American Journal of Modern Physics
2013; 2(5): 255-263
Published online August 20, 2013 (http://www.sciencepublishinggroup.com/j/ajmp)
doi: 10.11648/j.ajmp.20130205.14

Modification of Einstein's E= mc^2 to E = $\frac{1}{22}$mc^2

L. Marek-Crnjac

Technical School Center, Maribor, Slovenia

Email address:
leila.marek@guest.arnes.si

To cite this article:
L. Marek-Crnjac. Modification of Einstein's E= mc^2 to E = $\frac{1}{22}$ mc^2. *American Journal of Modern Physicy.*
Vol. 2, No. 5, 2013, pp. 255-263. doi: 10.11648/j.ajmp.20130205.14

Abstract: The Egyptian engineering scientist and theoretical physicist Mohamed El Naschie has found a definite resolution to the missing dark energy of the cosmos based on a revision of the theory of Relativity. Einstein's equation of special relativity $E = mc^2$, where m is the controversial rest mass and c is the velocity of light developed in smooth *4D* space-time was transferred by El Naschie to a rugged Calabi-Yau and *K3* fuzzy Kähler manifold. The result is an accurate, effective quantum gravity energy-mass relation which correctly predicts that 95.4915028% of the energy in the cosmos is the missing hypothetical dark energy. The agreement with WMAP and supernova measurements is astounding. Different theories are used by El Naschie to check the calculations and all lead to the same quantitative result. Thus the theories of varying speed of light, scale relativity, E-infinity theory, M-theory, Heterotic super strings, quantum field in curved space-time, Veneziano's dual resonance model and Nash's Euclidean embedding all reinforce, without any reservation, the above mentioned theoretical result of El Naschie which in turn is in total agreement with the most sophisticated cosmological measurement. Incidentally these experimental measurements and analysis were awarded the 2011 Nobel Prize in Physics to Adam Riess, Brian Schmidt, and Saul Perlmutter.

Keywords: Dark Matter, Homology of Fuzzy Kähler, Betti Numbers, Heterotic Strings, New Special Relativity Theory

1. Introduction

Special relativity presupposes a smooth space-time with Lorentzian symmetry group invariance [1]. Quantum space-time on the other hand is modelled via radically different geometrical realization [1-8]. In string theory, M-theory and super gravity one uses various types of Calabi-Yau and complex Kähler manifolds for space-time extra dimensions [9-17]. Consequently requiring Poincaré invariance in a complex space with extra dimensions will most surely lead to a different energy-mass relation than the classical equation of special relativity. Should the principle of scale relativity hold, then one would expect to retrieve Einstein's familiar formula in a scaled form [3-5]. Noting that for a continuous manifold the Betti number b_2 which counts the three dimensional holes in a manifold is given by $b_2 = 1$ and that the same Betti number for a *K3* Kähler is $b_2 = 22$, it is possible to show that $E = mc^2$ may be elevated to a quantum relativity, i.e. a quantum gravity equation when scaled by $\lambda_{QR} = b_2(S^3)/b_2(K3) = 1/22$.

This prior intuitive expectation noted first by El Naschie was confirmed later on by him on two counts, namely first experimentally using the cosmic measurement of Ries, Schmidt and Perlmutter [4] and second theoretically using numerous sophisticated established theories, all leading to the same robust result, namely $\lambda = 1/22$.

In this paper we show following El Naschie that for a fuzzy Kähler [10, 13], the scaling factor changes from $\frac{1}{22}$ to $\frac{1}{(22+k)} = \frac{1}{22.18033989}$. In addition to giving a derivation of $E_{QR} = \lambda_{QR}(mc^2)$ where m is the controversial rest mass and c is the speed of light, we show that this result is in exquisite agreement with the cosmological measurement of COBE and WMAP as well as the analysis of certain supernovas which led to the award of last year's 2011 Nobel Prize in Physics [4]. Based on *K3* fuzzy Kähler, one can predict with very high precision that the percentage of hypothetical dark energy missing in the universe is 95.4915028 percent. This is a potentially unprecedented agreement between theory and measurement in cosmology, if not in all of theoretical physics [1]. We probably will know for sure when the Planck measurement project is completed. However, this particular result of El Naschie has in one giant leap unified many theories, old and new, and reconciled theory with measurement [16-19]. In Table 1 we summarize the results of various theories and methods leading to the same energy reduction factor of almost $\frac{1}{22}$.

2. Homology of a Space-Time Based on Crisp *K3* Kähler Manifold

Following super strings and related theories [12] we look first at the possibility of a quantum gravity space-time based upon a *K3* Kähler manifold [13]. We start with a non-fuzzy crisp Kähler then look at the fractal-like fuzzy case.

2.1. Classical Non-Fuzzy Kähler

We consider a *K3* Kähler manifold with four complex dimensions used extensively in theories with hidden dimensions particularly super and heterotic string theory [12, 13]. The manifold is fixed by the Betti numbers which determine the Euler characteristic and the signature. In case of non-fuzzy (crisp) *K3* the Betti numbers are [10, 13]

$$b_0 = b_4 = 1,\ b_1 = b_3 = 0,\ b_2^- = 19 \ \text{ and } \ b_2^+ = 3 \tag{1}$$

It follows then that the Euler characteristic is [10, 13]

$$\begin{aligned} \chi &= b_0 + b_4 + b_2^- + b_2^+ \\ &= 1 + 1 + 19 + 3 \\ &= 240 \end{aligned} \tag{2}$$

while [10,13]

$$\begin{aligned} b_2 &= b_2^+ + b_2^- \\ &= 3 + 1 \\ &= 22 \end{aligned} \tag{3}$$

and the signature is [10,13]

$$\begin{aligned} X &= b_2^+ - b_2^- \\ &= 3 - 19 \\ &= -16. \end{aligned} \tag{4}$$

We stress once more that b_2 counts the 3 dimensional holes in *K3* and will play a crucial role in our derivation.

2.2. Fuzzy, Fractal-Like K3 Kähler

Now we look at an even more exotic version of *K3* [13]. With that we mean El Naschie's fuzzy Kähler which he used in earlier studies in a slightly modified form [13, 14]. The El Naschie Kähler we construct here is a fuzzy version of the one considered above. The Kähler in question is given by the same b_0, b_4, b_1 and b_3 as the previous crisp Kähler. Only b_2^- and b_2^+ which measure a sort of average number of *3D* fractal voids are given by [13, 14]

$$b_2^- = 19 - \phi^6 \ \text{ and } \ b_2^+ = 3 + \phi^3 \tag{5}$$

where $\phi = (\sqrt{5} - 1)/2$. It follows then that [13, 14]

$$\begin{aligned} b_2 &= (19 - \phi^6) + (3 + \phi^3) \\ &= 22 + k \\ &= 22.18033989. \end{aligned} \tag{6}$$

It is important to note the following: The small numbers $\phi^6 = 0.05572809014$ as well as $\phi^3 = 0.236067977$ and $k = \phi^3\ (1 - \phi^3) = 0.18033989$ all have various physical, topological and geometrical interpretations. For instance ϕ^6 is the exact value of the vital Immirzi parameter of loop quantum gravity without which nothing would fit in this theory [15]. In addition and as realized for the first time by El Naschie ϕ^6 may be viewed as the probability for quantum entanglement of three quantum particles while ϕ^5 is the well known Hardy's generic probability of quantum entanglement [16,17] for two particles which was also confirmed experimentally. The ϕ^3 on the other hand is the generic probability of a Cantorian space-time with a core Hausdorff-Besicovitch dimension equal to $(4 + \phi^3) = 4 + \bar{4}$ and is directly related to the famous Unruh temperature as demonstrated by El Naschie in some of his unpublished papers and lectures. Finally,

$$\frac{1+k}{10} = \frac{\phi^3}{2}. \tag{7}$$

That means

$$k = 5\phi^3 - 1 \tag{8}$$

which is a deep and useful relation utilized in various E-Infinity derivations.

3. Elevating Einstein's Relativistic Equation to a Quantum Gravity Energy-Mass Relation

We said that b_2 is an important homological invariant of a manifold [9-11] and that it basically counts the 3 dimensional voids in the manifold [9, 14]. For a two sphere S^2 or any connected manifold b_2 is equal to unity $b_2 = 1$. On the other hand, for our classical Kähler $b_2 = 3 + 19 = 22$, and this number already indicates that this manifold is almost a Swiss cheese full of 3 dimensional holes [10, 13]. Compared to the smooth S^2 manifold akin to the space-time of Einstein, *K3* has 22 times less space-time and following general relativity, less energy. Now following, for instance, Nottale's scale relativity principle, we could define a scaling λ_{QR} to be:

$$\lambda_{QR} = \frac{b_2(Einstein\ space)}{b_2(Kähler)} = \frac{1}{22} \tag{9}$$

and use it to scale $E = mc^2$ to

$$E_{QR} = \lambda_{QR}(mc^2) = \frac{1}{22}(mc^2) = 0.0454545\,(mc^2). \tag{10}$$

This implies that the missing hypothetical dark energy is

$$E(Dark) = \left(1 - \frac{1}{22}\right)(100) = 95.454545\%. \tag{11}$$

This is extremely close to the cosmological measurement [4]. Even better still, if we use the fuzzy version we arrive at a mathematically exact equation

$$E(Dark) = \left(1 - \frac{1}{22+k}\right)(100) = 95.49150281\%. \tag{12}$$

In fact when using the fuzzy Kähler we notice immediately a quantum mechanical interpretation of the result because

$$E_{QR} = \left(\frac{1}{22+k}\right)(mc^2) \tag{13}$$

means that

$$E_{QR} = \frac{1}{2}\left(\phi^5\right)(mc^2). \tag{14}$$

However, ϕ^5 is nothing else but Hardy's generic quantum entanglement [16,17] so that our λ_{QR} may be viewed as the screening of a substantial part of the energy in the cosmos by quantum entanglement reducing the Newtonian action at distance by as much as $(1 - \phi^5/2)(100) = 95.4915\%$.

4. Quantum Entanglement as a Consequence of a Zero Measure Fractal Geometry

The totally incomprehensive riddle of spatial separation in quantum mechanics may easily be resolved using the property of zero Lebesque measure of all totally disjointed Cantor sets [16, 17]. There is irrefutable theoretical and experimental proof for this E-infinity based proposal [16, 17]. The story goes as follows: Using an ingenious Gedanken experiment L. Hardy [18] was able to establish via Dirac's orthodox quantum mechanical computation that the probability for quantum entanglement of two quantum particles is given by $P \cong 9\%$. On close examination by first Mermin [19] and then the second author [17], it becomes evident that Hardy rounded off the result concealing its exact numerical magnitude namely that [16, 17, 19]

$$P(Hardy) = \phi^5 \tag{15}$$

where $\phi = \frac{\sqrt{5}-1}{2}$ is the golden mean. The E-infinity interpretation stems from the general E-infinity formula for the probability of quantum entanglement [12]

$$P = P_1 P_2 = \phi^n \frac{1-\phi}{1+\phi} \tag{16}$$

where n is the number of quantum particles and $\frac{1-\phi}{1+\phi}$ is the inverse of the Hausdorff-Besicovitch dimension of the E-infinity fractal space-time core [7, 20]

$$\langle n \rangle = \frac{1-\phi}{1+\phi} = \phi^3 = \frac{1}{4+\phi^3}. \tag{17}$$

For two particles this means

$$P = (P_1 = \phi^2)(P_2 = \phi^3) = \phi^5. \tag{18}$$

Seen that way quantum entanglement can be understood as a natural consequence of the zero length (i.e. zero measure) of a Cantor set and the problem of spatial separation in quantum mechanics is swept away. In a zero measure space-time manifold there is simply no meaning for spatial separation [16]. This incredible result of Hardy-Mermin and El Naschie was experimentally confirmed using various accurate methods in many international laboratories [16, 18, 19].

5. The Missing Energy of the Universe

At present the problem, of dark energy or the missing energy in the universe, constitutes the most challenging problem in physics and cosmology a-like [4, 21, 22]. Accurate measurement has shown that only 4.5% of the total energy thought to be contained in the universe is detectable [4, 21]. The simple conclusion for these results which were awarded the 2011 Nobel Prize in Physics is that either Einstein's equation contains some error or 95.5% of the energy in the universe is due to mysterious dark matter and dark energy which cannot be detected with any known methods [4, 21, 22]. The nonlinear-dynamical fractal resolution of this problem however is unbelievably simple, more than one could imagine [4, 21]. The rationale behind this is as follows: If space-time itself is a real Cantorian fractal then it resembles an unimaginably large cotton candy [7, 16, 21, 23]. The majority of this cosmic cotton candy is naturally voids containing nothing, not even space or time. Consequently, Einstein's famous equation [21]

$$E = mc^2 \tag{19}$$

must be modified to take all these fractal voids into consideration. This can be done in various equivalent ways. The simplest is to take bosonic strings compactified "dark" dimensions into account in the form of a Weyl-Nottale scaling factor. Since bosonic string space has 26 dimensions and Einstein's relativity is only 4 dimensional then the "dark" dimensions are 26 – 4 = 22 and our scaling factor must be [21]

$$\lambda = \frac{1}{26-4} = \frac{1}{22}. \tag{20}$$

Consequently, the revised E is

$$E_{QR} = \frac{mc^2}{22}. \tag{21}$$

Noting that $\frac{1}{22} \cong 4.5\%$, we see that the new E_{QR} accurately accounts for the cosmological measurements [4].

Another way to come to the same conclusion is to reason that from high energy particle physics point of view $E = mc^2$ is based on the existence of one messenger particle, namely the photon (γ). However, this was in 1905 when Einstein conceived his theory. In the meantime we know that we have 12 messenger photon-like particles given by the Lie symmetry groups of the standard model of particle physics [7, 20]

$$| SU(3)\ SU(2)\ U(1) | = 8 + 3 + 1 = 12. \tag{22}$$

Consequently, by inserting $\lambda_0 = \frac{1}{12-1} = \frac{1}{11}$ in Newton's kinetic energy and letting $v \to c$ one finds

$$\begin{aligned} E_N &= \lambda_0\left(\frac{1}{2}mc^2\right) \\ &= \frac{1}{11}\left(\frac{1}{2}m(v \to c)^2\right) \\ &= \frac{mc^2}{22} = E_{QR} \end{aligned} \tag{23}$$

exactly as in the first derivation, namely equation (21).

6. Unifying Relativity and Quantum Theory via Zero Measure Fractals

The previous derivations of the revised Einstein equation $E = \frac{mc^2}{22}$ were only very accurate approximations. However, an exact derivation can be obtained when taking the exact fractal nature of quantum entanglement in deriving E_{QR}. Again this could be done in several equivalent ways. Here we give two methods only. The first is based upon formal analogy between the E-formula of the theory of varying speeds of light [5, 6]

$$E = \frac{mc^2}{1 + \frac{mc^2}{E_P}} \tag{24}$$

and the Cantor set unit interval physics of Ultimate L and F and Taiji-El Naschie theory [24, 25]. Here E_P denotes the Planck energy [5, 19]. Now within Taiji-El Naschie theory E_P is simply equal to $P(Hardy) = \phi^5$ while the devisor m is the five dimensionality of Kaluza-Klein and similarly the devisor c is Sigalotti's critical speed [26, 27, 28] $c = \phi$. Inserting in E one finds

$$\begin{aligned} E &= \frac{mc^2}{1 + \frac{(5)(\phi^2)}{\phi^5}} \\ &= \frac{mc^2}{1 + 21.18033989} \\ &= \frac{mc^2}{22 + \phi^3(1-\phi^3)} \\ &= \frac{\phi^5}{2}mc^2. \end{aligned} \tag{25}$$

In other words, the exact E_{QR} of quantum relativity is equal Einstein $E = mc^2$ multiplied with half of Hardy's $P = \phi^5$ [16-19]. That means

$$\begin{aligned} E_{QR} &= \frac{\phi^5}{2}mc^2 \\ &\cong \frac{mc^2}{22}. \end{aligned} \tag{26}$$

Seen that way the reduction of E from the 100% of Einstein's theory to the 4.5% of the exact quantum relativity theory is due to quantum entanglement at the Hubble cosmic distances which could be explained rationally via the zero measure of fractal Cantorian geometry [7, 16, 20]. The second method we will use to derive the same previous formula is to go back to relativistic boost and then connect it to the random Cantor sets topology. We start with the three well documented relativistic effects namely time dilation, shortening of spatial extension and mass increase at $v \to c$. That means [21, 28]

$$\begin{aligned} t &\to t(1+\beta) \\ x &\to x(1-\beta) \\ m &\to m(1+\beta) \end{aligned} \tag{27}$$

where β is a boost which needs not be specified at this point. Setting in Newton's kinetic energy one finds [21]

$$E = \frac{1}{2} m(v \to c)^2 (1+\beta)\left(\frac{1-\beta}{1+\beta}\right)^2. \tag{28}$$

Taking β to be Sigalotti's critical value $\beta = \phi$ [26, 27] one finds the same previous result

$$E_{QR} = \frac{\phi^5}{2} mc^2. \tag{29}$$

7. Theory

The analysis generalizing $E = mc^2$ of special relativity to quantum relativity [29, 30] i.e. effective quantum gravity formula $E_{QR} = (mc^2)/(22.1803989)$ consists of three main steps. The first is to transform space, time and mass to a probabilistic space, time and mass using quantum mechanics leading to $E_P = (P/2)mc^2$ where P is a quantum entanglement probability. Second, we devise a special form of $E_R = \gamma mc^2$ where γ is a function of a unit interval boost β. Third, we equate E_P to E_R and find the exact value of β for which E becomes a maximum.

7.1. Probabilistic Quantum Entanglement

In [19] Mermin gives unrivalled lucid derivations and interpretations of quantum non-locality and entanglement of two quantum particles relevant to the movement from a point 1 to a point 2. The probability P of the generic Hardy entanglement [19, 31] is given by equation (10) of [19] as

$$P = \frac{p_1 p_2 (1-p_1)(1-p_2)}{1-p_1 p_2} \tag{30}$$

For $p_1 = p_2 = d$ one finds

$$P = d^2\left(\frac{1-d}{1+d}\right). \tag{31}$$

Now we introduce the following probabilistic transformation [29, 30]

$$\begin{aligned} \text{Space } (X) &\longrightarrow xp \\ \text{Time } (T) &\longrightarrow tp \\ \text{Mass } (M) &\longrightarrow mp. \end{aligned} \tag{32}$$

Inserting into Newton's kinetic energy one finds the following probabilistic energy for $v \to c$

$$\begin{aligned} E_P &= \frac{1}{2} mp\left(\frac{xp}{tp}\right)^2 \\ &= \frac{mp}{2}(v \to c)^2. \end{aligned} \tag{33}$$

That means [12, 13]

$$E_P = \frac{1}{2} d^2 \left(\frac{1-d}{1+d}\right) mc^2. \tag{34}$$

7.2. Determining the Magnitude Probabilistic Quantum Entanglement d and the Relativistic β

The next step in our strategy to arrive at an effective quantum gravity E is to require that both E_P and E_R be equal. That means

$$E_P = E_R. \tag{35}$$

Therefore we have

$$\frac{mc^2}{2} d^2 \frac{1-d}{1+d} = \frac{mc^2}{2} \frac{(1-\beta)^2}{1+\beta}. \tag{36}$$

Clearly this is only possible for $d = \beta$ and inserting back in (36) one finds that

$$\beta^2 \frac{1-\beta}{1+\beta} = \frac{(1-\beta)^2}{1+\beta}. \tag{37}$$

This leads to a simple quadratic equation

$$\beta^2 + \beta - 1 = 0, \tag{38}$$

with the well known and rather expected solutions

$$\beta_1 = \phi,\ \beta_2 = -\frac{1}{\phi} \tag{39}$$

where $\phi = \frac{\sqrt{5}-1}{2}$ is the golden mean as in the work of Mermin [19] and Styer [31].

8. Discussion

For the last thirty years or so nonlinear dynamics became an indispensible tool for countless branches of engineering and applied sciences as well as mathematics [16]. By comparison high energy and quantum physics was slow to utilize the tremendous possibilities offered by deterministic chaos and fractal geometry [7, 16, 20, 32]. The situation changed radically in the last five years or so. In particular the success of resolving fundamental problems such as the mystery of dark energy and quantum entanglement is paving the way towards a reappraisal of many fundamental problems in theoretical physics and cosmology from the point of view of nonlinear dynamics, chaos and fractals [7, 16, 32, 33]. It is an accurate statement to claim that the word

Notion and the concept of self-similarity and self-affinity became indispensable tools of exact science only after the rise to prominence of non-linear dynamics, chaos and fractals some three decades ago [3, 35]. This is what made it possible to apply global analysis in conjunction with fractal geometry in relativistic quantum cosmology and discover that Newton's kinetic energy $E = \frac{1}{2}mv^2$ as well as Einstein's relativistic formula $E = mc^2$ and the new quantum relativistic energy-mass equivalence equation $E_{QR} ; mc^2/22$ are merely self-similar scaling of each other in the sense of modern nonlinear dynamical theories [28]. We could go even several steps further and realize that a fractal form of Legendre transformation leads us to recognize that the energy formula for dark energy is given directly by

$$E(Dark) = (\frac{1}{2}mc^2)(5\phi^2); mc^2 / \left(1 + \frac{1}{21}\right). \tag{40}$$

This obviously is the complementary energy of the ordinary energy

$$E(Ordinary) = (\frac{1}{2}mc^2)(\phi^3\phi^2); mc^2 / 22. \tag{41}$$

Adding both expressions we find that

$$E(Total) = E(Einstein) = mc^2. \tag{42}$$

We draw here attention to the T- duality and the unit interval physics behind these incredibly simple and elegant relations reconciling classical physics with relativity and quantum theory. It is remarkable that the same physics behind the very large meets at "infinity" with physics of the extremely small unifying high energy with cosmology and all via the magnificent concept and mathematics of renormalization. In turn this mathematics is nothing more than taming all singularities using fractal self-similarity [32, 33].

This conclusion has momentous ramifications going as far as showing the existence of negative gravity as well as explaining the fractal rationale behind the mystery of the constancy of the speed of light and negative absolute temperature [34].

9. Resolution of the Missing Hypothetical Dark Energy Using Scale Relativity and E-infinity

Scale relativity puts the running value of $\bar{\alpha}_0$ at 10^{16} GeV of scale relativity [3, 35, 36] for $\bar{\alpha}_{GUT}$ = 105. Clearly at $\bar{\alpha}_{GUT}$ we have everything except gravity. Scaling 105 logarithmically and squaring it gives us now a measure for the error in Einstein's special relativity energy mass resolution when applied at ultra high energy and distances.

That way we find the scaling exponent needed for $E = mc^2$, namely

$$\begin{aligned} \lambda &= \frac{1}{\left(\ln \bar{\alpha}_{GUT}\right)^2} = \frac{1}{(4.65396036)} \\ &= \frac{1}{21.65934694} \\ &= 0.04616944. \end{aligned} \tag{43}$$

Einstein's energy-mass equation now reads as follows:

$$E = \gamma mc^2 \tag{44}$$

where

$$\gamma = \frac{1}{21.65934694}. \tag{45}$$

The corresponding dark energy is therefore

$$\begin{aligned} E(Dark) &\cong \left(1 - \frac{1}{21.65934694}\right)(100) \\ &= 95.383\%. \end{aligned} \tag{46}$$

Before giving an exact interpretation for this approximate result let us first revise the numeric. The value which should have been used for $\bar{\alpha}_{GUT}$ is (10)(D_F^{11}) which means [7, 20]

$$\begin{aligned} \bar{\alpha}_{GUT} &= (10)\left(\frac{1}{\phi^5}\right) \\ &= (10)(11.09016995) \\ &= 110.9016995. \end{aligned} \tag{47}$$

Logarithmic scaling and squaring then leads to

$$\begin{aligned} \frac{1}{\lambda} &= (\ln 110.9016995)^2 \\ &= (4.70864419)^2 \\ &= 22.17133038 \\ &\approx 22. \end{aligned} \tag{48}$$

The result is almost the exact one, namely (22 + k) where k = 0.18033989 as we can show using exact methods. In other words, $\lambda \approx \frac{1}{22}$ is the reciprocal value of the non-visible "dark" dimension of our bosonic section of the transfinite version of heterotic string theory. That means for "dark" dimensions we have [15]

$$\begin{aligned} D(Dark) &= \text{The total number of the dimensions} - \text{space-time dimensions} \\ &= (26 + k) - (4) \\ &= 22 + k \\ &= 22.18033939. \end{aligned} \tag{49}$$

E-infinity scaling reaches the exact result without logarithmic scaling. Let us first recall that the entire heterotic superstrings dimensional hierarchy is readily found for $\bar{\alpha}_0$ for a Cooper pair as follows, starting from [7, 8, 20, 37, 38]

$$\left(\frac{\bar{\alpha}_0}{2}\right)(\phi)^n = (68.54101966)(\phi)^n \tag{50}$$

and setting n = 1, 2, 3 … one finds [7]

$$\begin{aligned} 42+2k &= 42.36067977 \\ 26+k &= 26.18033939 \\ 16+k &= 16.18033939 \\ 10 & \\ 6+k &= 6.18033939 \\ 4-k &= 3.819660122. \end{aligned} \tag{51}$$

Setting $X \pm k \approx X$ one finds the classical heterotic string dimensional hierarchy 26, 16, 10, 6 and 4. This was a down scaling of $\frac{\bar{\alpha}_0}{2}$. Now the up scaling leads to the following term $\left(\frac{\bar{\alpha}_0}{2}\right)\left(\frac{1}{\phi}\right)^n$. For n = 1 one finds [4]

$$\left(\frac{\bar{\alpha}_0}{2}\right)\left(\frac{1}{\phi}\right) = \left(11+\phi^5\right)(10) = 110.9016995 \tag{52}$$

$$= \bar{\alpha}_{GUT}.$$

Dividing through all the five interactions using the D_T = 5 one finds [7]

$$\frac{\bar{\alpha}_{GUT}}{5} = 22 + k$$
$$= 22.18033989. \tag{53}$$

This is of course the exact result and shows the high quality of accuracy in the Nottale method. Should we have used the fractal weight $5+\phi^3$ rather than 5 we would have found [7, 20]

$$\frac{\bar{\alpha}_{GUT}}{5+\phi^3} = 21+k \tag{54}$$
$$= 21.18033989.$$

In the first case we look at an Einstein 4 dimensional space-time with 22 + k "dark" dimensions while in the second case we have a 5 dimensional Klein-Kaluza space-time with only 21 + k "dark" dimensions. Based on this analysis our tangible space is exactly four dimensional topologically and $4+\phi^3$ = 4.23606799 Hausdorffly. However, it is the larger $11+\phi^5$ core of our space which encapsules the $4+\phi^3$ smaller core which decides on Hardy's quantum entanglement [16, 17] being exactly $\frac{1}{11+\phi^5} = \phi^5$ and also decides on the reduction factor or the scaling exponent $\lambda = 2\phi^5 = \frac{1}{22+k}$ of Einstein's equation $E = mc^2$. The scaled new quantum relativity or effective quantum gravity equation

$$E = \left(\frac{1}{2}\right)\left(\frac{1}{11+\phi^5}\right)\left(mc^2\right) = \frac{mc^2}{22+k} \tag{55}$$

predicts that we have a missing dark energy of exactly *E(Dark)* = 95.49150281%, almost the same as in the approximate scale relativity analysis following Nottale's theory. This reduction could be interpreted in a variety of intuitive ways which will be discussed in the Table 1.

10. Discussion

Following the picture adopted by heterotic string theory compactified on a Calabi-Yau manifold, every point in our space-time is joined to a Calabi-Yau 6 dimensional real manifold containing internal symmetry and compactified dimensions [11]. On this account we would have all in all (4)(6) = 24 dimensions and adding the string world sheet to it arrives at the 24 + 2 = 26 bosonic dimension. These dimensions move in the opposite direction of another 16 Fermionic dimensions from which one finds 26 - 16 = 10 super symmetric dimensions. However, in our transfinite version of heterotic strings we do not need the 2 dimensional world sheet to arrive at 26. This is because the Hausdorff-Besicovitch dimension of our core space is not 4 but 4 + ϕ^3 = 4.23606799 and the 6 dimensions of the Calabi-Yau manifold [11] are not 6 but 6 + k = 6.18033898. Consequently, the total dimension is given by

$$\begin{aligned} D_s(Heterotic) &= (4+\phi^3)(6+k) \\ &= 26+k \\ &= 26.18033989. \end{aligned} \tag{56}$$

Now Einstein's energy-mass equation was based on a mere 4 dimensional flat non-fractal, non-fuzzy Euclidean manifold. Subtracting these 4 dimensions from D_s = 26 + k we are left with $26 + k - 4 = 22 + k$ hidden dimensions.

This is a wonderfully simple and intuitive picture and is numerically identical with our analysis which was based on superficially completely different theories such as Nottale's scale relativity [3, 35, 36, 38] or E-infinity theory [7, 17, 20, 35]. It is now clear that $E = mc^2$ must be scaled using

$$\lambda = \gamma = \frac{1}{22+k} = 0.0450849718 \tag{57}$$

which fully agrees with the measurement of WMAP and supernova analysis by predicting that exactly 95.4915028% of the energy of the cosmos must be dark energy [2, 4].

***Table 1**. Results of some of the various theories applied to dark energy by El Naschie*

Theory	**Mass-energy equation**	**Remarks**
General relativity plus holographic boundary	$E=\left(\frac{R^{(4)}-D^{(4)}}{SL(2,7)+\left(R^{(4)}-D^{(4)}\right)}\right)mc^2$ $=\frac{(20-4)}{336+(20-4)}mc^2$ $=\frac{1}{22}mc^2$	$R^{(4)}$ is the number of independent components of the Riemannian tensor in $D = 4$ or the degrees of freedom of pure gravity in $D = 8$ thus we have $R^{(n)}=\frac{n^2\left(n^2-1\right)}{12}$ $=\frac{4^2\left(4^2-1\right)}{12}=20$
General relativity plus *6D* Calabi-Yau manifold	$E=\frac{1}{\left(R^{(4)}+D^{(6)}\right)-D^{(4)}}mc^2$ $=\frac{1}{(20+6)-4}mc^2$ $=\frac{1}{22}mc^2$	Calabi-Yau manifold has 6 real dimensions and is used as *K3* Kähler in superstring theories. By contrast *K3* Kähler has 4 dimensions only but they are complex dimensions, not real.
Special relativity in a hyper *4D* J. Huan He - Hilbert cube given by $D=4+\cfrac{1}{4+\cfrac{1}{4.......}}=4+\phi^3=4.2360679$	$E=\frac{1}{2}\frac{(1-\phi)^2}{(1+\phi)}mc^2=\frac{\phi^5}{2}mc^2$	We introduced on light cone speed $\frac{(1-\phi)}{(1+\phi)}$ as well as a light cone mass $m(1+\phi)$ and utilize Hardy's quantum entanglement.
Nottale's scale relativity	$E=\left(\frac{1}{\ln\bar{\alpha}_{GUT}}\right)mc^2$ where $\bar{\alpha}_{GUT}\cong 110$ is the inverse coupling constant of grand unification of all non-gravitational forces. Thus $E=\frac{mc^2}{(4.70042)^2}$ $=\frac{1}{22.09}mc^2\approx\frac{1}{22}mc^2$	Scaling as a gauge theory is an idea due to Herman Weyl. This idea leads to physical contradiction unless space-time is a fractal devoid of any natural scale such as all non-Archimedean geometrical and P-Adic theories .

11. Conclusion

The homology of *K3* Kähler and what El Naschie calls extra "dark" dimensions is the definite cause behind what we call the missing dark energy [4]. To arrive at the correct quantitative result and reconcile theory with experiments we need to scale the classical $E=mc^2$ by a scale relativity factor λ_{QR} defined as the ratio of two second Betti numbers [10, 11]**.** Since the Betti number of fuzzy Kähler b_2 is 22 + k and since b_2 = 1 for Einstein space of special relativity our λ_{QR} becomes equal to $1/(22+k)$ and one finds $E_{QR}=\lambda_{QR}(mc^2)$ [9]**.** This means the so-called missing dark energy of the cosmos is exactly equal to $(1-\lambda_{QR})(100)$ = 95.4915028%. It is almost surreal how close the results of cosmic measurement are to this percentage [4]. Noting that λ_{QR} may also be written as $\phi^5/2$ that means half Hardy's quantum entanglement probability found using orthodox quantum mechanics and confirmed through sophisticated quantum information experiments, we feel that the ordinary sharp non-fuzzy *K3* Kähler manifold approximates quantum gravity space-time geometry and topology to an astonishing extend and must be real.

Scale relativity gives yet another very constructive mental picture to understand what the missing dark energy means apart of answering the quantum question quantitatively with remarkable accuracy. Scale relativity is completely embedded in the scale invariance of fractal geometry [3, 7, 17, 20, 35, 36]. We do not need to go from general relativity via quantum mechanics to arrive at quantum gravity. We could do the same by starting with special relativity however after freeing it from traditional prejudice and putting it in the right space-time setting, namely fractal geometry.

El Naschie's has checked the results using at least 10 different theories including Nottale's scale relativity, Magueijo and Smolin's varying speed of light theory, Witten's M-theory, Veneziano's dual resonance theory and quantum field Yang-Mills theory in curved space-time and obtained exactly the same result reported [21]. Table 1 gives an overview of some of these results. With that we feel quite confident that the mystery of the dark energy has been solved at least in principle by Mohamed El Naschie and that it is essentially not a mystery any more.

References

[1] R. Penrose, The Road to Reality. Jonathan Cape: London, 2004.

[2] Y. Baryshev and P. Teerikorpi, Discovery of Cosmic Fractals. World Scientific: Singapore, 2011.

[3] L. Nottale, Scale Relativity. Imperial College Press: London, 2011.

[4] L. Amendola and S. Tsujikawa, Dark Energy, Theory and Observations. Cambridge University Press: Cambridge, 2010.

[5] J. Mageuijo and L. Smolin, "Lorentz invariance with an invariant energy scale," arXiv: hep-th/0112090V2, 18 December, 2001.

[6] J. Mageuijo, Faster than the Speed of Light. William Heinemann: London, 2003.

[7] M.S. El Naschie, "The theory of Cantorian space-time and high energy particle physics," (An informal review), Chaos, Solitons & Fractals, 41, 2009, pp. 2635-2646.

[8] M.S. El Naschie, "The discrete charm of certain eleven dimensional space-time theory," Int. J. Nonlinear Sci. & Num Simulation, 7(4), 2006, pp. 477-481.

[9] C. Nash and S. Sen, Topology and Geometry for Physicists. Academic Press: San Diego, 1983.

[10] D. Joyce, Compact Manifolds with Special Holonomy. Oxford Press: Oxford, 2003.

[11] S. Yau and S. Nadis, The Shape of Inner Space. Perseus Book Group: New York, 2010.

[12] J. Polchinski, String Theory, Vol. I and II. Cambridge University Press: Cambridge, 1999.

[13] M.S. El Naschie, "On a class of fuzzy Kähler-like manifolds," Chaos, Solitons & Fractals, 26, 2005, pp. 477-481.

[14] M.S. El Naschie, "E-Infinity-Some recent results and new interpretations," Chaos, Solitons & Fractals, 29, 2006, pp. 845-853.

[15] C. Rovelli, Quantum Gravity. Cambridge Press: Cambridge, 2004.

[16] M.S. El Naschie, "Quantum entanglement as a consequence of a Cantorian micro space-time geometry," J. of Quantum Info. Sci., Vol. 1, 2011, pp. 50-53.

[17] Ji-Huan He, L. Marek-Crnjac, M. A. Helal, S. I. Nada and O. E. Rössler, "Quantum golden mean entanglement test as the signature of the fractality of micro space-time," Nonlinear Sci. Lett B, 1(2), 2011, pp. 45-50.

[18] L. Hardy, "Non-locality of two particles without inequalities for almost all entangled states," Phys. Rev. Lett., 71(11), 1993, pp. 1665-1668.

[19] D. Mermin, "Quantum mysteries refined," American J. Phys., 62, (10), 1994, pp. 880-887.

[20] M. S. El Naschie, "A review of E-infinity and the mass spectrum of high energy particle physics," Chaos, Solitons & Fractals, 19, 2004, pp. 209-236.

[21] M. S. El Naschie and L. Marek-Crnjac, "Deriving the exact percentage of dark energy using a Transfinite version of Nottale's scale relativity," International Journal of Modern Nonlinear Theory and Application, Vol 1, No 4, 2012, pp. 118-124.

[22] E. J. Copeland, M. Sami, Shinji Tsujikawa, "Dynamics of dark energy, " arXiv: hep-th/0603057V3, 2006.

[23] G. Ord, M. S. El Naschie and Ji-Huan He (editors), "Fractal Space-time and Non Commutative Geometry in High Energy Physics," A new Journal by Asian Academic Publishing Ltd, Hong Kong, China, Vol. 2, No. 1, 2012, pp. 1-79.

[24] M. S. El Naschie, "A unified Newtonian-relativistic quantum resolution of the supposedly missing dark energy of the cosmos and the constancy of the speed of light," International Journal of Modern Nonlinear Theory and Application, Vol 2, No1, 2013, pp. 43-54.

[25] R. Elwes, "Ultimate L," The New Scientist, July 30, 2011, pp. 30-33.

[26] S. Hendi and M. Sharifzadeh, "Special relativity and the golden mean," J. of Theor. Phys., 1, 2012, pp. 37-45.

[27] E. Wit and J. McClure, Statistics for Microarrays: Design, Analysis, and Inference, 5th Edition. John Wiley & Sons Ltd.: Chichester, 2004.

[28] L. Sigalotti, A. Mejias, "The golden mean in special relativity," Chaos, Solitons & Fractals, 30, 2006, pp. 521-524.

[29] W. Rindler, Relativity. Oxford University Press: Oxford, (in particular pp. 111, 112), 2001,

[30] W. Rindler, "General Relativity before Special Relativity: An unconventional overview of Relativity Theory," American Journal of Physics, 62(10), October 1994.

[31] D. F. Styer, The Strange World of Quantum Mechanics. Cambridge University Press: Cambridge, (in particular pp. 54-55), 2000.

[32] M. S. El Naschie, "Elementary prerequisites for E-Infinity," Chaos, Solitons & Fractals, Vol. 30, 2006, pp. 579-605.

[33] J. P. Gollub and P. C. Hohenberg, "Summary session," Phys. Scr. T9, 1985, pp. 209-216.

[34] J.-Ping Hsu and Leonardo Hsu, A Broader view of Relativity, 2nd edition.World Scientific: Singapore, 2006.

[35] M. S. El Naschie, L. Nottale, S. Al Athel and G. Ord (Editors), "Fractal Space -Time and Cantorian Geometry in Quantum Mechanics", Special Issue, Chaos, Solitons & Fractals, 7(6), 1996.

[36] L. Nottale, Fractal Space-Time and Micro Physics. World Scientific: Singapore, 1993.

[37] A. Connes, Non-commutative geometry. Academic Press: San Diego, 1994.

[38] M. S. El Naschie, "Transfinite harmonization by taking the dissonance out of the quantum field symphony", Chaos, Solitons & Fractals, 36, 2008, pp. 781-786.