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The Mechanical Invariance Factor in Musical Acoustics and Perception (Revisited)

Akpan Jimmy Essien

Member of the Acoustical Society of Nigeria, University of Nigeria, Nsukka, Nigeria

Email Address:
ajessien@ajessien.com

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Abstract: Mechanical, acoustical, and neurophysiological investigations in music, acoustics, and auditory perception repose on the Pythagorean string ratio theory of musical pitch intervals (6^{th} century B.C). Recently, the mechanical validity of the string ratio theory and its psychological import have been challenged and denied on grounds of invariance. In this regard, Essien (2014) demonstrated experimentally that, contrary to established tradition in physics of sound, the tension of a string is not constant when string length is modified even though the balanced-force exerted on the string is held constant. The data revealed the existence of two sources of force in a vibrating string: (1) The oppositely-directed force applied externally to the string (labelled *F*ex); (2) The force that is the intrinsic property of the string (labelled *F*in). The latter is the missing parameter in Pythagorean auditory psychophysics. The omission lured researchers into acoustics and neurophysiology of pitch without an invariant physical correlate of pitch. Essien's (2014) data showed that all transformations to string length or the balanced-force exerted on a string are various ways to modify the string's resistance to deformation. Thus, the force in a string varies inversely with string length even though *F*ex is held constant. In the present paper, string length is shown to have very little or no effect at all on a string's vibrational frequency and subjective pitch. Because psychoacoustic theories of hearing are founded on the string ratio theory, the data not only offer the missing psychological element that deprived the string ratio theory of a scientific status, but also refute both Ohm's acoustical law (1843) and Helmholtz's resonance theory (1877). The force in a string is portrayed as the mechanical parameter in control of pitch regardless of vibrational frequency or spectral structure. Implications for future research in musical acoustics and auditory perception are discussed.

Keywords: Invariance, String Tension, String Ratios, Pitch Perception, Mechanics of Spectral Change

1. Introduction

Acoustic cues for perception "represent a narrow-minded way of thinking which leads us into a blind alley when faced with the problem of discovering the real nature of the auditory mechanism." [1].

The Pythagorean string ratio theory is the pivot around which revolve all work in hearing [2; 3]. In this regard, Stephen & Bates [4]: noted: "Pythagoras' discovery, instead of acting as a stimulus for further experiments became the basis of fantastic philosophical and mathematical speculations such as the famous "harmony of spheres", and so for another 2000 years, sound was mainly involved in a semi-mystical arithmetic of music." The situation has not changed today. After all, Ohm's Acoustical Law [5], and Helmholtz's Resonance/Place Theory [6], both reflecting perception by ratios of stimulus magnitude, are the acoustical and neurophysiological facets of the string ratio theory. The difficulties encountered in the search for acoustic cues to perception, whether in music or speech, have led many investigators to label the objective "elusive" [7] or "ambitious" [8]. The ecological approach is presumably promising [9, 10, 11, 12]; and recent researchers, following in the footsteps of Pythagoras, have conducted mechanically-based investigations into pitch perception [13, 14, 15, 16, 17]. However, rather than seek out the *invariant* property of the sound source that underlies subjective pitch, modern investigators focus the effects of individual mechanical parameters of the sound source on frequency of vibration, or the relationship between the vibratory modes of the entire sound source and pitch, all as though vibrational frequency were invariant with pitch. [13, 14, 15, 16, 17, 18, 19, 20]. By neglecting the significance of invariance in auditory psychophysics, current ecological procedures, like psychoacoustics, have deviated widely from the principle of auditory psychophysics [21]. Therefore, hearing research cannot attain the status of a science in the absence of invariance in the formulation of its foundation [22]. In this regard, Essien [23] pointed out the existence of a force that is the inherent property of a string (hereafter *F*in) outside the oppositely-directed force that is applied externally to a string (hereafter *F*ex). The former was shown to vary inversely with string length, such that the force in the string, in reality, is not constant but varies with pitch even though *F*ex (called tension in traditional string mechanics) is held constant. This missing mechanical property of a string deprived Pythagorean psychophysics of a mechanical invariant parameter in pitch production; it lured hearing research into premature acoustic studies of music in the absence of an invariant physical correlate of subjective pitch. This paper will present four experimental demonstrations that undercut current theories and practices in auditory perception research: (1) The missing parameter in the mechanics of pitch production; (2), That the length of a string has very little or no effect at all on a string's vibrational frequency or subjective pitch; (3) That the force in a string is not constant when string length is modified even though the balanced-force exerted on the string is held constant; and (4) That the force in a string is in control of subjective pitch regardless of a string's vibrational frequency, spectral structure, and change. Implications for future research in pitch production, musical acoustics, and perception are discussed.

2. Mechanics of a String: The Missing Parameter

In prehistoric physics, the tension of a string is the balanced-force exerted on the string. That this concept of tension in strings is *prehistoric* is evident from Helmholtz's report [6] that Pythagoras might have acquired this knowledge from Egyptian priests, yet no one knows when it was established in the remote antiquity (p. 1). Today, the tension of a string is quantified in terms of the oppositely-directed force acting on a string regardless of the physical dimensions (or other properties) of the string. According to this provision, musical pitch (and pitch intervals) is said to be controlled by sub-lengths of a string. Consequently, doubling the length of a string halves the vibrational frequency of a string and reduces the pitch by one Octave; or, halving the length of a string doubles the string's vibrational frequency and increases pitch by one Octave, and so forth. To this day, upon this tenet of the string ratio theory hang Ohm's Law, Helmholtz's Resonance/Place Theory, and all facets of psychoacoustic and neurophysiological theories of hearing. Abundant experimental evidences testify to the falsity of both Ohm's and Helmholtz's theories. Because both theories were founded on Pythagorean string ratios, a question arises: Since Ohm and Helmholtz got it wrong, was Pythagoras right? To address this fundamental question, we need to revisit the mechanics of a string to identify the missing factor that undermined the scientificity of Pythagorean string ratios, Ohm's, and Helmholtz's theories of pitch.

In traditional physics of sound, outside differences in the material of which a string is made, a string is known to have two primary physical dimensions, i.e. length, and density. The third parameter—tension—is induced by the force applied to the string from outside the string itself. Therefore, the tension of a string is identified as the balanced-force exerted on a string; it is purportedly constant as long as the balanced-force is constant, and it is quantified in terms of the size of the force, regardless of the physical dimensions (length and density) of the string. To appreciate the error of this conception of string tension, let us consider the behaviours of strings under a constant force.

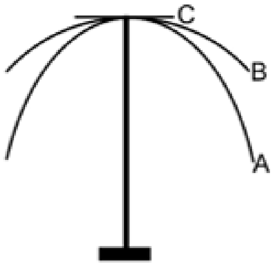


Figure 1. *The Resistance of Strings to Deformation.*

Figure 1 presents three strings A, B, and C of the same material and density, the only difference is in their lengths. They are balanced on a fulcrum, and are affected by the same force of gravity which attempts to deform them. The degree of deformation is indicative of each string's inherent force of resistance to the downward pull. Two behaviours are observable: (1) The longest string A is the weakest, and this weakness is manifest in the degree of deformation caused by the force of gravity. (2) The shorter the string the higher is its resistance to the downward pull. If we were to help each of the three strings against the downward pull, the amount of external force required would decrease the shorter the string. In fact, the shortest string C would not require any force at all from outside the string because it is strong enough (sufficiently stiff or sufficiently tense?) to resist the deformation without any external assistance in the form of the so-called tension. Thus, by way of compensation, we can circumvent the need for oppositely-directed force by simply cutting the string shorter, and shorter, and yet shorter. At each cutting, the string emerges stronger. The only source of the force of resistance is the string itself since it does not benefit from any source of force from outside it.

The above observations may be made from a different standpoint. For example, rather than assist the strings to resist the deformation, we might view them in terms of the size of force that would bend each of them. Judging by the responses of the strings, we would require less force to bend the longest string A; the force required to bend these strings would vary inversely with length of string. Thus, if we consider the force required by string A, for example, to resist the deformation, it would be high; but if we consider the force required to bend it, the force would be low. Either way, we would attain the same conclusion that the strings in figure 1 possess inherent force that varies in size as a function of the string's physical and mechanical properties.

The above observations lead to the following deduction: Whether the force in a string is supplied externally to the string, or whether it is an inherent property of the string, it restores the string to its original configuration following a deformation [24, 25, 26]. The balanced-force augments the force that is the inherent property of the string. Therefore, transformations to string length or the balanced-force are various ways to modify the force in the string. Because the force in a string has always been expressed in terms of the balanced-force exerted on the body [27, 28, 29, 30], the actual force in a string does not seem to have ever been measured in the history of hearing research. The attempt made below aims to quantify the force arising from transformations to string length even though the size of the balanced-force is held constant. Furthermore, we will examine the impact of the force on the string's vibrational frequency, spectral structure, and subjective pitch.

3. Experiment

3.1. Equipment and Method

The equipment used in this study was the sonometer illustrated in figure 2. Figure 2(a) presents the complete acoustic system. The most conspicuous component is the resonator. Its dimensions in mm are L1,465, W280, and H100. The system was equipped with three strings in three different gauges (or densities) i.e. B130, E105, and A85, from a *Rotosound* 45-5 set, long scale, round-wound nickel on steel. The maximum length of string was 1,100 mm, of which the maximum effective length was 870 mm. The balanced-force exerted on each string was adjustable via the tuning head such as a guitar has, as shown in figure 2(b). Unlike a guitar, though, the apparatus was equipped with three spring balances (*Super Samson*, 25 kg) shown in figure 2(c), a spring balance for each string. Thus, the size of the balanced-force which induces tension in each string could be measured in kg. A fixed bridge shown in figure 2(d) served all three strings. Besides, there were three mobile bridges, one for each string, for adjusting the effective length of each string independently of other strings. A ruler fitted along the edge of the resonator was graduated in mm; it allowed for reading off the effective length of string at the position of each mobile bridge. The device was equipped with a pick-up mechanism to capture the vibrations of the strings. A sound-recording equipment was plucked into the pick-up socket (shown in figure 2(d)) to capture the signals produced by each string. Thus, the acoustic system places string length, string density, the balanced-force acting on each string, and the signals produced, under the control of the experimenter. The set-up facilitated, not only observations of interactions between different mechanical parameters of each string and across strings, but also convertibility of one parameter of each string to another, and across strings. The present experiment focuses the relationship between string length and pitch, and the convertibility of the *inverse* relationship between string length and pitch to a *direct* relationship with pitch [23]. In this experiment, only the A85 string, with the effective length set to 860 mm, was used.

Two informants (TH, a male, and YI, a female) took part in this experiment. Only the performances of the female informant will be discussed here for reasons given later. The female informant was an absolute pitch, professional classical violinist, who is identified in this work as YI. She was taught music from infancy, and had practised the art for 23 years at the time of the experiment. She held a Master's degree from the Royal Academy of Music, London, and was teacher of music (classical violin) at Pimlico Academy, London, UK. The task was formulated as follows:

Given the length of the full string as 860 mm,

1. Adjust the balanced-force until the string sounds the musical tone A3;
2. Reduce string length to 645 mm, and tune it to sound the musical tone A3;
3. Reduce string length to 430 mm; and tune it to sound the musical tone A3;
4. Reduce string length to 215 mm; and tune it to sound the musical tone A3;
5. Reduce string length to 107 mm; and tune it to sound the musical tone A3;
6. Reduce string length to 53 mm; and tune it to sound the musical tone A3;

As the above sequence of steps in the performance of this task shows, the experiment involves sub-lengths of the same string. Nevertheless, it is readily noticeable that it does not aim to explain pitch intervals (which is arrived at through the perception of the pitches of *two* different stimuli). Rather, the experiment addresses the perception of pitch *per se*. Because the full string and its sub-lengths are tuned to the *same* subjective pitch, the experiment operates on the platform of invariance, seeking to detect, from among the different mechanical configurations of the string, the parameter of the sound source, or of the physical manifestations of the signal, that remains inseparably tied to pitch; or as Fechner [21] expressed it (p.47), “as effectively as the length of the yard is tied to the material of the yard-stick.” In simple, psychophysical terms, the relationship is *invariant*, because one cannot change a parameter in the relationship without changing the other, or hold one parameter in the relationship constant and yet change the other. It is only the detection of a mechanical parameter that maintains such a relationship with pitch that can demonstrate the link between the sound source and the sensation pitch, leading, hopefully, to clearer insights on the principle of the auditory mechanism in musical pitch perception. The six signals produced by each of the six string configurations were recorded on tape using an AIWA hi-fi cassette deck.

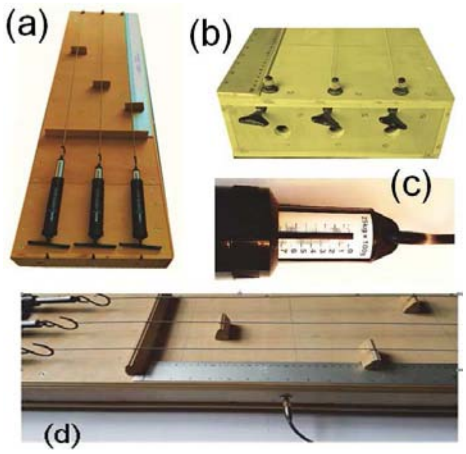


Figure 2. *The Multichord String Tuner. (a) The Complete Acoustic System. (b) Tension Heads for Adjusting the Size of Balanced-Force Exerted on Individual Strings. (c) Spring Balance for Recording the Size of Force Exerted on a String. (d) Fixed, and Mobile Bridges; Ruler; and Pick-up Socket for Sound-Recording.*

3.2. Results

One major shortfall of traditional mechanical approach to hearing was the premature passage from mechanical parameters in pitch production to acoustics, and subsequently to neurophysiology of pitch perception. It was *premature* because Pythagorean psychophysics did not provide the *invariant* mechanical parameter in pitch control. All mechanical parameters of a sound source—in the present case a string—are variants that may be compensated for by another mechanical property of the string. Thus, if the balanced-force, for example, is held constant as did Pythagoras, string length (or density) may be used to modify pitch. And if string length is held constant, the balanced-force is also a potential candidate for generating different pitches on the *same* string length. The concept of invariance, which seems to have been misconstrued in auditory psychophysics, argues against the founding of a theory of auditory perception on any parameter, whether acoustic or mechanical, that may be compensated for [31]. Thus, over the past 25 centuries since Pythagoras, hearing sciences have had their share of controversies that arise from this premature passage from mechanics to acoustics without mechanical invariance. To address and highlight the prematurity—indeed a fundamental error in hearing research—the following results of the above experiment will be examined in two stages: (1) Mechanical compensations between string length and the balanced-force. (2) The impact of the mechanical parameter in pitch control on the acoustic manifestations of the signals.

3.2.1. Mechanical Compensations

Mechanical compensations between string length and the balanced-force was addressed by pitch-matching across sub-lengths of string. For the experiment, test subject TH brought in his own guitar to serve as an anchor for tuning sub-lengths of the experimental string A85 to the required musical pitch A3. Nevertheless, he could not do more than the first four steps in the above task; he found it difficult to match the pitches as the string got shorter and shorter, probably due to confusion between timbre and pitch. However, the results of what he could do were in perfect agreement with those of the absolute pitch test subject YI. For the present investigation which aims primarily at the principle or law rather than inter-subject variability, only the performances of the absolute pitch test subject YI will be reported and commented on in the present paper.

The effective length of the full string (as stated in the test instructions) was 860 mm. The oppositely-directed force applied to this string to produce pitch A3 (or A220) was 22 kg. When string length was reduced to 645 mm, the pitch rose. For the 645-mm string to produce the same subjective pitch A3 as the full string, the oppositely-directed force was reduced from 22 kg to 13.2 kg. By following the 6 steps outlined in the task above, the sizes of balanced-force needed for the full string (860 mm) and its sub-lengths (645, 430, 215, 107, and 53 mm) to sound the same pitch A3 were 22 kg; 13.2 kg; 6 kg; 2 kg, 1.6 kg, and 0 kg. respectively. Thus, the sizes of force displaced for decreasing string sub-lengths to sound the same pitch as the full string were 8.8 kg; 16 kg; 20 kg; 20.4 kg and 22 kg. Let us analyse these figures with a theory of pitch production and perception in mind.

It is necessary to refine terminology and bring it in line with the findings of our investigations. We note distinction between the force exerted externally on a string and the force that is the inherent property of the string. Henceforth, to preclude terminological ambiguity, we shall refer to the oppositely-directed force applied externally to the string as *F*ex; and the force which is the inherent property of the string as *F*in. When length of string is shortened, or when size of force is reduced, the reduction is here considered as length of string or size of force *displaced*.

Figure 3 is a summary of compensations between string length and the balanced-force exerted on the string. There are four elements involved: (1) The auditory signal (A3) which is a constant. (2) The balanced-force acting on the string (*F*ex). (3) The force that is the inherent property of the string (*F*in), and (4) String Length. (String density is not considered here since it is constant across sub-lengths). The 22-kg force is the reference point for monitoring changes in the size of force in the string. As noted earlier, the full string (860 mm) sounded the tone A_3 at 22 kg. When string length was reduced to 645 mm, pitch rose. The pitch rise is of no interest to the present work which aims for the mechanical property of the string that generates the target musical pitch A_3. For the string to sound the same pitch A_3, *F*ex was reduced to 13.2 kg. The size of *F*ex displaced (8.8 kg) is equivalent to the increment in *F*in because of the length of string displaced (215 mm). The higher pitch following the reduction of string length to 645 mm is attributed to increased *F*in, which had to be displaced for the relatively shorter string to sound the same pitch as the full string. When string length was further reduced to 430 mm, pitch rose. Again, 7.2 kg of *F*ex was further displaced for the 430 mm sub-length to sound the tone A_3. Thus, a total of 16 kg of *F*ex was displaced for the 430-mm long string (half of the full string) to sound the same musical tone A_3 as the full string. In this way, *F*in is shown to increase with decreasing string length, resulting in decreasing *F*ex to produce the same subjective pitch on relatively shorter sub-lengths of string. The data show also diminishing size of Fex displaced as more string length is displaced. As noted earlier, the shortest sub-length measuring 53 mm required no *F*ex at all to produce the same pitch as its longer counterparts. In other words, as string length reduces and *F*in rises, the force that is the intrinsic property of the 53 mm long sub-length is sufficient for the string to generate the target pitch. Therefore, this sub-length does not call for any supplement (*F*ex) from outside the string (compare figure 1).

The above data are based on pitch rather than pitch intervals. From the standpoint of the string ratio theory of pitch intervals, the pull force in the string here under consideration would be constant, fixed at 22 kg for the full string and for all the sub-lengths. In contrast, the above data show that the force the string acquires because of transformations to its length is tremendous, indeed. By halving an 860-mm long string, it acquires 16 kg of force, which is 72.72727% of the balanced-force exerted on the string. In accordance with the focus of this work on invariance, we will not dwell on ratios and statistics here. Nevertheless, it is worth remarking that this fact is diametrically opposed to the constant-tension hypothesis according to which the force in the string is constant as long as *F*ex is held constant. Some sources acknowledge *slight variations in tension when the string is in vibration*. However, the observed changes are far from *slight*. This experiment shows that the tension of a string is jointly determined by the force that is applied externally to the string (*F*ex), and the force that is the inherent property of the string (*F*in). Also, the two sources of force are shown to be complementary and convertible. The experiment offers also insight into the parameter that seems to control pitch. In this regard, the results show that when string length is shortened, *F*in increases; the increased *F*in accounts for the pitch rise although *F*ex is held constant. For sub-lengths of string to sound the same pitch as the full string, the increased force arising from increased *F*in, which in turn arises from reduction in string length, must be displaced. Implications of these findings on the construction and features of musical instruments help us appreciate the art of the music instrument-maker. This aspect of the research is developed in detail elsewhere [31]. They provide solid evidence that the tension of a string comprises two sources of force, i.e., the force that is applied externally to the string (*F*ex), and the force that is the inherent property of the string (*F*in). The latter complements the former and compensates for it when it is held constant while string length undergoes transformations.

We know that the balanced-force exerted on a string is the only mechanical property of a vibrating string that maintains a *direct* relationship with pitch. The convertibility of string length to force converts the *inverse* relationship between string length and pitch to a *direct* relationship. The data show that string length *L*, and the oppositely-directed force impressed on the string *F*ex, are two different forms of the same thing, or two different ways to the same goal. Hence, the one can compensate for the other. Thus, we can comfortably label both as different sources of *force*. Mechanically, then, all transformations to the balanced-force exerted on a string, or to the length of a string, are various ways to modify the force in a string. Whether a balanced-force is applied to a string or not, we cannot change the length of a string and yet hold the force in the string constant (see figure 1). This finding refutes prehistoric concept of tension in strings as being exclusively determined by the balanced-force exerted on the string. If proponents of prehistoric physics insist on equating the force in a string with the balanced-force exerted on a string, the choice might serve a useful purpose in physics, but it would be detrimental to auditory psychophysics where there is the need to establish an unchanging relationship between the magnitude of the physical stimulus and the magnitude of sensation. In this regard, terminology requires some refinement to meet the needs of each science. Since the force in a string is shown to control pitch (regardless of its source), the following examination of the acoustic characteristics of the signals aims to highlight the impact of string length and force on vibrational frequency, spectral structure, and subjective pitch of signals.

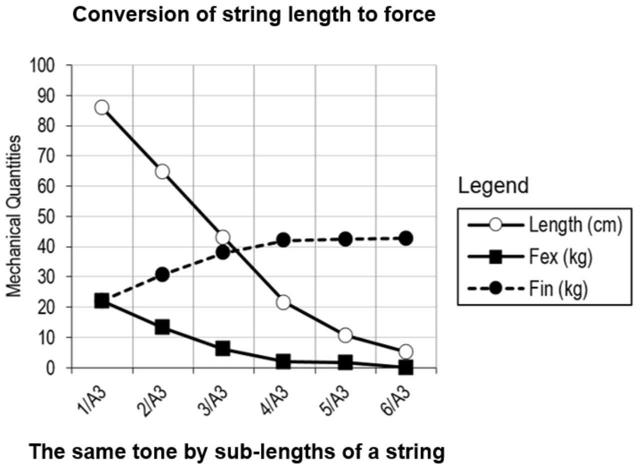


Figure 3. *Conversion of String Length to Force. As Length of String Decreases, the Force that is the Inherent Property of the String (Fin) Increases. Therefore, String Tension is the Joint Product of Fin and Fex. Thus, the Force in a String, Which Determines the Tension of the String, is not Constant When String Length is Modified Although the Force Applied Externally to the String (Fex) is Held Constant.*

3.2.2. Acoustic Analysis

The signal analysing software *Sigview* was used for the spectral analyses reported in this study. It is necessary to go through some samples of signal analysis for better acquaintance with terminology and clearer understanding of the summary of results. Consider, for example, the two signals in figure 4. The signal (bottom) is a portion extracted from the signal which was produced by the full string (860-mm long); the complete signal is 12.2 secs in duration. The signal (top), which was produced by the 53-mm long sub-length of the same string is 0.3 secs long. Nevertheless, these signals elicit the perception of the same subjective musical pitch A_3. Let us consider the spectral characteristics of the two signals.

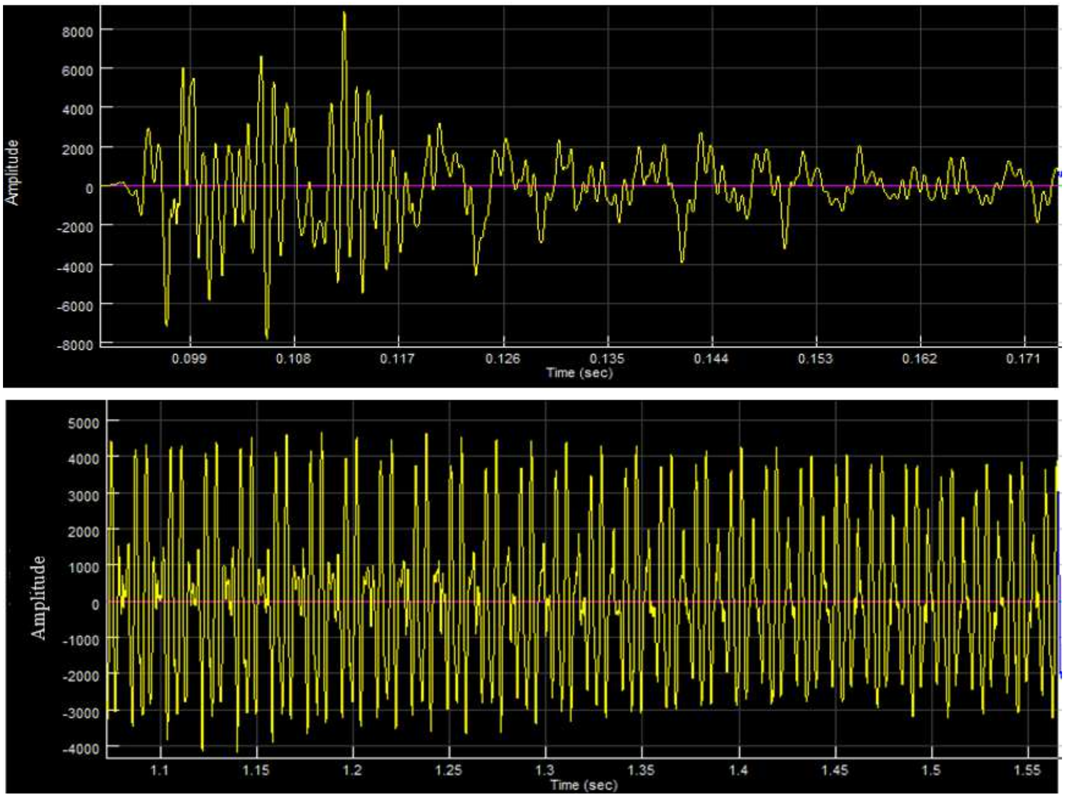


Figure 4. *The Impact of Mechanical Properties of a String on the Waveform of Naturally-Produced Stimuli. The Two Signals Above were Produced by Different Mechanical Configurations of the Same String; They Generate the Same Subjective Musical Pitch A3.*

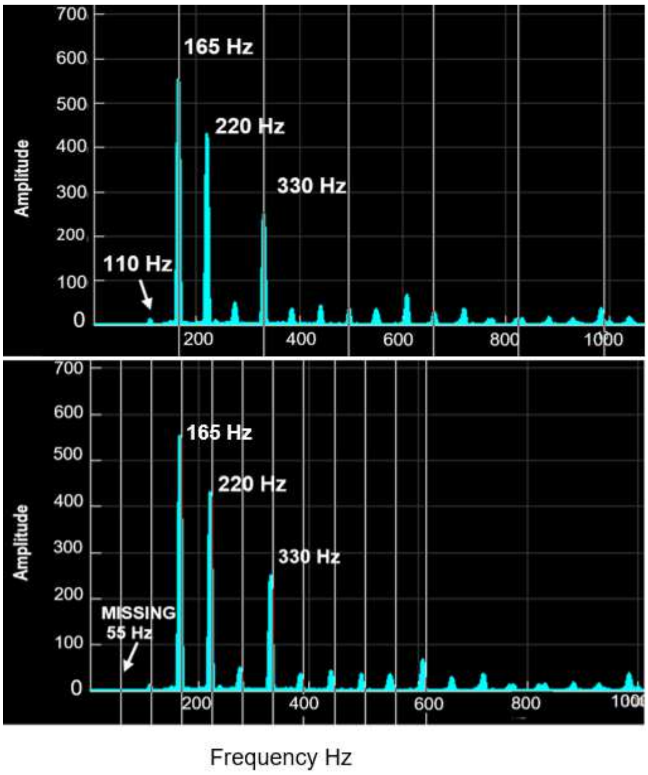


Figure 5. *Illustration of the Missing Fundamental in a Naturally Produced String Signal. In the Spectrum (top), the Resonant Component is the 165 Hz Partial, When the Harmonic Ruler is Placed at This Point, it Picks up Every Third Partial in the Spectrum. If Placed at the 110 Hz Component Which Has no Energy, the Ruler Picks up Every Second Partial. When Placed at the Point Marked 'MISSING 55 Hz' in the Bottom Image, the Ruler Picks up Every Partial, Showing That the Spectrum is Perfectly Harmonic. However, the Fundamental (55 Hz.) is Missing Along with the first Harmonic at 110 Hz. Thus, the Imposing 165 Hz. Component is not the Fundamental, but the Third Harmonic Component of This Signal.*

The vibrational frequency of the signal (bottom) as obtained by manual peak-to-peak estimation was 55.55 Hz. The result of FFT spectrum analysis of the same signal is presented in figure 5 (top). The 165 Hz. component with the highest intensity at 45.75 dB is considered as the resonant frequency of the signal, but it is *not* the fundamental because the other components are not integral multiples of the 165 Hz. partial. Therefore, if the harmonic ruler is placed at the 165 Hz. component as shown (top image), a harmonic marker falls on every third component of the spectrum, thus rendering the entire structure non-harmonic. If, however, the ruler is set further to the left as shown (bottom image), all the partials are in perfect harmonic arrangement. The component that makes this harmonic arrangement possible is the 55 Hz. component labelled 'MISSING'. Therefore, the fundamental for this sound is not the imposing 165 Hz partial, but the 55 Hz. component which, ironically, is missing from the spectral structure of the sound. Interestingly, the signal has a 220 Hz component which is neither the resonant frequency of the signal nor the fundamental. Later, we shall examine the contribution (if any) of these components to the perception of this signal as an A3 (or A220 Hz.) musical tone.

Figure 6 is the spectrum of the signal produced by the 53-mm long string. In contrast to the spectrum of the signal produced by the 860-mm string considered earlier above, this signal manifests the absence of periodic vibrations; it defies a fundamental acoustic description of musical sounds. To examine the components more closely, the presentation of the spectrum as histogram highlights one dominant peak at 164 Hz. The 220 Hz. component is an inconspicuous component in a valley of many others like it. The picture (bottom) presents the normalised FFT analysis of the same signal; it shows a resonant frequency component at 166 Hz., and other odd components at 228, 257, and 293 Hz. Interestingly, the resonant frequency values for the 860-mm string and the 53-mm sub-length are different by only 1 Hz. (165 and 166). What are the bearings of these components on the pitch of the signals? Let us examine this matter closely. To determine the contribution of the resonant frequencies, or the fundamental, or any other partial, to the unchanging musical pitch of the two stimuli, the following acoustic analysis will present their constancy (or otherwise) during each signal's evolution.

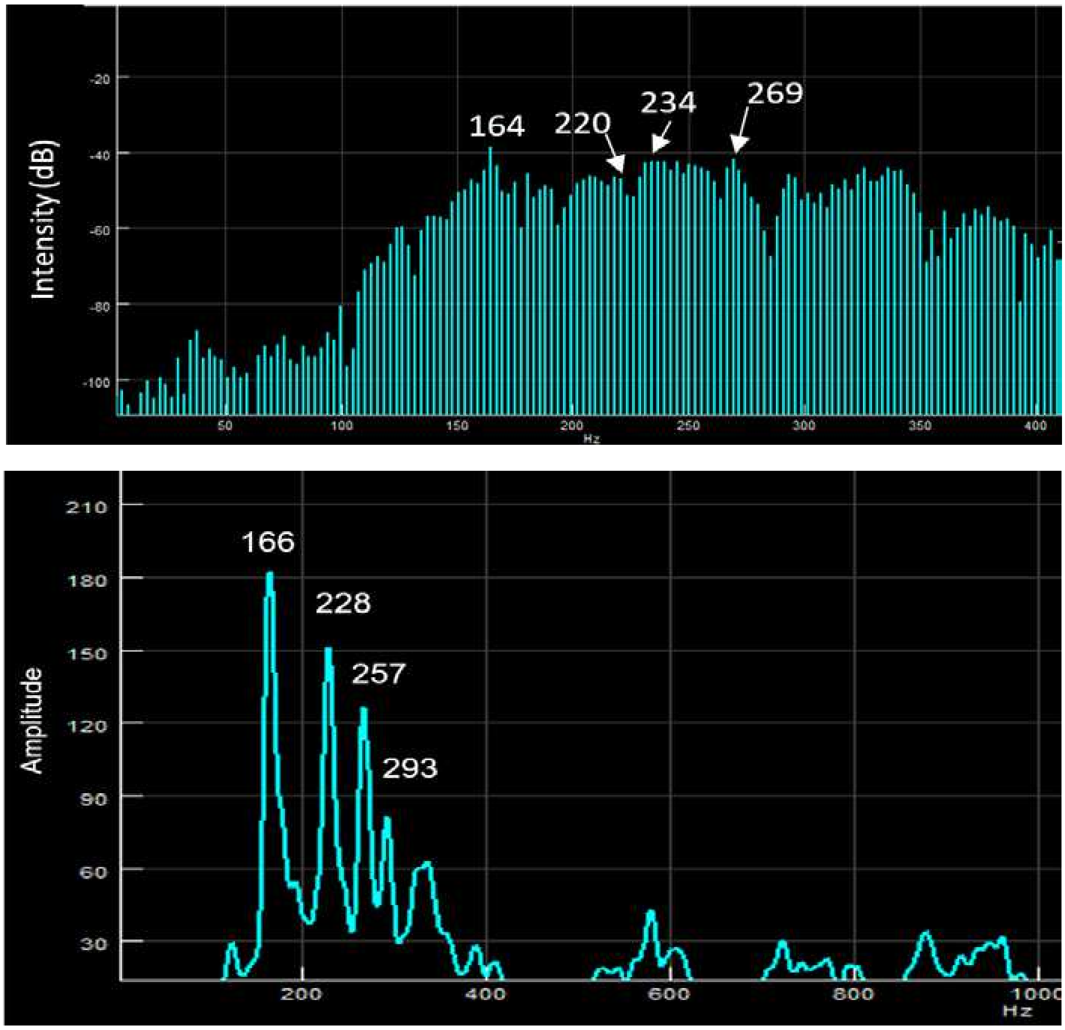


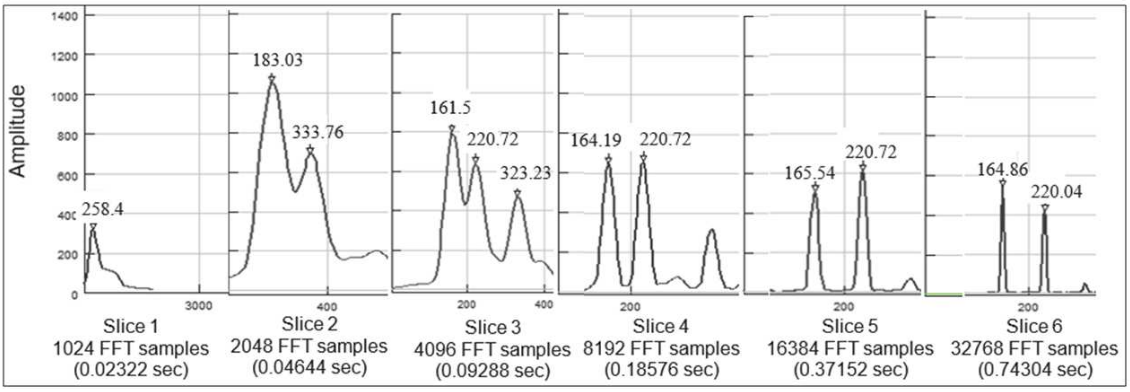
Figure 6. *Physical Manifestations of an A3 Musical Tone Produced on a 53-mm Long String. Although the Spectrum (Top) is Non-periodic, a 220 Hz. Frequency Component was Detected. The Normalised FFT Analysis (Bottom) Offers the Resonant Component at 166 Hz. Despite Non-periodic Spectral Structure, the Signal is, to the Ear, a Distinct Musical Tone A3.*

3.2.3. Evolution of Spectral Components and Structure

Naturally-produced sounds are characterised by change as a function of time; this instability poses a defiant problem for psychoacoustic theories of music and speech [32, 33, 34, 35]. In this examination, we shall scrutinise the behaviours of salient spectral components in the two signals above by means of spectral slices. To that end, all the spectral slices were adjusted to fit on a common amplitude scale for better observations of their relative intensity.

Figure 7(a) presents six spectral slices of the A3 musical tone produced by the 860-mm long string. The 220 Hz. partial appears in slice 3 as a subsidiary peak. In slice 4, this component is firmly established; it is in strong competition with the 164.19 Hz. partial. It dominates over the 165.54 Hz partial in slice 5, but is dominated over by the 164.86 Hz. component in slice 6. Figure 7(b) presents four slices showing the behaviours of the spectral components of the A3 tone produced by the 53-mm long string. The four spectral slices show no trace of the 220 Hz. component in figure 6 (top picture). However, slice 4 shows the resonant frequency component of 166 Hz in the 2D FFT presentation in figure 6 (bottom picture).

(a) Spectral slices of an A3 tone by the 860-mm long string



(b) Spectral slices of an A3 tone by the 53-mm long string

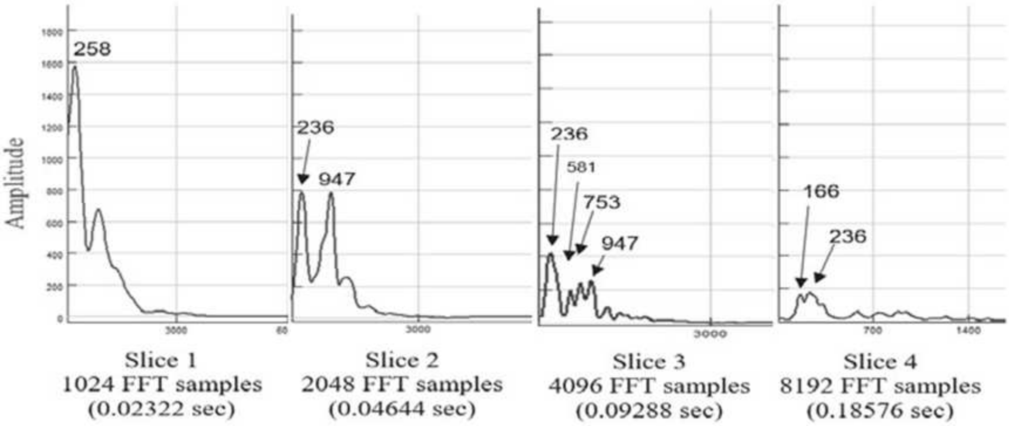


Figure 7. *(a) Shows That the 860-mm String Tone Attains Steady-State in Slice 4 with Two Spectral Components That Last Throughout the Rest of the Signal. In Contrast, the Tone from the 53-mm Long String in Figure 7. (b) Has No Steady-State Portion. The Resonant Frequency Component at 166 Hz. was Picked up Only at the End of the Signal in Slice 4.*

Spectral components and structures in 2D presentations offer only a partial view of the reality. For a better appraisal of the perceptual import of spectral components, it is necessary to observe their behaviours as a function of time. 3D representations of the signals assure such observations. Therefore, let us track changes in the spectral components of the two signals here under consideration. To that end, we shall examine the evolution of these spectral components in the effort to determine their contribution (if any) to the common pitch generated by the two signals.

Consider figure 8. The spectrogram (top) is the 3D shape of slice 4 in figure 7(a). The imposing spectral component is the 165 Hz. partial. This component is the dominant component of the signal in terms of intensity and duration. Strangely, the 220 Hz. partial next to it does not manifest the competitive intensity that was recorded in the 2D presentation in slice 4 of figure 7(a) even in the region of high energy. There is not much of this component thereafter. The high intensity observed in the 2D format characterised the region of high energy which is manifest in the form of the towering column at the beginning of the signal. Outside this region of high energy which is short-lived, the signal attains steady-state with decreasing intensity as presented in the middle picture. This spectrogram helps us appreciate the competition between the 165 Hz. and the 220 Hz. components. We observe that the high intensity of the 220 Hz. component drops sharply after the beginning of the slice, thus leaving the 165 Hz. component as the prime component of the signal right through to the end. Unlike the 165 Hz. partial with a slow decay rate, the 220 Hz. component decays very rapidly; only a little shoot remains at the end of the signal as shown in the bottom picture. Regarding the 53-mm string tone, the 220 Hz. frequency component was not manifest in any of the four spectral slices of the signal. The three slices of the signal in 3D analysis are shown in figure 9. The spectrogram (top) is the 3D format of slice 1 in figure 7(b). Recall that this signal had no steady-state portion. However, note the first component which runs from start to end of the signal. Apparently, the 166 Hz. frequency component, which is shown as the resonant frequency of the signal in figure 6, belongs to this frequency band and is picked up only in slice 4 at the end of the signal in figure 7 (bottom image).

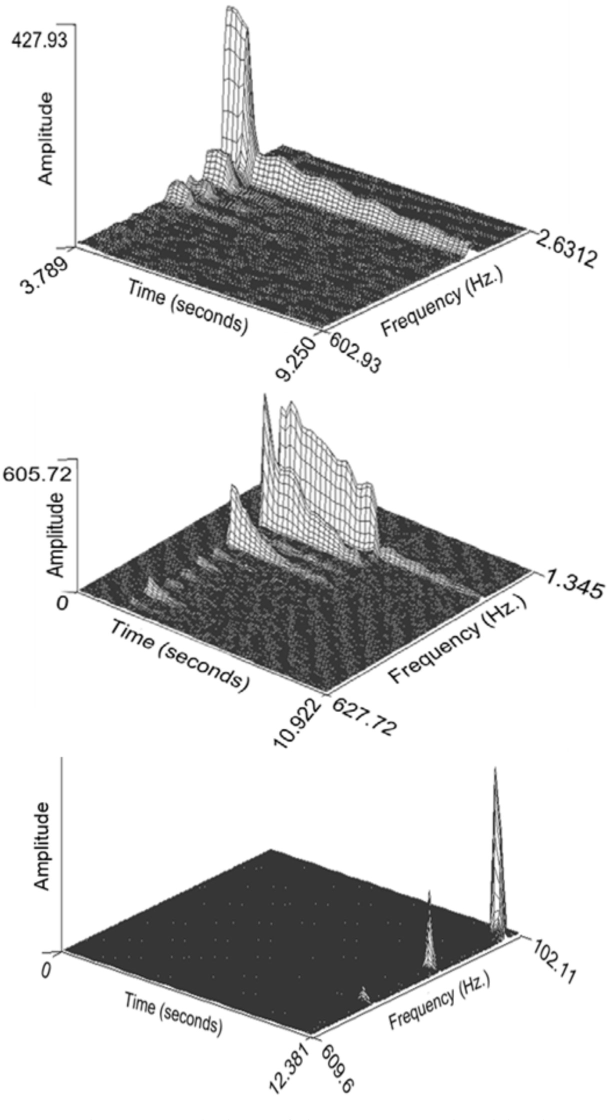


Figure 8. *Three Spectral Slices of the A$_3$ Tone Produced by the 860-mm Long String*

In the two signals under examination, the presence of the 220 Hz component is of interest, regardless of differences in their physical traits. Recall that the 220 Hz. component in the 53-mm string tone was traced and detected through some witch-hunting (as it were). In both cases, the component is neither the fundamental nor the resonant frequency of the signal. These facts make it difficult to attribute to it the pitch of the signal in accordance with Ohm's acoustical law [5] or Helmholtz's resonance/place theories [6].

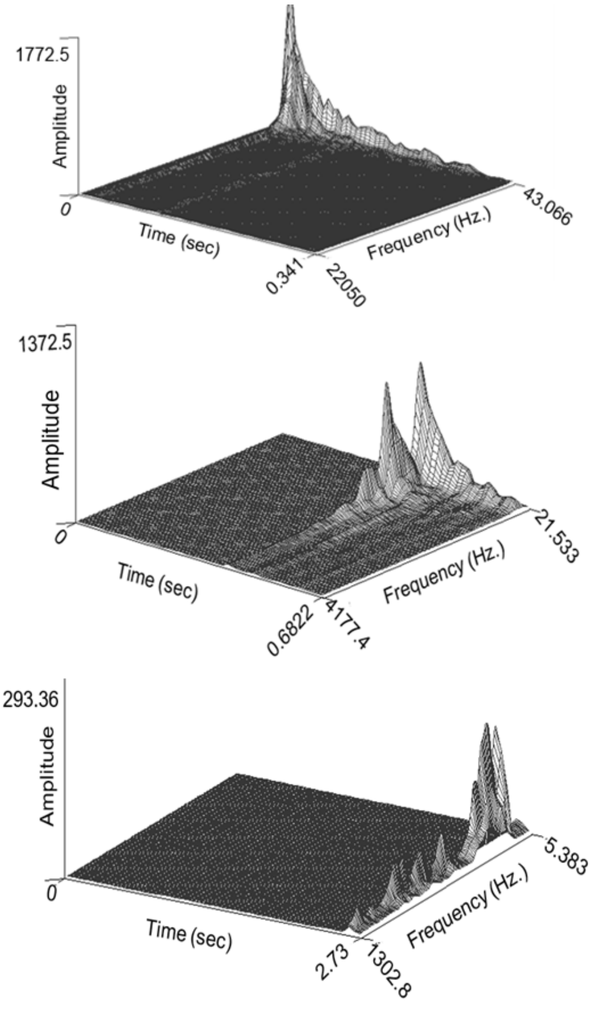


Figure 9. *Three Spectral Slices of the A$_3$ Tone Produced by the 53-mm Long String.*

In the case of the harmonic signal in figure 5, the 220 Hz. partial is the fourth component; and besides, it has very little energy and is short-lived. Why would the ear prefer the relatively weaker 220 Hz. tone to the 165 Hz partial which would undoubtedly produce maximum stimulation on the basilar membrane? The theoretical possibility to retrieve the 55 Hz. fundamental in this circumstance poses even a more difficult problem for psychoacoustic theories of pitch perception since the signal is defined in auditory terms as an A$_3$ musical tone; specified acoustically by the frequency of 220 Hz. The recorded f_0 of 55 Hz. should generate a pitch that is two octaves lower i.e. A$_1$ rather than A$_3$. The controversies arising from this phenomenon (called the missing fundamental, or residue pitch, or periodicity pitch, or low pitch) permeate every fibre of hearing research and raise questions that have never been answered, neither at the mechanical level [11, 12, 14, 15, 16, 17], nor at the acoustic level [3, 11. 36, 37, 38, 39, 40, 41, 42], nor at the neurophysiological level [1, 7, 8, 43, 44, 45]. Because the fundamental is missing, the perception cannot be direct but derived through some random computational procedures without a guiding principle [46]. However, it has been argued, and plausibly too, that the ear does not seem to function in that manner [1, 3, 31, 43, 45, 47, 48, 49, 50]. We cannot list here disillusioned researchers over the unbridged chasm between auditory experiences and physical characteristics of stimuli. Besides, it is not worthwhile here as all contrary facts and evidences have been exposed during the past 200 years since the young years of modern psychoacoustics from Seebeck, Ohm, and Helmholtz.

Nonetheless, the 220 Hz. component was present in the two signals examined above. Proponents of psychoacoustic theories might find in it a flimsy string on which to hang the weighty hope to explain music, speech, and hearing by purely psychoacoustic procedures. The present experiment furnishes mechanical and acoustic data that undercut such hope. To take a decisive stand on the matter of musical pitch production and perception, let us now view a synoptic presentation of the data gathered from the string experiment described above. We shall examine the presentation from two different viewpoints: (1) From the viewpoint of psychoacoustics; (2) From the viewpoint of invariance in mechanics of sound production.

4. Mechanics, Acoustics, and Musical Pitch Perception

4.1. The Problem of Non-Invariance in Psychoacoustics

Figure 10(a) summarizes the results of the string experiment based on the Pythagorean string ratio theory. String length was halved successively four times. The size of the oppositely-directed force (*F*ex) needed for the 860-mm long string to produce the musical tone A3 was 22 kg. This size of force was held constant in accord with the terms of the string ratio theory. Theoretically, each sub-length should produce a pitch an octave higher—A4, A5, etc. (see Column 3). The term *Resonant Frequency* (column 4) refers to the spectral component with the highest intensity, and *Partial* (column 5) specifies the position of the harmonic in the spectrum. The *F*o (column 6) denotes the lowest component of which all the other components in the spectrum are integral multiples as described earlier. Thus, entries in Row 1 state that the 860-mm long string, tensioned to 22 kg, has the most prominent partial at 165 Hz, and this component being the 3rd partial in the signal. the f_0 for this sound, as established earlier above, is 55 Hz. In this way, f_0 values were established for all signals (where possible). The data show rising resonant frequency values as string length reduces. This is a well-known phenomenon. However, Column 6 shows no doubling of f_0 when string length is reduced to 430 mm. Thereafter, however, the f_0 is shown to double with the halving of string length. The precision is astounding! We note also the tendency on the part of the resonant component to shift backward in the spectrum as length of string decreases. One might be tempted to draw precipitated conclusions based on these results and attribute the observed spectral behaviours of the signals to length of string according to the string ratio theory. Within the framework of invariance, however, everything that we have examined above in figure 10(a) is devoid of psychological essence because they are all based on variants—different string lengths, different resonant frequencies, different f_0 values, different pitches, etc. Furthermore, the signal produced by the 53-mm string had the first component at 166 Hz. and the others at 228 Hz; 257 HZ, and 293 Hz. In strict acoustic terms, this signal has no f_0; yet, it is a distinct musical tone to the ear.

	(a) String Ratio Approach						**(b) Mechanical Invariance Approach**					
Data Row	Length (mm)	Fex (kg)	Musical Pitch	Resonant Freq. (Hz)	Partial	Fo (Hz)	Length (mm)	Fex (kg)	Musical Pitch	Fin (kg)	Resonant Freq. (Hz)	Fo (Hz.)
1	860	22	A3	165	3rd	55	860	22.0	A3	N/A	165	55.0
2	430	22	A4	278	3rd	93	645	22.0	UP	N/A	211	70.0
3	215	22	A5	373	2nd	186	645	13.2	A3	8.8	167	55.7
4	107	22	A6	372	1st	372	430	13.2	UP	N/A	167	83.0
5	53	22	A7	845	2nd	N/A	430	6.0	A3	16.0	164	54.8
6							215	6.0	UP	N/A	222	111.0
7							215	2.0	A3	20.0	173	57.0
8							107	2.0	UP	N/A	161	N/A
9							107	1.6	A3	20.4	133	N/A
10							53	1.6	UP	N/A	227	N/A
11							53	0.0	A3	22.0	166	N/A

Figure 10. *Psychoacoustic VS Invariance-Based Ecological Approaches to Pitch and Hearing. The Data in (a) Illustrate the String Ratio Theory of Pitch Intervals, Which is the Mechanical Basis of Current Psychoacoustic Theories of Pitch and Hearing. The Data in (b) Illustrate the Invariance-Based Ecological Approach to Pitch and Hearing. They Show That Transformations to String Length Produce No Impact on Frequency of Vibration or Pitch. Rather, Frequency and Pitch are Impacted by Rise in Fin as String Length Decreases. Frequency and Pitch are Shown to Return to Their Original Levels When Increments in Fin, Arising from Reduced String Length, are Displaced.*

From the viewpoint of invariance, all sub-lengths of the string must produce the *same* pitch so that the pool of mechanical data manifest the parameter that controls pitch because of an unchanging relationship with the sensation pitch. Only the parameter that meets that criterion constitutes a psychological basis for a theory of auditory perception. Thus, any observed acoustic precision in figure 10(a) is, sadly, at the root of all the disputes and controversies and, indeed, the failure of psychoacoustics in music, speech, and hearing research since string length is not in control of frequency and pitch. Let us consider the results of the experiment from the viewpoint of mechanical invariance approach to pitch perception.

4.2. The Mechanical Invariance Factor

Figure 10(b) introduces invariance into the study. Therefore, all the different string lengths were tuned to the *same* musical pitch A_3. The new addition in figure 10(b) is the *missing mechanical parameter F*in in column 4. Entries in row 1 report that the 860-mm long string, tensioned to 22 kg, produced the musical pitch A220 (or A_3) at the resonant frequency of 165 Hz. The f_0 of the signal as established earlier is 55 Hz. We do not know the inherent force of the string (*F*in) at this point. Then, string length was shortened to 645 mm while *F*ex was held constant. The subjective pitch of signal rose as evidenced by the rise in frequency to 211 Hz. Because our focus is the mechanical determinant of the A_3 musical tone, the new and higher pitch is of no interest to the present experiment; therefore, it is labelled UP (Un-labelled Pitch). In row 3 the 645-mm long string was tuned to the *same* A_3 musical pitch. To achieve that goal, *F*ex was reduced to 13.2 kg. The size of *F*in is measured in terms of the force displaced. i.e. 8.8 kg, for the relatively shorter string to sound the same pitch A_3. The f_0 was established as 55.7 Hz.

A scrutiny of the data in this manner shows increased pitch and f_0 each time the string is shortened while *F*ex is held constant. However, whenever a sub-length of string was re-tuned to the same musical pitch A_3, *F*ex fell; and the f_0 (where applicable) returned to the same level, lying between 54.8 and 57 Hz. Interestingly, the shortest 53-mm sub-length required no *F*ex to produce the A_3 tone. Thus, the full string and its sub-lengths are potential generators of the same subjective pitch at about the *same* f_0. The data reveal that string length has no effect on a string's vibrational frequency because frequency values remain the same regardless of string length. These facts are diametrically opposed to those of the foundation of hearing sciences as portrayed in figure 10(a).

It is quite hard to address all the problems of hearing sciences in a research paper such as this. A more detailed presentation which encompasses mechanical, acoustic, and physiological aspects of auditory perception has been attempted elsewhere from where the present paper is extracted [31]. Nevertheless, the data exposed to account in this study show that efforts at finding pitch by current psychoacoustic procedures is, as Haggard [48] would put it, "the search for the […] spectre in the spectrum." Indeed, a complete rejection of theories and practices based on prehistoric physics of sound, Pythagorean string ratio theory, Ohm's acoustical law, and Helmholtz's resonant/place theory is compelling if auditory research is to attain the status of a behavioural science, and progress even by little from where Pythagoras left off some 2,500 years ago.

5. Summary and Conclusions

The string ratio theory is the hob around which turn all work in hearing research. In this regard, we have Ohm's Acoustic Law and Helmholtz's Resonance or Place Theory. These are the acoustic and neurophysiological extensions of the string ratio theory. Despite intensive and extensive research at the three levels of investigations, pitch remains a mystery. The pitch enigma suggests the existence of a debilitating error in the foundation of hearing sciences. Thus, this paper investigated the scientific plausibility of the string ratio theory as a foundation for hearing sciences, and the scientific status of psychoacoustics in speech and music perception research. The investigation focused the mechanical parameter of the sound source in pitch control. An earlier experiment [23] had demonstrated the existence of a force that is the inherent property of a string (*F*in) besides the balanced-force that is exerted on the string (*F*ex). The present study confirmed that the force in a string increases as string length decreases even when no balanced-force is exerted on the string. The experimental data showed increased vibrational frequency and pitch each time string length was reduced. Upon this knowledge hangs the colossal edifice of hearing sciences according to the provisions of the string ratio theory, Ohm's Law, and Helmholtz's Resonance Theory. However, for any sub-length of string to sound the same pitch as the full string, the increased force in string arising from reduction in string length must be displaced. When the appropriate size of force was displaced, f_0 and pitch returned to their previous values regardless of string length. The data show that the tension of a string is not synonymous with the balanced-force exerted on a string, but is jointly determined by the balanced-force and the force that is the intrinsic property of the string. Therefore, the tension of a string may be changed by changing the length (or other properties of the string) even though the balanced-force acting on a string is held constant. Future work will examine the role of string density in pitch production following the pattern set in the present work. Meanwhile, the following five critical conclusions are arrived at: (1) The higher f_0 and pitch arising from sub-lengths of a string are *not* determined by string length but by the rise in the force which is the intrinsic property of the string (*F*in). (2) The enigmatic nature of the sensation pitch in music or speech is attributable to the founding of hearing sciences on a variant physical parameter of sound rather than the invariant property of the sound source that underlies the auditory sensation pitch. (3) The existence of an invariant mechanical parameter of sound sources that underlies pitch despite acoustic variability is a potential case for direct perception in opposition to computational theories of perception. (4) Until the invariant property of the sound source that underlies a given auditory sensation is detected, isolated, and given acoustic representation, the search for the functional codes in music and speech in the physical representations of sound is premature. (5) From the standpoint of invariance, the quest for the principle of the auditory mechanism based on current psychoacoustic theories and practices without invariance, has been, is, and will forever be subject to futility, unless we turn around and learn to do hearing sciences scientifically [31].

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