
Fluidification of Stochastic Petri Net by Non Linear Timed Continuous Petri Net

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Abstract: Reliability analysis is often based on stochastic discrete event models like Markov models or stochastic Petri nets. For complex dynamical systems with numerous components, analytical expressions of the steady state are tedious to work out because of the combinatory explosion with discrete models. Moreover, the convergence of stochastic estimators is slow. For these reasons, fluidification can be investigated to estimate the asymptotic behaviour of stochastic processes with timed continuous Petri nets. The contributions of this paper are to sum up some properties of the asymptotic mean marking and average throughputs of stochastic and timed continuous Petri nets, then to point out the limits of the fluidification in the context of the stochastic steady state approximation. To overcome these limitations, the new semantic and the condition for convergence is proposed: fluid Petri nets with Non Linear Timed Continuous Petri Net (NL-CPN).

Keywords: Fluidification, Stochastic Petri Nets, Continuous Petri Nets, Steady State, Reliability Analysis

1. Introduction

Reliability analysis is a major challenge to improve the safety of industrial processes. For complex dynamical systems with numerous interdependent components, such studies are mainly based on stochastic discrete event models like Markov models or stochastic Petri nets (SPNs) [8]. Such models are mathematically well founded and can be investigated in order to work out either analytical results or numerical simulations. But in case of large systems, the combinatory explosion limits their use. In this context, fluidification can be discussed as a relaxation method.

The aim of this paper is to discuss the estimation of the SPNs asymptotic mean markings by using fluid models and to introduce new semantics of Continuous Petri nets (CPNs) for that issue. The fluidification of discrete Petri nets has been investigated first for the analysis and performance evaluation of discrete event systems (DES) [3], [4], [11], then for control applications [12], [7]. Recently fluidification has been also introduced for SPNs [13], [14]. In this paper, the limits of the fluidification of SPNs with timed CPNs with constant firing rates and infinite server semantic are first discussed. The set of asymptotic mean markings (reached from an identical initial marking) are sometimes different for

SPNs and CPNs with same structures.

The paper has five sections. The section two introduces PNs, SPNs, CPNs. Then, the section three discusses the fluidification of SPNs according to CPNs. The section four introduces Non Linear Timed Continuous Petri Net (NL-CPN) is proposed and discussed. The last section gives some conclusions

2. Stochastic and Continuous Petri Nets

2.1. Petri Nets (PN)

A Petri net (PN) is defined as $\langle P, T, WPR, WPO \rangle$ where $P = \{P_i\}$ is a set of n places and $T = \{T_j\}$ is a set of q transitions. $WPR = (w^{PR}_{ij}) \in (Z^+)^{n \times q}$ is the pre-incidence matrix where Z^+ is the set of non negative integer numbers (w^{PR}_{ij} is the weight of arc from place P_i to transition T_j) and $n \times q$ are the dimensions of matrix WPR . $WPO = (w^{PO}_{ij}) \in (Z^+)^{n \times q}$ is the post-incidence one (w^{PO}_{ij} is the weight of arc from transition T_j to place P_i) [4]. The PN incidence matrix W is defined as $W = WPO - WPR \in (Z)^{n \times q}$. The PN marking vector $M(t) = (m_i(t))$, $i = 1, \dots, n$, is an application from the set of non negative real numbers R^+ to $(Z^+)^n$ such that, for each place $P_i \in P$, $m_i(t) = M(P_i, t)$ is the number of tokens in place P_i at time t . M_1 is the PN initial marking and $M(t) = (m_i(t))$, i

$= 1, \dots, n$ is the mean marking vector at time t (i.e. $m_i(t)$ is the mean number of tokens in place P_i during the time interval $[0, t]$). Each transition T_j fires according to the round towards zeros (i.e. "floor" function) of its enabling degree $n_j(M(t))$ defined for marking $M(t)$ by equation (1):

$$n_j(M) = \min (m_i / w_{ij}^{PR}) \text{ for all } P_i \in {}^\circ T_j \quad (1)$$

where ${}^\circ T_j$ stands for the set of T_j upstream places. The place P_i such that $i = \text{argmin} (m_k(t) / w_{kj}^{PR})$ for all $P_k \in {}^\circ T_j$ is the critical place for transition T_j at time t . Right and left natural annullers of the token flow matrix W are called T-semiflows and P-semi flows, respectively. When $Y^T \cdot W = 0$, the net is said to be conservative and when $W \cdot x = 0$. The net is said to be consistent. In a consistent net, a vector x such that $W \cdot x = 0$ represents a repetitive sequence, or in other words, a potential steady-state behaviour of the system in which all transitions are fired [17].

2.2. Stochastic Petri Nets (SPN)

A stochastic Petri net is defined as a timed PN whose transitions firing periods are characterized by random distributions. The basic model (SPN) has been introduced by [8], [9] and several extensions have been developed [1], [5] for the reliability analysis of repairable systems. Basically, a SPN, $\langle \text{PN}, \mu \rangle$, is a PN with a firing rate vector $\mu = (\mu_j) \in (\mathbb{R}^+)^q$. Each transitions T_j is characterized by the firing rate μ_j so that $\mu_j \cdot dt$ is the probability that the transition T_j will fire between t and $t+dt$ when the transition T_j has been enabled, with degree 1 at t . The marking process of a SPN is characterized according to the PN incidence matrices, the initial marking, the firing rates, the firing policy, the server policy and the execution policy [2], [5]. The average throughput, marking and mean marking vectors of a SPN at time t will be referred as $X_s(t)$, $M_s(t)$. The SPNs that are considered in this paper satisfy the assumptions (H_1) to (H_5) :

(H_1) the marked SPNs are bounded.

(H_2) the marked SPNs are reinitialisable.

(H_3) the firing policy is a race policy: the transition whose firing time elapses first is assumed to be the one that will fire next.

(H_4) the server policy is of type infinite server: the minimal period of each transition T_j is defined with a stochastic duration which is characterized according to an exponential distribution of varying parameter: floor ($n_j(M(t)) \cdot \mu_j$).

(H_5) the execution policy is of type resampling memory: at the entrance in a marking, the remaining firing time of all transitions that were enabled is reset.

As a consequence, the considered SPNs have a reachability graph $R(\text{SPN}, M_1)$ with a finite number N of states and their marking process is mapped into a Markov model with state space isomorphic to $R(\text{SPN}, M_1)$ [8], [2]. In that case, the asymptotic behaviour of SPNs can be worked out according to the steady state probabilities of the Markov model.

2.3. Continuous Petri Nets

CPNs have been developed in order to provide continuous approximations of the discrete behaviours of PNs. The marking of each place is a continuous non negative real valued function of time [11]. A CPN is defined as $\langle \text{PN}, X_{\max} \rangle$ where PN is a Petri nets and $X_{\max} = \text{diag} (x_{\max_j}) \in (\mathbb{R}^+)^{q \times q}$ is the diagonal matrix of maximal firing speeds x_{\max_j} , $j = 1, \dots, q$ [4]. $M_c(t)$ is the marking vector and $X_c(t) = (x_{c_j}(t)) \in (\mathbb{R}^+)^q$ is the firing speeds vector at time t . The marking variation is given by (2):

$$dM_c(t) / dt = W \cdot X_c(t), M_c(0) = M_1 \quad (2)$$

Finite server (i.e. constant speeds) and infinite server (i.e. variable speeds) semantics exist for CPN. In this paper only the infinite server semantic is considered. $X_c(t)$ depends continuously on the marking of the places according to (3):

$$x_{c_j}(t) = x_{\max_j} \cdot n_j(M_c(t)) \quad (3)$$

2.4. Steady States of SPNs

For live SPN that satisfy hypotheses (H_1) to (H_5) and with a finite number of states in the reachability graph, the marking process is mapped into a Markov model with state space isomorphic to the reachability graph of the SPN model [2]; [9]; [13] In that case the steady state of SPN can be worked out according to the steady state probabilities of the Markov model.

Let define $X_s = (x_{sj}) \in (\mathbb{R}^+)^q$ as the average throughputs vector and $M_s = (m_{si}) \in (\mathbb{R}^+)^n$ as the mean markings vector of SPN and $\Pi_{ss} = (\pi_{ssk}) \in [0, 1]^{1 \times N}$ as the steady state probabilities vector of the associated Markov model with N states. Let define $A(\mu) \in (\mathbb{R}^+)^{N \times N}$ as the generator of the underlying continuous time Markov model (i.e. the continuous time Markov model that defines the variations of the state probability vector according to the Petri net reachability graph) and $1_N = (1, \dots, 1)^T \in (\mathbb{R}^+)^N$ as the column vector of size N with all entries equal to 1. It is well known that Π_{ss} is the single solution of equations (4) [7], [10]:

$$\begin{aligned} \Pi_{ss} \cdot A(\mu) &= 0 \\ \Pi_{ss} \cdot 1_N &= 1 \end{aligned} \quad (4)$$

As a consequence, X_s and M_s are obtained according to equations (5) and (6):

$$x_{sj} = \mu_j \cdot \left(\sum_{k=1 \dots N} n_j(M_k) \cdot \pi_k \right) \quad (5)$$

$$m_{si} = \sum_{k=1 \dots N} m_{ki} \cdot \pi_k \quad (6)$$

where $M_k = (m_{ki}) \in (\mathbb{R}^+)^n$ stands for the marking vector corresponding to the state k of the Markov model. This method gives an analytical solution of the SPN steady state in case of ergodic Markov models, but requires the computation of the transition matrix $A(\mu)$ and as a consequence the reachability graph $R(\text{PN}, M_1)$ of the SPN. For large systems, the computational time and memory requirements necessary

to work out $R(PN, M_1)$ become important as long as the number N of states increases exponentially. In that sense, SPN can be considered as a stochastic estimator for the Markov model. The advantage of this estimator is that the determination of $R(PN, M_1)$ is no longer required, the drawback is the slow convergence of the stochastic estimator, particularly in case of rare events.

The system in Figure 1 models a simple manufacturing system. The final product is composed of two different parts, A and B, that are processed in machines M_1 and M_2 (represented by transitions T_1 and T_2), and stored in buffers P_4 and P_6 , respectively. Then, they are assembled by M_3 (i.e. transition T_3), and processed in M_4 (i.e. transition T_4).

Finally, M_5 (i.e. transition T_5) packages them. During the processing of parts A and B, tool1 (tokens in place P_5) and tool2 (tokens in place P_7) are needed. Also tool3 (tokens in place P_3) has to be used in the three final operations. The machines M_1, M_2, M_4 and M_5 are assumed to be reliable and an active redundancy ($n = 3$) is considered for the assembly machine M_3 that is assumed to have failure and repair rates $\lambda = 1.5e^{-2} TU^{-1}$ and $\mu = 1e^{-1} TU^{-1}$. To achieve high availability requirements 3 active redundancies are considered for M_3 (place P_{10}). The productivity of the workshop is evaluated with the computation of the output flow x_5 with respect to the number k of pallets and tools: $M_1 = (2k \ 2k \ k \ 0 \ k \ 0 \ 0 \ 3 \ 0)^T$

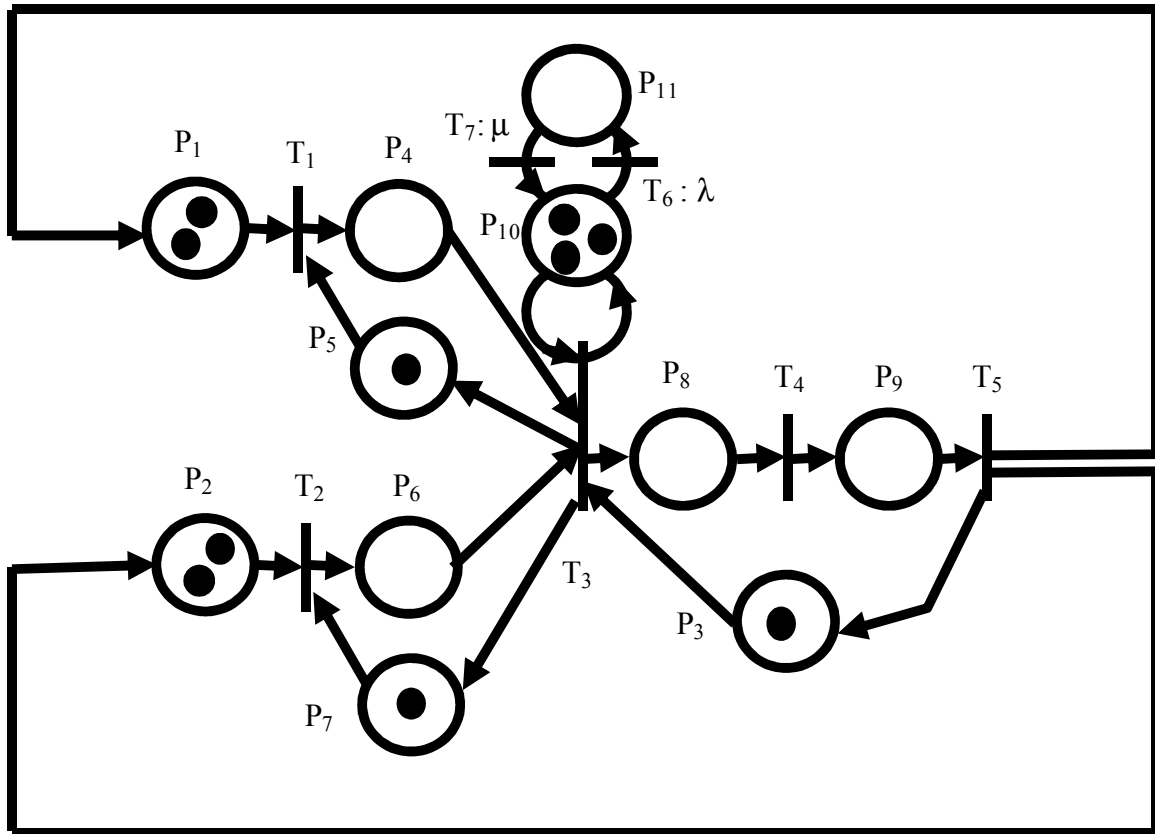


Figure 1. Assembly workshop.

Only the flow of transition T_5 and the marking of places P_1, P_2, P_8 and P_9 will be considered. Other flows and markings are given by (7):

$$\begin{aligned}
 x_1 &= x_2 = x_3 = x_4 = 2 \cdot x_5 \\
 m_4 &= y_4 \cdot M_1 - m_1 - m_8 - m_9 \\
 m_6 &= y_5 \cdot M_1 - m_2 - m_8 - m_9 \\
 m_3 &= y_3 \cdot M_1 - m_8 - m_9 \\
 m_5 &= y_1 \cdot M_1 - y_4 \cdot M_1 + m_1 + m_8 + m_9 \\
 m_7 &= y_2 \cdot M_1 - y_5 \cdot M_1 + m_2 + m_8 + m_9
 \end{aligned}
 \tag{7}$$

Equations (7) hold for SPN and CPN. Steady state markings and flows are reported in Table 1 with respect to

initial marking.

Table 1. Number of states, average throughputs and mean markings of SPN from Figure 1 in function of k .

k	N	$x_{s,5}$	$m_{s,1}$	$m_{s,2}$	$m_{s,8}$	$m_{s,9}$
1	205	0.50	1.75	1.75	1.00	1.48
2	1885	1.13	3.51	3.51	2.25	2.75
3	7796	1.77	5.20	5.21	3.54	4.07
4	22187	2.42	6.97	6.91	4.84	5.36
...
∞	∞	∞	∞	∞	∞	∞

The productivity of the system in Figure 1 is evaluated with the computation of the output flow $x_{s5}(t)$ with respect to the number of pallets and tools. The results obtained with Markov models and SPNs simulation over a time interval of $D = 1000 TU$ are summed up in Table 2 and Table 3.

Table 2. Performance evaluation with Markov models.

k	N	$x_5(t)$	Markov model	Computational effort (TU)
1	48	0.29		0.1
2	216	0.61		0.9
3	640	0.93		12
4	1500	1.25		108
...

For $k > 4$, the computational effort becomes heavy because of the large number N of states and the performance evaluation with Markov model analysis is no longer computable [17].

Table 3. Performance evaluation with SPNs.

k	$x_{s5}(t)$ SPN	Computational effort (TU)
1	0.30	0.59
2	0.61	2.0
3	0.94	4.0
4	1.24	6.8
5	1.55	11
10	2.41	30

Simulation with SPN can be used to overcome the computational limitation with Markov model. The simulation error does not exceed 1%. One can also notice that the computational effort increases but remains acceptable up to $k=10$. For $k > 10$, fluidification must be introduced.

3. Fluidification of SPN

The aim of this section is to pinpoint the limit of the fluidification of SPNs by CPNs with same structure and same initial marking. The several example for which CPNs do not converge to the asymptotic mean marking of SPNs is presented.

3.1. Trivial Fluidification of SPNs

The steady state of a SPN = $\langle \text{PN}, \mu \rangle$ and the one resulting of a CPN = $\langle \text{PN}, X_{\max} \rangle$ with same structure, initial marking and $X_{\max} = \mu$ are in general case not identical. For example, the steady state of the PN in Figure 1 considered as continuous is given in Table 4. It can also be noticed that the difference between SPN and CPN steady state decreases with respect to the marking. More generally, a PN can be live as discrete and non-live as continuous. In a similar way, a discrete bounded system may be unbounded as continuous [11]. Moreover, like in discrete nets, the throughput of a CPN does not fulfil in general any monotonicity property, neither with respect to the initial marking, nor with respect to the structure of the net, nor with respect to the transitions rates. [12]. The throughput of a CPN is not in general an upper bound of the throughput of the discrete PN [6].

Table 4. Average throughputs and mean markings of CPN from Figure 1 in function of k .

k	x_{e5}	m_{e1}	m_{e2}	m_{e8}	m_{e9}
1	0.67	1.67	1.67	1.33	1.33
2	1.33	3.33	3.33	2.67	2.67
3	2	5	5	4	4
4	2.67	6.67	6.67	5.33	5.33
...
∞	∞	∞	∞	∞	∞

3.2. Regions in the Reachability Space

According to the function “min(.)” in the expression of the enabling degree (1), the marking space of CPNs is divided into K regions A_k with $K = |\circ T_1| \dots |\circ T_q|$. Each region A_k is defined by its configuration [7], also called PT-set [6] defined by (8):

$$\text{PT-set}(A_k) = \{(P_i, T_j) \text{ s. t.}$$

$$\forall M_c(t) \in A_k, x_{c_j}(t) = x_{\max_j}(t). m_{c_i}(t)/w_{ij}^{\text{PR}}\} \quad (8)$$

The place P_i such that $i = \text{argmin}(m_k(t) / w_{ij}^{\text{PR}})$ for all $P_k \in \circ T_j$ is the critical place for transition T_j at time t . PT-set(A_k) represents the set of all pairs (P_i, T_j) , $j = 1, \dots, q$ such that P_i is the critical place of transition T_j in region A_k .

According to the PT-sets. A region A_k is critical if there exists two transitions T_j and T_m that have the same critical place P_i in region A_k (i.e. $(P_i, T_j) \in \text{PT-set}(A_k)$ and $(P_i, T_m) \in \text{PT-set}(A_k)$). In a critical region A_k , a single place drives the throughputs of at least two distinct transitions and the rank of matrix A_k is less than the rank of the constraint matrices in other regions.

Each r -region A_k is characterized by a constraint matrix $A_k = (a_{ij}^k) \in (\mathbb{R}^+)^{q \times n}$, $k = 1, \dots, K$, $i = 1, \dots, q$ and $j = 1, \dots, n$:

- $a_{ji}^k = 1/w_{i(k,j)}^{\text{PR}}$ for all $T_j \in T$,
- $a_{ji}^k = 0$ otherwise.

As a consequence, in each region A_k , the firing speed vector can be written as $X_c(t) = X_{\max} \cdot A_k \cdot M_c(t)$, equation (9) holds:

$$\forall M_c(t) \in A_k, dM_c(t) / dt = W \cdot X_{\max} \cdot A_k \cdot M_c(t) \quad (9)$$

3.3. Problem Statement

Fluidification of discrete PNs leads to some unexpected results and numerous structural and behavioural properties are not preserved with fluidification. In particular, a PN can be live as discrete and non-live as continuous and a discrete bounded system may be unbounded as continuous [12]. The throughput of a CPN is mainly not identical to the throughput of a discrete PN. It is often but not always an upper bound of this throughput [6], [18].

Concerning SPNs, the approximation of the steady state by CPNs is of particular interest as long as the estimation of the asymptotic mean markings can be used to work out the availability, the mean time between failures, and other indicators for reliability. Large approximation errors appear frequently even if such errors decrease with respect to the initial marking. The asymptotic mean markings of SPNs can be approximated (with an acceptable accuracy) with the steady state of CPNs if (1) all transitions remain enabled with degree at least 1 in long run; (2) the marking vector does not leave the region of initial marking in long run. Markovian Continuous Petri Nets (i.e. CPNs with an additive normally distributed noise) have been investigated to relax the condition (2) [13]. In addition, a partial relaxation of SPNs leads to Hybrid Markovian Continuous Petri Nets and can be used to relax condition (1) [14], [19], [21]. But in the former works the continuous models are non-deterministic and the steady state is no longer easy to work out.

4. Fluidification with NL-CPN

In this section, continuous Petri nets with adaptive maximal firing speeds are proposed to converge to the asymptotic mean marking of SPNs also in critical regions [18]. The maximal firing speeds are considered as functions of time and updated in order to compensate on line the errors on the throughputs and markings regarding the asymptotic stochastic average throughputs and mean markings to be reached. Non linear timed continuous Petri nets with adaptive maximal firing speeds are defined as a consequence.

4.1. Definition

A Non Linear Timed Continuous Petri Net: NL-CPN = $\langle P, T, WPR, WPO, M_s, X_s \rangle$ is defined by a set of $n + q$ differential equations defined by (10):

$$dM_c(t)/dt = W \cdot X_c(t) \tag{10}$$

$$dX_{max}/dt = \eta \cdot \text{diag}(\beta \cdot W^T \cdot (M_s - M_c(t)) + (1 - \beta) \cdot (X_s - X_c(t)))$$

under the constraint $X_{max} \geq 0$.

η is the adaptation parameter, and $\beta \in [0, 1]$ is a ratio that balances the weight of the marking and throughput errors in the variation of X_{maxj} . The initial marking is M_i and the initial maximal firing speeds are given by $X_{max}(0) = \text{diag}(\mu)$. In order to motivate the updating equation (10) of the maximal firing speeds, let consider a transition T_j and define w_j as the column j of incidence matrix W . If at time t , for any places $P_i \in {}^\circ T_j$, the marking $m_{ci}(t)$ satisfies $m_{si} - m_{ci}(t) > 0$ (resp. $m_{si} - m_{ci}(t) < 0$) and for any places $P_k \in T_j^\circ$, the marking $m_{ck}(t)$ satisfies $m_{sk} - m_{ck}(t) < 0$ (resp. $m_{sk} - m_{ck}(t) > 0$) then $w_j^T(M_s - M_c(t)) < 0$ (resp. $w_j^T(M_s - M_c(t)) > 0$) and x_{maxj} decreases (resp. x_{maxj} increases). The variation of X_{max} is also driven according to the firing speed $x_{cj}(t)$ of transition T_j : If $x_{sj} - x_{cj}(t) < 0$ (resp. $x_{sj} - x_{cj}(t) > 0$) then x_{maxj} decreases (resp. x_{maxj} increases). But when the preceding conditions are not simultaneously satisfied, the convergence of $M_c(t)$ to M_s is no longer monotonic.

4.2. Introductive Example

Consider for example the SPN described in Figure 2 [6]. This PN has 1 T-semiflow $x = (1 \ 1 \ 1 \ 1)^T$ and 2 P-semiflows $y_1 = (0 \ 0 \ 0 \ 1 \ 1)$, $y_2 = (1 \ 1 \ 2 \ 1 \ 0)$. As a consequence only the flow of transition T_1 and the marking of places P_1, P_2 and P_4 will be considered.

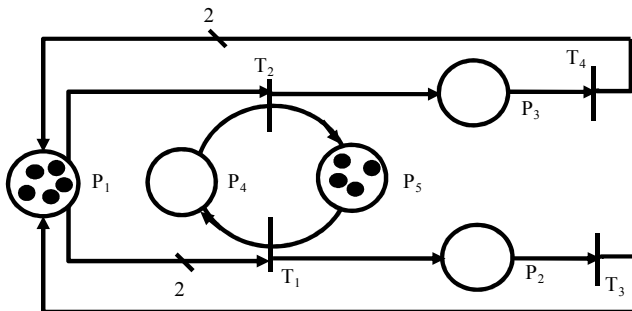


Figure 2. An SPN with firing rate vector $\mu = (3 \ 1 \ 1 \ 10)^T$ and initial marking $M_i = (5 \ 0 \ 0 \ 0 \ 4)^T$.

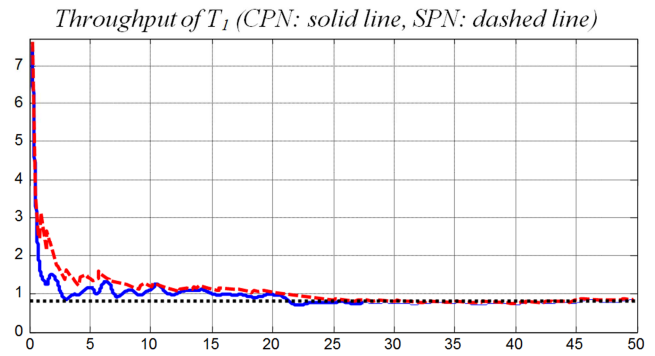


Figure 3. Throughput evolution of CPN (solid line) and SPN (dashed line) in function of time of Figure 2.

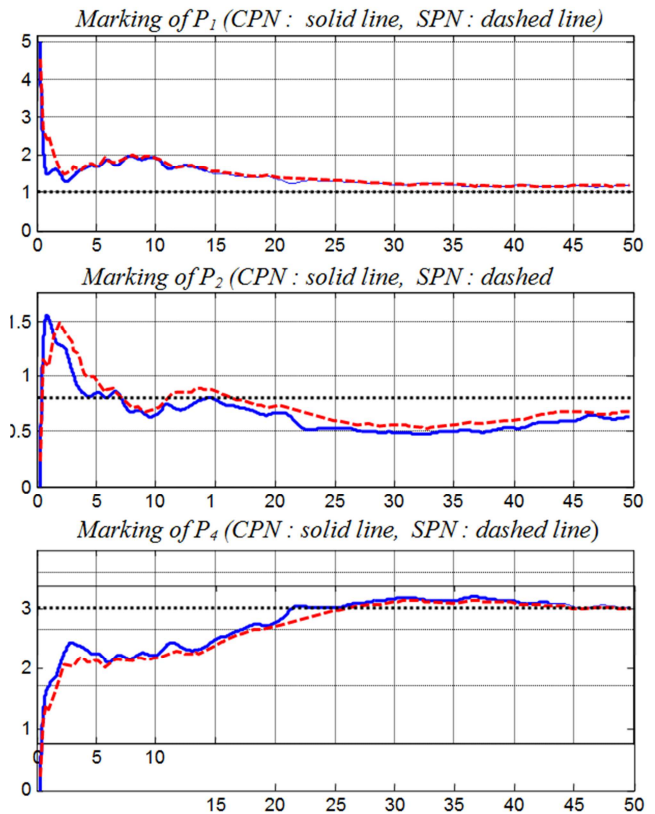


Figure 4. Markings evolution of CPN (solid line) and SPN (dashed line) in function of time of Figure 2.

under the constraint $X_{max} \geq 0$, where η is the adaptation parameter arbitrary fixed to 0.1, $\text{diag}(\mu) \in (\mathbb{R}^+)^q \times \mathbb{R}^q$ is the diagonal matrix with SPN firing rates. As a consequence only the flow of transition T_1 and the marking of places P_1, P_2 and P_4 will be considered (Figure 3 and Figure 4).

5. Conclusions

This paper has pointed out that the asymptotic behaviour of SPNs cannot be trivially approximated with CPNs. Moreover it is not always possible to find any CPN with same structure and same initial marking that converges to the asymptotic mean marking of SPN in long run. In the present form, the proposed semantic cannot be used immediately for the performances evaluation of SPNs but provide an

innovative point of view for the investigations of stochastic systems and may be used for controllers. Our future works are to continue the study of CPN. We will also study continuous approximations directly derived from the SPNs transition firing rates.

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