefficients for each arc $e \in E$. Also, it is desired to maximize dynamic flow on N so that the cost of circulation is minimum and that all flow arrives at the sink within time T. Thus, it is required to model time horizon T in the solution techniques of MCC problem to be allowed to transfer the results of this problem to maximum dynamic flow problem. For this, an additional arc (d, s) from the sink d to the source s with sufficient capacity and transit time $-(T+1)$, is inserted on original network N.

We assign infinite capacity to the artificial arc in the transformed network N^+. However, flow along it is regulated by its adjacent real arc with capacity equal to that of the arc before it was split. Also, being zero transit time on this arc, the optimal MDCF computed on N^+ is not different with the optimal MDCF on original network N. The MDCF computation idea of [11] is applicable on N^+ that gives optimal solution and runs in strongly polynomial time. The order of time complexity of the algorithm is not affected by the increment of the size of the transformed network. Moreover, network transformation process in step 2 can be accomplished in linear time. Thus, for network $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c}(\mathrm{e}), \tau(\mathrm{e}), s, d, T)$ with not necessarily equal transit time on anti-parallel arcs and with integer inputs, Algorithm 1 solves discrete time maximum dynamic contraflow problem optimally in polynomial time, if the direction of the arcs are allowed to reverse only once at time zero.

Applying Algorithm 1 for the evacuation network N depicted in Figure 1. for time horizon $T = 10$, the maximum dynamic contraflow with discrete time setting is of value 52, whereas the maximum dynamic flow without contraflow is of value 30.

3.2. Maximum Dynamic Contraflow with Continuous Time Setting

Notion of natural transformation is helpful to generalize a discrete dynamic flow on a network $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c}(\mathrm{e}), \tau(\mathrm{e}), s, d, T)$ as a continuous dynamic flow [15]. The notion states that the amount of flow that arrives at node w through arc $e = (v, w) \in E$ at time step θ in discrete time setting is equal to the amount of flow arriving at w through arc $e = (v, w)$ during unit interval of time $[\theta, \theta + 1)$, i.e., $f_d(e, \theta) := f_c(e, [\theta, \theta + 1))$ for all $\theta \in \{0, 1, \ldots, T-1\}$. Here, capacity constraints for continuous dynamic flow f_c are obviously obeyed, since $f_d(e, \theta) \leq \mathrm{c}(\mathrm{e})$ implies $f_c(e, [\theta, \theta + 1)) \leq \mathrm{c}(\mathrm{e})$ for all time points in the interval $[\theta, \theta + 1)$. This transformation is a bidirectional, if T and all transit time are integral [25]. We propose a solution procedure based on this notion for continuous time maximum dynamic contraflow (CT-MDCF) problem modeled on network N with not necessarily equal transit time on anti-parallel arcs and with integer inputs, if the direction of the arcs are allowed to reverse only once at time zero. The procedure has been summarized in Algorithm 2.

Algorithm 2: Algorithm CT-MDCF

1. Given network $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c}(\mathrm{e}), \tau(\mathrm{e}), s, d, T)$ with not necessarily equal transit time on anti-parallel arcs and with integer inputs.
2. Transform network N into N^+ as in Algorithm 1.
3. Transform network N^+ into its auxiliary network $\widetilde{N^+}$ as in [11].
4. Compute the discrete dynamic, temporally repeated flow on network $\widetilde{\mathrm{N}}$ for time horizon $\mathrm{T} - 1$.
5. Transform the discrete dynamic flow into continuous dynamic flow using the natural transformation $\mathrm{f_d}(\mathrm{e}, \theta) := \mathrm{f_c}(\mathrm{e}, [\theta, \theta + 1))$ for all $\theta \in \{0, 1, \cdots, \mathrm{T} - 1\}$.
6. Perform the flow decomposition into chain and cycle flows of the maximum flow obtained from step 4 and remove all cycle flows.
7. Arc $\overleftarrow{\mathrm{e}} \in \mathrm{E}$ is reversed if and only if the flow along arc $\vec{\mathrm{e}} \in \mathrm{E}$ is greater than $\mathrm{c}(\mathrm{e})$ or if there is non-negative flow along arc $\mathrm{e} \notin \mathrm{E}$.
8. Get continuous time maximum dynamic contraflow on N for time horizon T.

Only the step 5 is additional effort in Algorithm 2 while comparing it with Algorithm 1. Since the transformation of $(T-1)$-horizon discrete time maximum dynamic flow yields a T-horizon continuous time maximum dynamic flow [15], the time complexity of finding a temporally repeated continuous flow is same to that in discrete case. Thus, for network $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c}(\mathrm{e}), \tau(\mathrm{e}), s, d, T)$ with not necessarily equal transit time on anti-parallel arcs and with integer inputs, Algorithm 2 solves continuous time maximum dynamic contraflow problem optimally in polynomial time, if the direction of the arcs are allowed to reverse only once at time zero.

Applying Algorithm 2 for the evacuation network N given in Figure 1. for time horizon $T = 10$, the maximum dynamic contraflow with continuous time setting is of value 45, whereas the maximum dynamic flow without contraflow is of value 26.

3.3. Earliest Arrival Contraflow on TTSP Network

Let $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c(e)}, \tau(\mathrm{e}), s, d, T)$ be a network with not necessarily equal transit time on anti-parallel arcs and with integer inputs. The discrete time earliest arrival contraflow (DT-EACF) problem, if the direction of the arcs are allowed to reverse, maximizes the feasible net flow from s to d at each time steps $\theta \in \{0, 1, \ldots, \mathrm{T}\}$. In the following we propose an efficient solution procedure for DT-EACF problem modeled on a special class of network known as two terminal series-parallel (TTSP) network N. A TTSP network N is a network with a single source s and a single sink d which has a single arc (s, d) together with source s and sink d or is obtained from two series parallel networks N_1 and N_2 by one of the following two operations:

(i) Parallel Composition: Merge source nodes s_1 of N_1 and s_2 of N_2 to form the source s of N and merge sink nodes d_1 of N_1 and d_2 of N_2 to form the sink d of N.
(ii) Series Composition: Merge the sink node d_1 of N_1 with the source node s_2 of N_2.

We apply minimum cost flow algorithm of [26] to solve the DT-EACF problem for TTSP network. The algorithm solves maximum dynamic flow problem using a temporally repeated flow over the time horizon T. In fact, this maximum dynamic flow has the earliest arrival property [26]. We claim that two terminal series parallel network N, after transforming into network N^+, remains two terminal series parallel network. That is, the following algorithm solves the earliest arrival contraflow problem with discrete time setting, if the direction of the arcs are allowed to reverse only once at time zero. Moreover, the time complexity of Algorithm 3 is dominated by the polynomial time complexity of algorithm in [26].

Algorithm 3: Algorithm DT-EACF

1. Given TTSP network $\mathrm{N} = (\mathrm{V}, \mathrm{E}, \mathrm{c(e)}, \tau(\mathrm{e}), s, d, T)$ with not necessarily equal transit time on anti-parallel arcs and with integer inputs.
2. Transform network N into N^+ as in Algorithm 1.
3. Transform network N^+ into its auxiliary network $\widetilde{N^+}$ as in [11].
4. Solve earliest arrival flow problem on $\widetilde{N^+}$ by using the algorithm in [26].
5. Arc $\tilde{e}$ is reversed if and only if the flow along arc $\vec{e}$ is greater than $\mathrm{c(e)}$ or if there is non-negative flow along arc $e \notin E$.
6. Get discrete time earliest arrival contraflow on N for time horizon T.

Together with the notion of natural transformation [15], Algorithm 3 solves continuous time earliest arrival contraflow problem on N when the arc reversal capability is allowed only once at time zero. Also, the solution to the problem is optimal and can be found in strongly polynomial time.

4. Conclusion

The importance and applicability of the idea of network contraflow especially in evacuation planning problem has been increasing due to its lane direction reversal capability. In the case of unequal to-and-fro transit time of oppositely directed lanes of a road, it is crucial to take network contraflow models with not necessarily equal transit time on anti-parallel arcs into account for preparing evacuation tasks. In this paper, we gave a network flow based evacuation model, an optimization model, capturing this situation. We studied the maximum dynamic contraflow problem and earliest arrival contraflow problem, and proposed strongly polynomial time algorithms as solution procedures, if the arcs are flipped only once at time zero, for discrete as well as continuous time setting. We also discussed about optimality and efficiency of the proposed algorithms, and present numerical examples that compares the optimal flow value with and without contraflow approach.

Applying the proposed model for real data-set and examine the performance of solution technique would enhance the scope of this work. Studying contraflow evacuation planning problems with other variants such as abstract contraflow, partial contraflow, lexicographically dynamic contraflows, etc., within the proposed model framework, are also crucial. Moreover, consideration of contraflow evacuation planning problems addressing the situation where multiple parallel lanes with different transit time exist on road topology would be further research work.

Acknowledgements

First author would like to thank University Grants Commission, Nepal for providing partial financial support (PhD Fellowship Award 2016) to conduct this research. Authors are grateful to anonymous reviewers whose comments significantly improved the quality of paper.

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