Mathematical Review and Circuit Analysis Software Development for Small-Signal Single-Stage Transistor Amplifier Using Hybrid Parameter

Henry Erialuode Amhenrior¹, *, Samuel Amhanyo Amhenrior²

¹Department of Electrical/Electronic Engineering, Faculty of Engineering, Enugu State University of Science and Technology, Enugu, Nigeria
²Department of Electrical/Electronic Engineering, Faculty of Engineering, Ambrose Alli University, Ekpoma, Nigeria

Email address:
henrin2003@yahoo.com (H. E. Amhenrior), samhanyo@hotmail.com (S. A. Amhenrior)

*Corresponding author


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Abstract: In this paper, a mathematical analysis of a Small-Signal Single-Stage Transistor Amplifier using Hybrid Parameter and the development of a software to aid this analysis was carried out. This was borne out of the desire to be able to correctly and accurately analysis small signal transistors to get the required performance quantities that can be used in the design of circuits. Also, the need to carry out this analysis with minimal error and time was of great motivation for this development. The methodology involved two aspects: the mathematical review of the transistor amplifier and the Software development. In the mathematical Analysis, mathematical tools such as matrix and determinant were used to analyze and evaluate the performance quantities namely the input resistance, the current and voltage gains; the output impedance and the power gain of the small-signal single-stage transistor using the generalized h-parameter configuration equations in a two-port system. Also, the conversion of the generalized h-parameter into the three transistor configurations and the conversion from one configuration to another were obtained. The software development was done in two stages which are the Programming of the Analysis Method and the Graphical User Interface (GUI) Development. The Analysis Method programming was done by the use of Visual Basic version 6 Programming Language through the use of Object Oriented Programming designed with a main program and subroutines namely the Mathematical Analysis, the h-parameter Conversion, the Circuit Diagram Simulation and the h-parameter Equivalent Circuit. These Subroutines are called into the main program whenever needed. The Graphical User Interface is a Menu based system for the administration of the Analysis Software Package. It was developed also by the use of Visual Basic version 6 Programming Language and Microsoft Access Office 2000. The software development is resident in a PC with a Windows Operating System. The test results show that the mathematical analysis was correctly done and the developed Software is very fast, efficient, accurate and reliable in analyzing small signal transistor amplifiers as designed.

Keywords: Analysis, Transistor, Hybrid Parameter, Single-Stage Amplifier, Performance Quantities

1. Introduction

The efficiency and reliability of any human activity depends on the ease with which the process can be carried out. This also applies to electronic circuit designs and analysis and indeed the analysis of a transistor amplifier. The failure of transistor amplifiers is a common occurrence in electronic equipment and gadgets. These failures have their adverse effect in having frequent down time of electronic system, thereby reducing production and profit of the establishment or the firm concerned due to unavailability of the equipment and gadgets concerned. Again, there is the very costly risk of endangering the life of the user, and the people around him and even those in the remote part where the effect of the system can be felt. As a result, some
manufacturers of semiconductor devices such as transistors usually advise that safety should be foremost in circuit designs, that even with maximum effort in producing semiconductor devices, failure may still occur and these failures may cause personal injury, fire or property damage [1]. This situation is worrisome to every individual and stakeholders concerned. These failures and unreliability of transistor amplifiers are usually due to incorrect circuit analysis done manually and repeatedly, thereby making the analysis to be prone to undetectable errors. These errors manifest only when the physical models are built. Again, the repetitive nature and the long time spent on the analysis of one circuit makes it monotonous and causes fatigue. However, there is the need to develop a process using computer that is not prone to errors in transistor circuit analysis; not affected by fatigue and monotony; very fast and very efficient in operation.

2. Literature Review

The strength of human being is very feeble in consideration to other mammals and universal forces. Similarly, most signals are usually weak for use in some applications. Sequel to this, some researchers have observed that, there is the need to advance devices that can aggregate signal so that very little physical ability is required to execute various task and these devices are referred to as amplifiers [2]. Signals and energies get fainted as they move along their transmission lines resulting from attenuation, interference, distortion, magnetic effect and other phenomena effects. One of such amplifier for small signals are the transistors.

The performance and failure of most semiconductor devices like the transistor depends on the design and analysis of its configuration. Anant et al., in their study recognized that an isolator can fail due to high voltage and current through circuits or components integrated in it and an optocoupler, a semiconductor device was used in their analysis [3]. Wu et al., observed that catastrophic failure mechanisms are closely associated with semiconductor physics and over-stress working conditions. These failures are hard to anticipate and as a result may cause serious effect in the application of IGBT in power electronic converters [4]. As appreciated by Mitsibushi Corporation, safety should be foremost in circuit designs to ensure optimum performance [1]. Sequel to the above, scientists, stakeholders and many authors have developed different ways to analyze the transistor amplifier over the years. Two main schools of thought exist in prominence today in respect to the equivalent circuit to be substituted for the transistor namely the hybrid parameter model and the $r_c$ model. For some time, the industrial and educational institutions depended greatly on the hybrid parameters [5, 6]. The hybrid parameter equivalent circuit continues to be very fashionable, nonetheless the presence of an equivalent circuit obtained exactly from the operating conditions of the transistor – the $r_c$ model [5]. Transistor hybrid parameters for a specific operating region have continually been indicated by transistor manufacturers on their datasheets. The parameters of the model can be gotten exactly from the hybrid parameters in this region. Nevertheless, the hybrid equivalent circuit has a drawback of being limited to a distinct set of working situations for it to be seen to be accurate. The parameter of the other equivalent circuit can be gotten for any region of operation within the active regions and are not restricted by the single set of parameters provided by the data sheet. However, the $r_c$ model fails to account for the output impedance level of the device and the feedback effect from output to input. In addition, some author have opined that $r_c$ model is the easiest to understand [7].

Electronic devices have been variously analyzed using the two-port model. Lobna proposed a two port network three impedance oscillators and analyzed all the attainable realizations using various arrangements. They studied the result of each arrangement on the oscillation condition and the frequency of oscillation. They then generate the transmission matrices for small-signal equivalent models of the BJT and MOS transistors [8]. Ahmed and Mohammad modeled all the feasible four-impedance settings that gives a good second-order two-stage Colpitts oscillator using two-port network transmission parameters and they tested these by the use of BJT and MOS transistors in an experimental case [9]. However, Ahmed addressed the task of Ahmed and Mohammad above for BJT and MOS in favour of two-port network transmission parameters for general design equations not limited to matched devices [10]. In another study, Ahmed proposed two instances of two-port analysis namely, an elementary and the advanced; he derived the transmission matrices for small-signal equivalent models of the BJT and MOS transistors. He suggested that his derived two-port network is especially good to employ in network analysis because the expression is only derived once and is independent of the complexity of any transistor, and it uses inter-network connectivity in which network could be divided into series, parallel or cascade interconnects [11].

Ankush et al., observed that time and cost are involved during failure in fault isolation and they therefore established a highly integrated data network with software analysis tool, the Synopsys Avalon to reduce the fault analysis time that will help in a better integrated circuit yield [12]. None of these authors analyzed the equivalent circuit of a transistor using h-parameters in matrix and determinant form. However, Eneh attempted the analysis of the transistor using h-parameter in matrix and determinant form, and this could be handled by computer programs [13]. Yet, one of the researchers analyzed a small – signal BJT amplifier using a software package such as Pspice [5]. The use of BASIC (Visual Basic) provides the opportunity for the user to define the scope and type of output for an analysis, while Pspice is limited to a specific list of output qualities.

2.1. A Transistor

The word ‘transistor,’ is a combination two words: ‘transfer’ and ‘resistor,’ and it mean an effective resistance, or transresistance of a piece of component when used as a
two-port, input/output device. A transistor is a three-terminal piece of component that gets its operation by using little amount of electric current or voltage to control great current [14].

A transistor is a three-layer semiconductor device made up of either two N-type and one P-type layers of material or two P-type and one N-type layers of material. The former is called an NPN transistor, while the latter is called a PNP transistor. Both are shown in Figure 1 (a) and (b) respectively with the proper D.C. biasing. The conventional representation of the transistor is as shown in Figure 2. It has three terminals labeled E for emitter, B for base and C for the collector. The emitter layer is heavily doped, the base is lightly doped and the collector is only lightly doped. The outer layer has widths much greater than the sandwiched P- or N-type terminal material with the ratio $0.150/0.001 = 150:1$ [5].

![NPN Transistor](image)

(a) NPN Transistor

![PNP Transistor](image)

(b) PNP Transistor

Figure 1. The three layers of transistor and DC biasing.

2.2. Development of Transistor

The development of the transistor emanated from the vacuum tube diode technology and applications which was launched in 1904 by J. A. Fleming [5]. This application was used by Thomas Edison in the production of electric light bulb [15]. This discovery led to more researches by adding more control grid to the vacuum tube. As a result, Lee De Forest in 1906, added a third element called the control grid to the vacuum diode resulting in the first amplifier, the triode. In the early 1930s the four-element tetrode and five-element pentode became popular in the tube industries [5]. The amplifying action of the first transistor was successfully demonstrated at Bell Laboratories in Murray Hill, New Jersey in December 23, 1947. Three persons were credited with the invention of the transistor namely William Shockley, John Bardeen and Walter Brattain. Shockley had started working in 1936 on the solid state physics theory that was the basis for the transistor [16]. This three-terminal solid-state device has several advantages over the tubes namely, small size and light weight; it had no heat requirement or heat loss; it had a very strong construction; it was more efficient as less power was absorbed for its own operation; there was no warm-up period required; and lower operating voltage was possible [5]. What appears to be the birth of modern transistor came in April 1954 when Adcock, Teal, and their team used a high purity silicon to fabricate an npn structure that has an emitter region that is carefully doped with impurity to increase current gain and a base layer thickness of 25 micrometers made of p–type material [17].

Also in 1954, Texas Instruments of Dallas, Texas commenced the commercial manufacturing of junction transistors for use in small radios, while in 1960, The Sony Company of Japan started the production of television sets from transistors after obtaining the right to produce transistors [16].

The transistor is still undergoing changes and improvements in its operations and applications through the efforts of researchers. Recently, Jose reported that a group of scientists had made a transistor that is able to mimic “some characteristics of neurons, such as counting, remembering and performing simple arithmetic operations [18].” At the moment, the potentiality of the transistor-type actions of ultracold atoms are being studied using various methods to
achieve an atomtronic transistor [14].

3. Methodology

In this section, the h-parameters are used in analyzing a single-stage transistor amplifier. The performance quantities are calculated for the general representation.

3.1. Analysis of the Single-stage Amplifier Using the General H-Parameter Equivalent Circuit

The h-parameter equivalent circuit of the transistor with applied voltage source and series resistance at the input and output circuits using the transistor two-port system is used for this mathematical review. This is as shown in the general h-parameter of Figure 3.

![Figure 3. Transistor General H-Parameter Equivalent Circuit.](image)

The five performance quantities to be calculated are:

1. Input resistance, $R_i = \frac{V_i}{I_i}$
2. Voltage gain, $A_v = \frac{V_o}{V_i}$
3. Current gain, $A_i = \frac{I_o}{I_i}$
4. Output resistance, $R_o = \frac{V_o}{I_o}$
5. Power gain, $A_p = P_o/P_i$ [19]

The generalized h-parameter equations can be written as

$$V_1 = h_{11}I_1 + h_{12}V_2$$  \hspace{1cm} (1)

$$I_2 = h_{21}I_1 + h_{22}V_2$$  \hspace{1cm} (2)

This can be represented in matrix form as follows

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From the output circuit,

$$I_2 = \frac{V_o - V_i}{R_o}$$  \hspace{1cm} (3)

Therefore (2) is equal to (3)

$$I_2 = h_{21}I_1 + h_{22}V_2 = \frac{V_o - V_i}{R_o}$$  \hspace{1cm} (4)

$$h_{22}R_oI_1 + h_{22}I_1V_2 = V_o - V_i$$

$$h_{22}R_oI_1 + (h_{22}R_o + 1)V_2 = V_i$$  \hspace{1cm} (5)

From (1) and (5) i.e.

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$V_o = h_{21}R_oI_1 + (h_{22}R_o + 1)V_2$$

We have the following matrix

$$\begin{bmatrix} V_1 \\ V_o \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21}R_o & (h_{22}R_o + 1) \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$I_1 = \frac{1}{D} \begin{bmatrix} V_1 \\ V_o \end{bmatrix}$$

$$I_2 = \frac{h_{12}}{V_2}$$

Where $D = \begin{vmatrix} h_{11} & h_{12} \\ h_{21}R_o & (h_{22}R_o + 1) \end{vmatrix}$

$$h_{11}(h_{22}R_o + 1) - h_{12}h_{21}R_o$$

$$h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\therefore D = h_{11} + \Delta hR_o$$

When $V_o = 0$

$$I_1 = \frac{1}{h_{11} + \Delta hR_o} \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{1}{h_{11} + \Delta hR_o} \begin{bmatrix} V_2 \end{bmatrix}$$

But $R_i = \frac{V_o}{I_o}$

$$R_i = \frac{V_o}{h_{21}I_1 + h_{22}V_2}$$

$$R_i = \frac{V_o}{h_{21} \left( h_{11}I_1 + h_{12}V_2 \right)}$$

$$R_i = \frac{V_o}{h_{11} + h_{12}V_2}$$

2. Voltage Gain $A_v$

When $V_o = 0$ from (1) and (5)

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$0 = h_{21}R_oI_1 + (h_{22}R_o + 1)V_2$$

$$V_2 = \frac{1}{D} \begin{bmatrix} h_{11} & V_1 \\ h_{21} & V_2 \end{bmatrix}$$

$$V_2 = \frac{-h_{21}R_oV_1}{h_{11} + \Delta hR_o}$$

Voltage gain, $A_v = \frac{V_o}{V_i}$

$$A_v = \frac{-h_{21}R_oV_1}{h_{11} + \Delta hR_o}$$

$$A_v = \frac{-h_{21}R_oV_1}{h_{11} + \Delta hR_o}$$
3. Current Gain \( A_i \)

In the output circuit of the Figure 3 with \( V_L = 0 \), (3) becomes

\[
I_2 = \frac{0 - V_2}{R_L} \\
V_2 = -I_2R_L
\]

Also, from (4),

\[
V_2 = \frac{I_2 - h_{21}I_1}{h_{22}}
\]

Therefore,

\[
-I_2R_L = \frac{I_2 - h_{21}I_1}{h_{22}}
\]

\[
-h_{22}I_2R_2 = I_2 - h_{21}I_1
\]

\[
-(1 + h_{22}R_2)I_2 = -h_{21}I_1
\]

\[
A_i = \frac{I_2}{I_1}
\]

\[
I_2 = \frac{-h_{21}}{-(1 + h_{22}R_1)} = \frac{h_{21}}{1 + h_{22}R_1}
\]

4. Output Resistance \( R_o \)

In the output circuit of Figure 3, when \( V_i = 0 \)

\[
I_1 = \frac{0 - V_i}{R_S}
\]

Therefore

\[
V_i = -I_1R_S
\]

Also, (1) can be rewritten as equals to the above (6). Therefore,

\[
-I_1R_S = h_{11}I_1 + h_{12}V_2
\]

\[
0 = (h_{11} + R_S)I_1 + h_{12}V_2
\]

Equation (2) remains unchanged

Hence, the following equations,

\[
0 = (h_{11} + R_S)I_1 + h_{12}V_2
\]

\[
I_2 = h_{21}I_1 + h_{22}V_2
\]

We have

\[
V_2 = \frac{[h_{11} + R_S] \begin{bmatrix} 0 \\ h_{21} \\ h_{12} \end{bmatrix}}{\begin{bmatrix} h_{11} + R_S & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}
\]

\[
V_2 = \frac{I_2(h_{11} + R_S)}{(h_{11}h_{22} - h_{12}h_{21}) + h_{22}R_S}
\]

5. Power Gain \( A_p \)

\[
V_2 = \frac{I_2(h_{11} + R_S)}{\Delta h + h_{22}R_S}
\]

But output resistance

\[
R_o = \frac{h_{11} + R_S}{\Delta h + h_{22}R_S}
\]

\[
A_p = A_i \times \frac{h_{21}}{h_{11} + \Delta hR_L}
\]

These performance quantities are as summarized in table 1.

### 3.2. Conversion of H-Parameters

Transistor data sheets generally specify the transistor in terms of its h-parameters for common base configuration i.e. \( h_{bb}, h_{bb}, h_{ab} \) and \( h_{ab} \). If we want to use the transistor in CE or CC configuration, we will have to convert the given set of parameters into a set of CE or CC parameters. Approximate conversion formula are tabulated in the Table 2 below.

#### Table 1. Performance quantities table.

<table>
<thead>
<tr>
<th>Performance Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Resistance</td>
<td>( R_i )</td>
<td>( \frac{h_{11} + \Delta hh_{L}}{1 + h_{22}R_L} )</td>
</tr>
<tr>
<td>Output Resistance</td>
<td>( R_o )</td>
<td>( \frac{h_{11} + R_S}{\Delta h + h_{22}R_S} )</td>
</tr>
<tr>
<td>Current Gain</td>
<td>( A_i )</td>
<td>( \frac{h_{21}}{1 + h_{22}R_1} )</td>
</tr>
<tr>
<td>Voltage Gain</td>
<td>( A_v )</td>
<td>( \frac{h_{11} + \Delta hh_{L}}{h_{21}} )</td>
</tr>
<tr>
<td>Power Gain</td>
<td>( A_p )</td>
<td>( \frac{h_{21}}{(h_{11} + \Delta hR_L)(1 + h_{22}R_L)} )</td>
</tr>
</tbody>
</table>

#### Table 2. Conversion of H-parameters.

<table>
<thead>
<tr>
<th>Number Parameter</th>
<th>From CB to CE</th>
<th>From CE to CB</th>
<th>From CE to CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{11} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
</tr>
<tr>
<td>( h_{12} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
</tr>
<tr>
<td>( h_{12} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
</tr>
<tr>
<td>( h_{21} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
</tr>
<tr>
<td>( h_{22} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
<td>( h_{ib} )</td>
</tr>
</tbody>
</table>

3.3. Analysis Software Package Development

The software aspect of this work involves coding in Visual Basic Programming Language Version 6.0 (VB6) with Microsoft Access application (MS Access office 2000) both running in windows 98 operating system for the implementation of the various aspects of the analysis. Visual
Basic is an efficient Object Oriented Programming Language which permits modular programming and enhances the development of subroutines. It is structured in a way that it has a main program flow which calls in any of the subroutines as need at a time. These subroutines are: the Mathematical Analysis; the Amplifier Circuit diagram Simulation; the h-parameter Equivalent Circuit and the h-parameter Conversions. It has a developed user-friendly Graphical User Interface (GUI) for each of the operation which can be selected from the main program menu also a graphical user interface. The flow chart for this process is as shown in Figure 4.

![Flow Chart of the Software Analysis](image)

**Figure 4. The flow Chart of the Software Analysis.**

When the software starts, the main menu is opened in a graphical user interface where all the operations that can be performed and Exit can be selected using their various buttons. Each selected operation executes the corresponding subroutine and prompt the user to provide the required data for the operation to be carried out. After the required data are provided and the “Compute” button is clicked, the data will be processed and the results are displayed. In each case, the results of h-parameter values and that of the mathematical analysis which are the performance quantities namely the input resistance, the output impedance, the current gain, the voltage and the power gain will always be displayed before proceeding to either the Amplifier Circuit diagram Simulation or the h-parameter Equivalent Circuit. In the case of h-parameter Conversions, the Common Emitter configuration values of the h-parameter are supplied and the configuration to be converted to will be selected. When the “Compute” button is clicked, then the conversion will be processed by the subroutine and the results displayed. To exit or close any of the submenu, the “Quit” button is clicked and to close the main menu, the “Exit” button is clicked. The screenshots of the Main Menu, the Mathematical Analysis Submenu and that of the h-parameter conversion Submenu are shown in Figures 5-7.
4. Test

After the analysis and software development, real values were used to test the software to ascertain its suitability for this analysis. The problem used for this analysis is as presented below.

**Problem:**

For the circuit in Figure 8, Q has the following values:

\[ h_{fe} = 110, \quad h_{ie} = 1.6k, \quad h_{re} = 2 \times 10^{-4}, \quad h_{oe} = 20 \text{ UA/V}. \]

Determine the performance quantities.

![Sample Test Circuit](image)

The software used these values entered and computed the performance quantities with the results shown as follows.

- Input resistance \( R_i = 1510 \)
- Voltage Gain \( A_v = -313.9 \)
- Current Gain \( A_i = 100.55 \)
- Output Resistance \( R_o = 86660 \)
- Power gain \( A_p = 31672.5 \)

The same values for common emitter transistor were used to stimulate the conversion by clicking CE to CB and the value displayed are as follows:

- Input resistance \( R_i = 1510 \)
- Voltage Gain \( A_v = -313.9 \)
- Current Gain \( A_i = 100.55 \)
- Output Resistance \( R_o = 86660 \)
- Power gain \( A_p = 31672.5 \)

The common emitter values were entered again and the simulation diagram as well as the h-parameter equivalent circuits where displayed appropriately with values.

5. Conclusion

This work has taken an in-depth look of the transistor performance quantities derivation using mathematical tools. The results of this analysis are quantities that can be computerized through programming. The review has expressed these quantities in the simplest terms revealing the
The efficiency and reliability of a small signal transistor amplifier analysis can now be achieved through the use of the circuit analysis package as developed in this work. Most of the ordeals in analyzing small signal transistor amplifier circuits manually can be eliminated by the use of this software analysis method. Again, the unnecessary time wastage in manual analysis has now been eliminated as the software only needs input values to process within some microseconds and produce the results of the analysis. This has gone a long way to reduce the tediousness of the calculations designers may need to contend with during such analysis. It also will help in correct and error free analysis of Small signal transistor amplifier if the correct values are supplied. This will ensure that failures and the accompanying risks and losses are greatly reduced in electronic equipment.

This computer aided circuit analysis for transistor amplifier is therefore an indispensable tool for engineers, industrialists, scientists and educational institution. Hence, it is highly recommended that stakeholders in these fields should be introduced to this method of analysis of amplifier for efficiency, speed, accuracy and ease of analysis. This is especially important in semiconductor circuitry design. Educational institutions will find the software package useful both in learning and practice to facilitate students understanding of transistor circuit analysis and software development.

References
[16] “The History of the Transistor,” San José State University, Silicon Valley, & Tornado Alley, USA. Available at: <http://www.sjsu.edu/faculty/watkins/transist.htm> [Accessed 29 August, 2017]