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Review Article

Algorithmic Thinking and Mathematical Learning Difficulties Classification

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Abstract: Learning difficulties research within the frame of dyscalculia has proceeded so far, nevertheless, they seem to fail in providing an overall conceptual map of the deficit. This paper objective is to propose a new classification in reference to dyscalculia features noticed at various ages. Although, there are several approaches on dyscalculia features, algorithmic thinking ability deficits are not taken into consideration. Authors focus on problem solving and algorithmic thinking difficulties within the frame of dyscalculia.

Keywords: Algorithmic Thinking, Dyscalculia, Learning Memory, Reasoning, Problem Solving, Spatial Perception, Visual Perception

1. Introduction

During last decades the challenge to describe difficulties in mathematical perception has risen. This paper deals with dyscalculia conceptual mapping and focuses on algorithmic thinking difficulties. Dyscalculia could be noticed on early age while difficulties in quantities direct estimation, in counting, in arithmetical symbols recognition, and in spatial concept awareness are rising. Dyscalculia could be identified due to difficulties noted in visual perception, in spatial number organization, in basic mathematical functions and in mathematical problem solving in general. People dealing with dyscalculia present difficulties in mathematical induction logic, in Euclidean and Non-Euclidean Geometry concepts perception, in Algebra and in Calculus. Moreover, shortage while dealing with algorithmic tasks are noticed.

2. Related Work

In reference to dyscalculia's classification several sub-typing models are proposed. Shalev et al, the sub-typing approach is mainly focused on potential coexisting reading disorders, number processing, and calculation difficulties and in the neuropsychological basis of dyscalculia. However, within this sub-typing, the specific features of children with dyscalculia were not classified [1]. According to Geary's grouping relative to preschoolers at risk of math difficulties, dyscalculia features are focused on three basic clusters. Dyscalculia is inferred initially as a persistent difficulty in learning or understanding number concepts, i.e. knowing basic number names and discriminating the larger/smaller number among others. Additionally, it is inferred to difficulties of perceiving counting principles, like finding the cardinality of a set and in counting numbers according to the basic number sequence. Finally, difficulties in arithmetic are referred, like troubles while solving simple arithmetic problems such as finger counting or recalling an answer. Authors also notice that dyscalculia should be evaluated overall, but this classification has not been analyzed in their work [2].

Karagiannakis et al used a multi-deficit neuro-cognitive approach and proposed a classification model describing four basic cognitive domains for mathematical learning difficulties, within which precise deficits may be included. Within the cognitive psychology frame Karagiannakis' and co-authors' classification deficits of core number; memory (retrieval and processing) inefficiency; reasoning shortage and visual-spatial perception difficulties are included [3].

3. Dyscalculia Definition

Researchers note that people dealing with learning difficulties present also difficulties in mathematics [4]. Dyscalculia is a disorder linked to numerical skills and refers to acquiring abilities in mathematics and it is encompassed to the "Specific Learning Disabilities” category [5]. Children facing dyscalculia deal with difficulties in perceiving mathematics in addition to all interrelated aspects [6]. Conferring R. Cohn’s definition about the dyscalculia term, specified in 1961, although students’ level of intelligence is not low, learning difficulties in mathematics are identified [7]. People present mathematical disability when their performance on calculation consistent assessments or on arithmetical reasoning tests is relatively low, given their age, education and cognitive aptitude. Low performance due to a cerebral trauma is called “acquired dyscalculia”. Mathematical learning difficulties with equally features nevertheless devoid of the evidence of a cerebral trauma are denoted as “Developmental Dyscalculia” [8]. According to the definition that Kosc (1974) proposed as “Developmental Dyscalculia”, dyscalculia is a structural disorder of mathematical abilities which has its origins in a genetic or congenital disorder. A disorder of those parts of the brain that are the direct anatomical physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions [9]. Summing up, the results from valid research and taking into account potential typical error and variation, the prevalence of people with dyscalculia is 5% - 6%, in reference to school-aged children [10]. In contrast with additional learning disabilities, dyscalculia affects both sexes equally [5].

4. Mathematical Learning Difficulties Classification

Dyscalculia features, irrespectively of the name used in each researcher approach in order to describe dyscalculia and cluster its features, according to authors’ perspective are integrated within the following categories: Visual Perception Difficulties, Spatial and Geometrical Concepts Perception Deficits, Time Perception Deficits, Calculation Deficits, Calculus Perception Difficulties and Algorithmic Thinking Difficulties. Dyscalculia features are classified and briefly described while a detailed analysis is to follow.

Table 1. *A classification model for mathematical learning difficulties, proposing 6 subtypes and typical mathematical perception deficits encountered.*

Visual Perception Difficulties
Number sense perception difficulties [11-13]
Subitizing ability deficits [14, 15]
Practical, pictorial, and symbolic number mode coding and decoding difficulties [11]
Enumeration difficulties [17, 18]
Quantitative numbers sequence perception [36, 19]
Digits permutations and omissions [39]
Digits replacements and reversals [40]
Patterns recognition difficulties [45]
Memory (retrieval - processing) inefficiency [1]
Pre-mathematical Skills [17]

Spatial and Geometrical Perception Deficits
Calculation symbols confusions [48]
Visual mathematical symbols and numbers confusions [46, 47]
Spatial number arrangement deficits [51]
Spatial orientation and direction concepts difficulties [9]
Cardinal directions confusions [55]
Clockwise motion perception [56]
Memory deficits [46]
Geometrical thinking deficits [57]
Difficulties in recognizing shapes and perceiving their properties [58]
Difficulties in definitions, theorems and proofs perception [59]
Mathematical induction and deduction logic perception deficits [62, 63]
Non-Euclidean geometry concepts difficulties [63]

Time Perception Deficits
Time perception and management deficits [12]
Schedule organization failure [13]
Deadlines failure [20]
Failure in recalling the sequence of past events [21]
Time concepts confusions [66]
Prior and after notion confusions [66]

Calculation Deficits
Pre-mathematical concepts perception deficits [73]
Memory deficits i.e. recalling simple arithmetic operations [10]
Mathematical symbols recognition deficiency [4]
Inadequate memorization of arithmetic tables [4]
Acalculia: Difficulties in performing simple mathematical tasks [68]
Noetic calculation estimation failures [71]
Failure in applying mathematical acts [75-77]
Integer, real and complex numbers perception difficulties [22]

Calculus Perception Difficulties
Math application and solving algebraic equations failure [14]
Variables and multivariable functions perception deficits [15]
Limit concept and infinite notion perception difficulties [15]
Differential calculus perception difficulties [16]
Integral calculus perception deficits [24]

Algorithmic Thinking Deficits
Algorithm perception, design and description difficulties [52, 82, 84]
Algorithm application procedure deficits [52, 82, 84]
Algorithm best solution evaluation difficulties [52]
Short and long term memory deficits [85, 52]
Memorization of algorithmic procedures difficulties [85]
Strategic thinking games difficulties [84]
Cognitive abilities deficits [80]

4.1. Visual Perception Difficulties

Some problems and difficulties in mathematics probably are due to the weakness of adequate visual perception of numbers as much as in the correct and absolute approach to number concepts [17]. This implies difficulties in conceptual ideas underlying all numerical concepts and especially difficulties in the conception of number notion [18]. Number concept is referred to the perception of the number as an absolute or ordinal number [19]. The perception of number three varies to the absolute number 3, that is referred to 3 items or to the ordinal number of 3 i.e. the sequence of numbers and specifically that of an object which is third in the row [20].

An additional problem related to number concept is difficulties in connecting number concept to quantitative and symbolic form [21]. Specifically, the quantitative form is related to the recognition, matching and direct estimation of quantities. The term subitizing is known as the status of finding the cardinality of set enclosing objects, which is an inherent property and evolves significantly over time. Specifically, infants up to three weeks are able to recognize automatically the cardinality of sets containing from one to three elements. This ability improves gradually in adults and reaches the instant recognition of cardinality of a set containing up to five components [22]. Furthermore, number concept difficulties, are related to difficulties in perceiving conservation concept i.e. the perception that the quantitative value of an object or a number of elements making up a set, remains unchanged irrespectively of the spatial organization [23]. Although those difficulties concern visual perception, simultaneously they are also related to the inability of spatial management and perception as well as Geometry concepts [24]. Respectively the number symbolic form term is referred to the inadequate link of number concept with words and symbols [25]. This is related to the specificity in numbers notion, to difficulties in coding and decoding number notion and is moreover related to shortness of proper conception of numbers in relevance to the way they are encoded and virtualized [26]. Thus, peculiarities while using mathematical vocabulary, as well as troubles in writing or reading numbers correctly are related to shortness of conceiving verbal code [27]. Especially number representation schemes are the practical, the pictorial, and the symbolic mode. The practical mode is related to the representation or description of a number with the use of objects such as sticks, rings in Abacus and fingers. The pictorial mode is connected to numbers representation with the use of shapes, images, drawings, lines and graphs [28]. The symbolic number representation mode concerns the display of a number with symbolic-schematic forms, such as Latin mode of representation [28]. Coding and decoding difficulties lead to troubles in perceiving both practical and the pictorial form of numbers [29]. This difficulty also affects the ability to represent a quantitative amount of objects into a symbolic mode [30]. For instance, people dealing with dyscalculia fail to conceive and pair a quantity or an object collection into symbolic form and vice versa [11]. Therefore, difficulties are noticed while recognizing quantities like finding the cardinality of a set or estimating correctly a number of items. This particularity is connected to the mental development of number sense [31].

The aforementioned difficulties are associated with pre-mathematical skills like the perception of quantities, classification of similar objects into groups, numbering objects, the ability to classify objects or distinguishing numbers as quantity and symbols [32]. Referring to troubles in enumeration, difficulties while writing numbers, in oral numbering, in verbal enumeration and in finding the cardinality of a set with or without counting are observed [33]. This is equivalent to difficulties on counting and matching number names to their value according to the number of objects related and linked to difficulties in recognizing numbers sequence [34]. In particular while mentioning troubles on quantities estimation, this includes difficulties while counting quantities as well as the lack of correct or/and automated estimation concerning the relation between several quantities [35]. Thus, those features are also displayed as difficulties in conceiving numbering system as well as in positioning numbers in numbers' axle [35].

Additionally, difficulties while numbering in ascending or descending order especially using a random starting point as well as while counting down numbers are also included [36]. Equivalent troubles are noticed while putting quantities in a row due to troubles in instant recognition and estimation of quantities which are related to subitizing ability [37]. An additional issue related to visual perception difficulties is the deficit of visually perceiving the conservation of the order of objects i.e. shortness of controlling and conceiving the absence of an element in a sequence of symbols or from a particular well-defined objects sequence [38].

Specifically while writing or reading numbers difficulties such as digits permutations or omissions are noticed e.g. the number 2305 is displayed as 235 etc [39]. Also, digits replacements and reversals are usually noted e.g. the number 683 is displayed as 663 and the number 235 is perceived as 253 respectively [40]. Furthermore, mistakes while mounting digits in columns of hundreds, tens and units such as writing a number digit in a wrong column are noticed [41]. This is due to problems perceiving number concept in addition to visual perception. Furthermore, there are failures while analyzing, breaking down and splitting a number into units, tens, thousands etc. [41]. Finally, the use of nonexistent digits is noticed, for instance, the number 145 is written as 1450 as well as permutations of letters while spelling words [42]. Those particularities could also be linked to algorithmic thinking and spatial perception difficulties. The correlation of these difficulties to algorithmic thinking is consistent due to the required awareness of number sequence and the ability to find, recognizing and recalling patterns due to memory deficits [43]. Additionally in reference to spatial perception, failure of perception of number digits is related to troubles in recalling or both imprinting a number's image [44]. Difficulties in measuring or assessing the number represented in a set of items and even more recognizing patterns and motifs are linked with the spatial perception difficulties as well [45].

4.2. Spatial and Geometrical Concepts Perception Deficits

4.2.1. Spatial Perception Difficulties

In the rank of spatial perception difficulties, troubles in the visual discrimination of symbols are noticed. Dyscalculic people are dealing with problems while copying characters and distinguishing or recognizing the distinction between two similar but in fact differing number symbols or alphabet characters [46]. Hence, they fail while copying the above-mentioned shapes and figures [46]. Especially confusions of symbolic representation of visually similar numbers, for instance, confusions among number "6" and number "9" are common [47]. In addition difficulties in distinguishing the difference between alphabet letters like for instance the letter "d" to "g" or the letter "p" to "q" are included [47]. Even more difficulties in distinguishing visually similar mathematical symbols or other characters e.g. "e" from "3" or "p" from "9" or even "q" from "9"[47]. These difficulties are also related to visual perception and memory dysfunctions. In particular trouble in information storage and therefore difficulties in recalling information obtained visually are noticed [46].

More specifically difficulty in the perception of symbolic form that is linked to faulty number sense perception is observed. In particular frequent confusions and difficulties in distinguishing math calculation symbols "+, -, *, :" in two dimension space as well as the distinction and the perception of the notion of mathematical symbols like "<, >, =, ≠, ≤, ≥" are noticed [48]. Those deficits are also connected to calculation disorders and concern visual perception difficulties additionally.

Incorrect reversing of number digits e.g. 082 instead of 802 or 820 could be integrated with both following dyscalculia's types: visual and spatial perception difficulties. Difficulties in understanding the positional value of the digit and furthermore troubles in grouping numbers in three digits groups' like 123.45 instead of 12.345 are noted due to inadequate spatial perception [49]. Referring to decimal numbers a common mistake noted is the placing of the decimal point in an incorrect decimal position [50].

A major topic concerning spatial learning difficulties is troubled in basic spatial arrangement i.e. classification of objects according to their quantitative value [51]. This signifies that people dealing with dyscalculia have difficulties in comparing quantities and shape sizes [25]. This leads to problems using and understanding the value of numbers depending on the position in number axis due to the inadequate perception of number digits' positional value. Therefore, they are facing a weakness in placing a number correctly in the number axis [52]. These difficulties are also related to visual perception deficits and trouble in number sense.

Difficulties in space are also related to problems on playing back or copying numerical data such as failure in copying simple numerical data of an array, a graph or statistical table and diagram [53]. Additionally difficulty in copying numerical data in a table is noted due to limited visual perception as well as the difficulty of replicating and retaining data correctly [54]. Additionally, data encoding and decoding are connected to difficulties in number reproduction which is related to visual perception difficulties. The above-mentioned difficulties are probably connected to visual perception and memory deficits [54].

Dyscalculic people face difficulties in orientation in regards to the person himself e.g. troubles in conceiving concepts like over me, under me, or on my right - left side, etc. [9]. In addition, they face difficulties in perceiving an object's position e.g. to identify if an object is lying on the right or left side in relevance to a steady object. On the other hand, they display difficulties in orientated aspects with respect to other objects e.g. this vase is on the table etc. [9]. In addition, they have difficulties in conceiving direction concepts and following instruction such as: go straight or turn left-right as well as conceiving the notion of up, down, next to or behind concepts in general [9]. Furthermore, they find it hard to conceive and distinguish cardinal directions namely North from South and East from West [55]. Generalizing the above characteristics, difficulties in regards to concepts like distance and speed are also encountered. Additionally, they have difficulties conceiving clockwise and anticlockwise motion namely they face difficulties in order to open a lock using a key turning it to the right [56]. They also have difficulties in naming the time on an analog clock. Additionally, difficulties in organizing movement activities in two and three dimensions are noted. Notably, they face difficulties while applying multiplication and division algorithms. This particularity concerns difficulties in spatial perception since students' eyes have to move up and down or right and left while the above mathematical calculation is performed.

4.2.2. Geometrical Concepts Perception Difficulties

In the range of spatial perception difficulties, troubles in geometrical thinking are included as well. According to Van Hiele's learning model, where the way that students perceive geometry is described, geometrical thinking evolves through five levels [57]. Referring to the geometrical development perception in early ages, identification level is the initial level in Van Hiele's learning model, where the enhancing of children's ability in perceiving shapes such as entities/totalities, without conceiving their features, is involved. Children begin forming concepts of shape long before they enter school. They may first learn to recognize shapes by their overall appearance, stating the name of the shape [58]. However, dyscalculic children appear to have difficulties in perceiving and representing shapes as a whole, although at this stage they don't consider that the shape is made up of separate components.

According to Van Hiele's learning model, descriptive, and theoretical abstraction level is linked to school aged children. "Descriptive level" or "analysis level" is the one where children are able to identify, recognize and state the features of shapes [58]. Children are able to logically classify the properties of figures, such as class inclusion, definitions and logical implications to gain a meaningful aspect. However, dyscalculic children face difficulties in perceiving the above-mentioned concepts and terms [57]. Initially, dyscalculic students are not capable of acquiring the necessary vocabulary to describe the attributes of a figure. Therefore, they show difficulties in perceiving and using terminology having to do with geometrical concepts as well as verbally describing figure properties [3]. The "theoretical abstraction" or "informal deduction" level is referred to as the stage where the rational layout of shapes is processed according to their features and the significance of well-given definitions is conceived [59]. Dyscalculic children in this stage of geometrical cognition development are not able to make logical arguments about the attributes themselves or relations among attributes [60]. While referring to older ages, geometrical thinking is developed according to Van Hiele's "formal logic" or "deduction" level [61]. This perception stage is related to the ability to sufficiently conceive and to use mathematical induction logic, as well as definitions, axioms, theorems and proof related to the conception of the Euclidean geometry. In the case of dyscalculic students, they encounter problems in applying geometrical methods in order to solve problems [62]. Thus, the role of deduction is not sufficiently conceived [63] and similar difficulties are noticed due to incomplete logical perception [59]. Finally, referring to the academic students, Van Hiele's fifth level is referred to the stage where students are sufficient in studying Non-Euclidean geometry, as well as unrealistic assumptions and definitions. The students analyze various deductive systems with a high degree of rigor, being able to understand the geometric methods and generalize the geometric concepts [63]. In the case of dyscalculic adults, Non-Euclidean geometry's assumptions, axioms, and theorems, where three-dimensional perception is required, are inefficiently conceived.

There are also difficulties in defining and perceiving the notion of slope or alignment. Concerning aspects in two or three-dimensional space such as length, width, height volume and so on, these are hardly conceived [64]. So recognition and discrimination of geometrical solids are hard to be achieved. Finally, there are difficulties while copying simple shapes or forming small straight lines without a ruler and even more a whole straight line [65]. Some problems are also observed due to lack of perceiving the conservation concept i.e. that size of an object or number that a set is representing remains unchanged irrespective of the spatial organization [23]. This is the reason why there are problems to perceive the size of objects and furthermore to make adequate comparisons.

4.3. Time Perception Deficits

People suffering from dyscalculia show difficulties in time perception. Difficulties in the perception of time and understanding concepts like before or after a fact are noticed. Additionally, troubles in perceiving and managing time are noted, such as estimating the time required in order to accomplish an assigned task and dealing with deadlines [12]. Furthermore, difficulties with time concepts are encountered, such as following a schedule consistently and difficulties concerning events sequence such as recalling sequences of past events [13]. Additionally, time-related difficulties appear, such as identifying and distinguishing time concepts i.e. hours, days, months etc. [66]. Therefore, there is confusion about the notion of prior and after.

Furthermore, dyscalculic people show the inability to identify and comprehend time concept while looking at an analog clock [67]. Those troubles are related to difficulties in perceiving the position and the motion of the indicators of a clock. These features are also related to difficulties on perceiving and sensing clockwise and anticlockwise notion [56]. This concerns both difficulties in visual perception, spatial perception, and inefficiency in conceptualizing and perceiving time. Those time perception distortions are also connected to slow spelling of numbers.

4.4. Calculation Deficits

A term usually used in order to describe difficulties in the calculation is the term Acalculia. Acalculia is an acquired impairment in which people have difficulties performing simple mathematical tasks, such as adding, subtracting, multiplying and even simply stating which of two numbers is larger [68]. Acalculia is distinguished from dyscalculia to the point that Acalculia is acquired due to neurological injury such as stroke, while dyscalculia is a specific developmental disorder first observed during the acquisition of mathematical knowledge [69]. The name comes from the Greek: "a" meaning "not" and Latin "calculare", which means "to count". Acalculia is distinguished to primary and secondary Acalculia. Primary Acalculia is related to difficulties in perceiving mathematical concepts and performing mathematical operations. Secondary Acalculia is the potential loss of calculation in relation with other cognitive difficulties [70].

Especially, difficulties in calculations are expressed in deficits related to short-term memory which implies an inability to calculate. These problems are linked with limited capability to make calculations and with difficulty in memorizing and recalling simple arithmetic operations [10]. Difficulties in recalling number facts and using algorithms in order to add, subtract, multiply and divide are noted. Problems in the versatility of noetic estimation are also noticed i.e. difficulties in mental representation and perception of algorithmic acts [71]. In addition to this, troubles appear in the correct use of arithmetic signs and adequate memorization of math facts, such as arithmetic and timetables [8]. Therefore, failures are noticed while adding, subtracting, multiplying and dividing two-digit numbers, three-digit numbers, decimals or fractions [8].

While referring to difficulties in the calculation there are lots of features related. Firstly deficiency while recognizing symbols mathematical symbols are noticed such as symbols of addition, subtraction, multiplication and division [8]. Difficulties in order to discriminate and conceive the notion mathematical operation symbols are also relevant to visual perception difficulties. Especially difficulties in order to recognize and distinguish the differentiation of inequality symbols disparity relations like bigger ">", smaller "<" equivalent "=" and approximately equal "≈ "are noted and lack of perception of the concepts of equality and inequality in general [72]. A difficulty in performing mathematical operations by hand as well as by memory is noted as well. That is due to the fact that mathematical operations are dealt mechanically without understanding the exact process. This is basically connected with deficit perception of pre-mathematical concepts [73].

Especially difficulties in adding are noted mainly due to troubles in immediate quantities estimation ability [74]. According to Engelhard classification, computational strategy errors are noted during adding mode [75]. Therefore, dyscalculic people often appear difficulties while implementing the addition algorithm properly. For example 54 +39 is computed to be 83 and 57 + 93 is computed to be 1410. Digits reversals are also often noted i.e. the sum 43+19 is considered to be 49+13 and the result finally computed is 62. Additionally confusions in applying the proper mathematical operation, for instance, the sum 14+1 is applied as subtraction and the final outcome is computed to be 13. Finally, weakness in recognizing and properly evaluating absurd results like the act 7+4 is summed up to be 3 [75]. The above-mentioned features according to Engelhard classification are noticed in subtraction as well.

Concerning difficulties in subtraction, dyscalculic people present incomplete perception of the evolved numerical data. Specifically, the difficulties noted are relative to the ability to distinguish the subtrahend from the minuend. That implies that there are difficulties in perceiving that the commutative property is not valid in subtraction mode and therefore subtraction algorithm is misapplied [76]. Moreover, there are troubles in understanding the concept in borrowing tens, unit etc. of the greater class digit.

Regarding multiplication, difficulties are noticed mainly due to inadequate memory ability. Equate short-term memory is necessary because the multiplication table memorization is required and is considered as a prerequisite for the adequate multiplication application [77]. Also, a common failure encountered is the carries omission in addition and subtraction due to memory deficits or limited concentration [34]. Additionally, according to Engelhard classification, defective algorithm applications are noted i.e. and the result of 123x42 is computed to be 186. For example, 36x4 is computed to be 1254 since 4x6 is equal to 24, 3+2 equals to 5 and 3x4 is equivalent to 12. This is due to the application of consecutive individual multiplications per column. Errors in basic numerical data and times table memorization are also noticed e.g. the result of 6x7 is considered to be 48, according to Engelhard classification. Moreover according to Engelhard computational strategies classification mistakes in applying mathematical formulas are noted e.g. the multiplication result of 5x1 is considered to be 1 as well as errors in multiplication with zero i.e. 3x0 is considered to be 3 [75].

In addition to multiplication difficulties, problems in applying division algorithm are noted as well. Especially difficulties in perceiving division symbol ÷ and therefore division is confused with either addition or multiplication. Additionally, division algorithm is regularly displayed due to the confusion of the connection between division and multiplication this is the reason why dyscalculic children fail to understand and solve simple division problems. Additionally, they face difficulties in the automatically retrieval of the solutions to simple division math problems and even more state a simple division problem in words [78]. Additionally, troubles at adequate problem-solving strategies like failure to recognize when the quotient is larger than both the numerator and denominator are commonly noted. Additionally, dyscalculic children are dealing with difficulties in applying a division with zero e.g. 0 ÷ P = P and notably, wrong placement of the quotient digits are noted [79].

4.5. Calculus Perception Difficulties

Referring to problems of high school students', difficulties in Algebra are observed. Under the term "Algebra", mathematical concepts such as main math concepts, math visualization, and math applications are included [14]. Under this perspective, difficulties in distinguishing the concepts of the integer, real and complex numbers, as well as difficulties in perceiving variables, are noticed [14]. Additionally, troubles in mathematical operations with variables are noted, not only, in addition, subtraction, multiplication, and division, but also in integer power root and non-integer power log [14]. Referring to math visualization, problems are noticed in regard to perceiving number concepts in number axle or complex plane (Argand diagram) or concepts related to Venn diagram [14]. Finally, referring to math applications, difficulties appear in solving algebraic equations i.e. in advanced mathematics, science, engineering, business, finance [14].

Referring to problems related to dyscalculia during academic ages, severe difficulties to calculus are observed [16]. "Calculus" is the mathematical study of change. It refers to any method or system of calculation guided by the symbolic manipulation of expressions and several math procedures are included [16]. Calculus has two major branches, the differential calculus (concerning rates of change and slopes of curves), and the integral calculus (concerning accumulation of quantities and the areas under curves) [16]. While dealing with calculus, students are confronted with the concept of limit, involving calculations that are no longer performed by simple Arithmetic and Algebra, and infinite processes that can only be carried out by indirect arguments [15]. In particular, some of the calculus basic topics are hard to be conceived by adults suffering from dyscalculia, such as limits, differential and integral calculus lists and tables, as well as multivariable functions. Especially, the concept of limit creates a number of several cognitive difficulties, including difficulties embodied in the language; terms like "limit", "tends to", "approaches", "as small as we please" have powerful colloquial meanings that conflict with the formal concepts [15]. Additionally, the idea of "N getting arbitrarily large", implicitly suggests conceptions of infinite numbers [15]. This implies that there is confusion over the passage from finite to infinite, in understanding "what happens at infinity", a concept hard to be conceived from dyscalculic people [15].

4.6. Algorithmic Thinking Difficulties

It is worth mentioning that some difficulties in mathematics are linked to weaknesses in algorithmic problem solving. The skill to design algorithms in order to solve a problem pre-requires and enforces cognitive abilities [80]. Adults dealing with dyscalculia mainly have troubles with algorithmic thinking expressed by limited ability in the algorithmic processing of stimuli [81]. “Algorithmic thinking ability” is a term which describes the capacity needed in order to complete a task using a series of default actions, aiming to complete a process [82]. This term is somehow a group of skills, concerning the way that an algorithm is conceived, designed, applied and evaluated. The term algorithm describes a finite sequence of actions, which describe how to solve a given problem [82]. Basic algorithmic principles contribute to the development of logical thinking and learning methodologies, which are needed to solve problems [83]. Basic algorithmic principles contribute to the development of logical thinking and learning methodologies, which are needed to solve problems [81]. Some of the basic principles needed are the ability to conceive the given problem and the ability to use techniques to describe the problem briefly [82]. Moreover, the application of techniques which separates a problem into smaller pieces, as well as the design, the description and the application of strategies are needed in order to solve a problem [82]. Some of the basic principles needed are the ability to conceive the given problem and to use techniques to describe the problem briefly [84]. Moreover, application of techniques, which separate a problem into smaller pieces, as well as design, description and application of strategies, are needed in order to solve a problem [84]. Algorithmic thinking is an ability evolving over time and more severe problems are displayed and noticed at older ages.

Algorithmic thinking difficulty is separate but not irrelevant of dyscalculia. People suffering from DAT (Difficulties in Algorithmic Thinking) are facing problems in scrolling memory and memorization of algorithmic procedures [85]. Additionally, difficulties in implementing algorithmic forms, which require sequential and analytical approach, have been observed [83]. On the other hand, difficulties in processing, remembering and organizing information are also noticed [86]. Furthermore, dysfunction of long-term memory, concerning mathematical procedures, is noticed i.e. difficulties in perception and solution of complex problems [85]. This means that people with DAT face difficulties to recall and to make successive steps to solve algorithmic problems and therefore inability in procedures, which require quality memory [52]. Consequently, pre-required skills, in order to solve mathematical problems, are not efficiently developed and therefore people with DAT usually avoid procedures even games, which require strategic thinking [84]. Time perception inefficiency is also observed because there are problems on time estimation, which is required in order to complete a task [66]. Finally, difficulties in space perception, such as orientation and giving directions, are also often noticed [87].

Specifically, concerning problem-solving difficulties, DAT disability focuses on three points Perception; Procedure and Best solution Difficulty. Firstly, difficulties referring to mathematical problem conception are noticed i.e. failure in proper perception, evaluation and problem decoding (Perception Difficulty). Secondly, due to poor short-term memory, and / or problems in processing memory are noticed while students apply algorithmic steps needed to solve the problem (Procedure Difficulty). On the other hand, there are difficulties in distinguishing the best problem solution, due to lack of perception of the optimized solution (Best solution Difficulty) [52]. These difficulties are expressed by the limited capacity of some people in the algorithmic processing of several stimuli [88]. Difficulties in algorithmic thinking are probably caused due to problems of long-term memory, thus inefficiency, while conceiving and solving a complex problem, is noticed [85]. Specifically, there are problems in serial memory and memorization procedures and therefore difficulty in applying algorithms in which sequential and analytical approach is needed [85]. Additionally, difficulties are noticed in the process, the memorization and the organization of information, which appears some weakness in the mental representation of the problem. That occurs due to dysfunction in the long-term memory, concerning mathematical procedures, resulting in difficulties in understanding and solving complex problems [85]. This means incomplete solving capabilities in order to solve an algorithmic problem in which the implementation and the revocation of algorithmic steps are needed [82].

Moreover, due to weaknesses in memorization dyscalculic people tend to avoid games which require strategic thinking [84]. Additionally, they avoid keeping score during games and they lose track of whose turn it is during board games and card games [84]. Dyscalculic adults also encounter problems in realistic contexts in everyday issues, particularly in conceiving the value of money, in using change or in estimating the cost of a product properly [89]. On the other hand, algorithmic thinking difficulties are also connected to poor sense of direction, expressed with troubles in managing space, i.e. in orientation and in giving directions, in turning left or right or in perceiving the points of the horizon [69]

5. Conclusion

This review includes an assessment of the basic theoretical background in learning difficulties in mathematical reasoning, known as dyscalculia are analyzed. In specific, a conceptual map of dyscalculia is provided, including difficulties in number sense, spatial and geometrical concepts perception deficits, time perception difficulties, and difficulties in the calculation. Difficulties in calculus perception referring to troubles in applying specified methodologies as an advanced stage of reasoning are also discussed. The difficulties noticed in the reasoning level related to the ability to create methodologies and developing strategies in order to deal with generalized problems, namely in algorithmic thinking abilities are analyzed in depth. Algorithmic problem solving requires follow-up procedures, including procedures for recognizing the problem objective, plan a solution algorithm taken into consideration alternative perspectives and reflection on problem-solving processes evaluating as well the process result. Therefore reasoning abilities augmentation is essential to be reconsidered with the frame of cognition and cognitive science.

6. Discussion

In this paper, a conceptual map is proposed in order to ensure efficiency of dyscalculia's identification procedure. Research in dyscalculia is still in its emergent stage, lagging behind other learning disabilities. In each case, algorithmic thinking difficulties are not evaluated and there is not yet an overall diagnostic screener dealing with the range of dyscalculia. Future directions include the suggestion of a screener, in order to deal with dyscalculia's total evaluation, in respect to the proposed feature classification, emphasizing additionally on algorithmic thinking difficulties as well as the connection among their features. The use of EEG brain imaging technique is also proposed as a future work in order to evaluate the connection of the above-mentioned difficulties with cognitive neuroscience aspects. EEG analysis results are about to be useful in order provide educational proposals to enhance the learning procedure in the frame of a recent lighted scientific field known as Neuroeducation. Additional work, as well as analysis of the proposed classification procedure, is on-going.

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