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A Generalization of Some Lag Synchronization of System with Parabolic Partial Differential Equation

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Abstract: In this paper, we study generalized adaptive synchronization of Lorenz chaotic system with parabolic partial differential equation. Systems with three uncertain parameters and the non-linear adaptive feedback control technique are considered. Moreover, a systematic design process of parameters identification and Lag synchronization of chaotic system is considered. Finally, a sufficient condition is given for Lyapunov stability.

Keywords: Lag Synchronization, Parabolic Partial Differential Equation, Uncertain Parameters, Adaptive Technique, Lorenz Chaotic System

1. Introduction

In the past two decades, many schemes for chase synchronization have been proposed, including linear and non-linear, such as in [1, 18-21]. At present, the researchers are concentrating on the following types of synchronization phenomena [23-34]. In this paper, we study generalized adaptive of Lorenz chaotic system with parabolic partial differential equation and with three uncertain coefficients, (see [2-17]).

We investigate the lag synchronization of Lorenz parabolic partial differential chaotic systems with uncertain three coefficients. Based on the generalized adaptive technique, a new controller and coefficient adaptive laws are designed such that coefficients identification is realized and lag synchronization of Lorenz parabolic partial differential chaotic system is achieved simultaneously.

2. A general Chaotic Problem

Let us consider the following generalized chaotic problem:

$$\left.\begin{aligned} \frac{\partial u_1(x,t)}{\partial t} &= \frac{\partial^2 u_1(x,t)}{\partial x^2} + a[u_2(x,t) - u_1(x,t)], \\ \frac{\partial u_2(x,t)}{\partial t} &= \frac{\partial^2 u_2(x,t)}{\partial x^2} + Cu_1(x,t) - u_1(x,t)u_3(x,t) - u_2(x,t), \\ \frac{\partial u_3(x,t)}{\partial t} &= \frac{\partial^2 u_3(x,t)}{\partial x^2} + u_1(x,t)u_2(x,t) - bu_3(x,t). \end{aligned}\right\} \quad (1)$$

$$u_1(x,0) = \varphi_1(x),$$

$$u_2(x,0) = \varphi_2(x),\ u_3(x,0) = \varphi_3(x),$$

$\varphi_1, \varphi_2,$ and φ_3 are given bounded continuous functions on $(-\infty, \infty)$,

(a, b, and c are given positive numbers).

The response system is controlled Lorenz Chaotic system as following

$$\left.\begin{array}{c}\frac{\partial v_1(x,t)}{\partial t}=\frac{\partial^2 v_1(x,t)}{\partial x^2}+\\ a_s(x,t)[v_2(x,t)-v_1(x,t)]+w_1(x,t),\\ \frac{\partial v_2(x,t)}{\partial t}=\frac{\partial^2 v_2(x,t)}{\partial x^2}+\\ C_s(x,t)v_1(x,t)-v_1(x,t)v_3(x,t)\\ -v(x,t)+w_2(x,t),\\ \frac{\partial v_3(x,t)}{\partial t}=\frac{\partial^2 v_3(x,t)}{\partial x^2}+\\ v_1(x,t)v_2(x,t)-b_s(x,t)v_3(x,t)+w_3(x,t).\end{array}\right\} \quad (2)$$

$$v_1(x,0)=\varphi_1(x), v_2(x,0)=\varphi_2(x), v_3(x,0)=\varphi_3(x).$$

Where a_s, b_s and c_s, of (2) are unknown functions, which need to be identified in the response system.

It is easy to see that

$$\left.\begin{array}{c}u_1(x,t)=\varphi_1^*(x,t)+\\ aP[u_2(x,t)-u_1(x,t)],\\ u_2(x,t)=\varphi_2^*(x,t)+\\ P\begin{bmatrix}Cu_1(x,t)-u_1(x,t)u_3(x,t)\\ -u_2(x,t)\end{bmatrix},\\ u_3(x,t)=\varphi_3^*(x,t)+\\ P[u_1(x,t)u_2(x,t)-bu_3(x,t)].\end{array}\right\} \quad (3)$$

Where,

$$\left.\begin{array}{c}\varphi_i^*(x,t)=\int_{-\infty}^{\infty}\frac{e^{\frac{-(x-y)^2}{4t}}}{\sqrt{4\pi t}}\varphi_i(y)dy,\\ i=1,2,3.\\ Pf=\int_0^t\int_{-\infty}^{\infty}\frac{e^{\frac{-(x-y)^2}{4t}}}{\sqrt{4\pi t}}f(y,\theta)dyd\theta,\end{array}\right\}$$

Also,

$$\left.\begin{array}{c}v_1(x,t)=\varphi_1^*(x,t)+\\ Pa_s[v_2(x,t)-v_1(x,t)]+Pw_1(x,t),\\ v_2(x,t)=\varphi_2^*(x,t)+\\ P\begin{bmatrix}C_s v_1(x,t)-v_1(x,t)v_3(x,t)\\ -v_2(x,t)+w_2(x,t)]\end{bmatrix},\\ v_3(x,t)=\varphi_3^*(x,t)+\\ P\begin{bmatrix}v_1(x,t)v_2(x,t)-b_s v_3(x,t)\\ +w_3(x,t)]\end{bmatrix}.\end{array}\right\} \quad (4)$$

Differentiating equation (3) with respect to t, one gets:

$$\left.\begin{array}{c}\frac{\partial u_1(x,t)}{\partial t}=\frac{\partial\varphi_1^*(x,t)}{\partial t}\\ +(a+P^*)[u_2(x,t)-u_1(x,t)],\\ \frac{\partial u_2(x,t)}{\partial t}=\frac{\partial\varphi_2^*(x,t)}{\partial t}\\ +(C+P^*)u_1(x,t)\\ -(I+P^*)u_1(x,t)u_3(x,t)\\ -(I+P^*)u_2(x,t),\\ \frac{\partial u_3(x,t)}{\partial t}=\frac{\partial\varphi_3^*(x,t)}{\partial t}\\ +(I+P^*)u_1(x,t)u_2(x,t)\\ -(bI+P^*)u_3(x,t).\end{array}\right\} \quad (5)$$

Also:

$$\left.\begin{array}{c}\frac{\partial v_1(x,t)}{\partial t}=\frac{\partial\varphi_1^*(x,t)}{\partial t}+\\ (a_s+P^*)[v_2(x,t)-v_1(x,t)]\\ +w_1(x,t)+P^*w_1(x,t),\\ \frac{\partial v_2(x,t)}{\partial t}=\frac{\partial\varphi_2^*(x,t)}{\partial t}+\\ (C_s+P^*)v_1(x,t)\\ -(I+P^*)v_1(x,t)v_3(x,t)\\ -(I+P^*)v_2(x,t)\\ +w_2(x,t)+P^*w_2(x,t),\\ \frac{\partial v_3(x,t)}{\partial t}=\frac{\partial\varphi_3^*(x,t)}{\partial t}+\\ (I+P^*)v_1(x,t)v_2(x,t)\\ -(b_s+P^*)v_3(x,t)+w_3(x,t)\\ +P^*w_3(x,t).\end{array}\right\} \quad (6)$$

Where:

$$P^*f=\int_0^t\int_{-\infty}^{\infty}\frac{\partial}{\partial t}\frac{e^{\frac{-(x-y)^2}{4(t-\theta)}}}{\sqrt{4\pi(t-\theta)}}f(y,\theta)dyd\theta,$$

$$w(x,t)=(w_1(x,t),w_2(x,t),w_3(x,t))^T$$

is the controller which should be designed such that two systems can be Lag synchronized.

Let

$$\left.\begin{array}{c}e_1(x,t)=v_1(x,t)-u_1(x,t-\tau),\\ e_2(x,t)=v_2(x,t)-u_2(x,t-\tau),\\ e_3(x,t)=v_3(x,t)-u_3(x,t-\tau).\end{array}\right\} \quad (7)$$

Where $\tau>0$ is the time delay for the error dynamical system. Therefor the goal of parameters identification Lag synchronization is to find an appropriate controller function $w(x,t)$ and parameter adaptive laws a_s, b_s and c_s, such that the synchronization errors.

$$e_1(x,t)\to 0, e_2(x,t)\to 0, e_3(x,t),\to 0 \; as \; t\to\infty \; \forall x\in (\infty,-\infty)\} \quad (8)$$

and the unknown parameters.

$$\lim_{t\to\infty} a_s=a, \lim_{t\to\infty} b_s=b, \lim_{t\to\infty} c_s=c. \quad (9)$$

3. Lag Synchronization of Lorenz Chaotic System and the Errors

In This section, we shall study the systems of errors (10) and the appearance of the lag synchronization of systems (1) and (2).

From systems (5) and (6), we get the following errors dynamical systems:

$$\left.\begin{aligned}
&\frac{\partial e_1(x,t)}{\partial t}=\frac{\partial \varphi_1^*(x,t)}{\partial t}\\
&-\frac{\partial \varphi_1^*(x,t-\tau)}{\partial t}-ae_1(x,t)\\
&+a_s v_1(x,t)-(a_s-a)v_1(x,t)\\
&+w_1(x,t)+P^* w_1(x,t)\\
&-au_2(x,t-\tau)\\
&+P^*[v_2(x,t)-v_1(x,t)]\\
&-P^*[u_2(x,t-\tau)-u_1(x,t-\tau)],\\
&\frac{\partial e_2(x,t)}{\partial t}=\frac{\partial \varphi_2^*(x,t)}{\partial t}\\
&-\frac{\partial \varphi_2^*(x,t-\tau)}{\partial t}-(c_s-c)v_1(x,t)\\
&+v_1(x,t)v_3(x,t)\\
&-P^* v_1(x,t)v_3(x,t)\\
&+w_2(x,t)+P^* w_2(x,t)\\
&-e_2(x,t)\\
&+ce_1(x,t)+P^* e_1(x,t)\\
&-P^* e_2(x,t)+u_1(x,t-\tau)\\
&-u_3(x,t-\tau)\\
&+P^*[u_1(x,t-\tau)-u_3(x,t-\tau)],\\
&\frac{\partial e_3(x,t)}{\partial t}=\frac{\partial \varphi_3^*(x,t)}{\partial t}\\
&-\frac{\partial \varphi_3^*(x,t-\tau)}{\partial t}-be_3(x,t)\\
&+P^* e_3(x,t)-(b_s-b)v_3(x,t)\\
&+w_3(x,t)+P^* w_3(x,t)\\
&-au_2(x,t-\tau)\\
&-[u_1(x,t-\tau)u_2(x,t-\tau)]\\
&-P^*[u_1(x,t-\tau)u_2(x,t-\tau)].
\end{aligned}\right\}\qquad (10)$$

Obviously, Lag synchronization of system (5) and (6) appears if the errors dynamical system (10) has an asymptotically stable equilibrium point, $e(x,t)=(0,0)$, where $e(x,t)=(e_1(x,t),e_2(x,t),e_3(x,t))^T$

Theorem1. Assuming that the Lorenz chaotic system (5) derives the controlled Lorenz chaotic system (6), take

$$\left.\begin{aligned}
&w_1(x,t)=-\frac{\partial \varphi_1^*(x,t)}{\partial t}\\
&+\frac{\partial \varphi_1^*(x,t-\tau)}{\partial t}+ae_1(x,t)\\
&+a_s v_1(x,t)+(a_s-a)v_1(x,t)\\
&-P^* w_1(x,t)+au_2(x,t-\tau)\\
&-P^*[v_2(x,t)-v_1(x,t)]\\
&+P^*\begin{bmatrix}u_2(x,t-\tau)-\\ u_1(x,t-\tau)\end{bmatrix},\\
&w_2(x,t)=-\frac{\partial \varphi_2^*(x,t)}{\partial t}\\
&+\frac{\partial \varphi_2^*(x,t-\tau)}{\partial t}+(c_s-c)v_1(x,t)\\
&-v_1(x,t)v_3(x,t)\\
&+P^* v_1(x,t)v_3(x,t)\\
&-P^* w_2(x,t)+e_2(x,t)-ce_1(x,t)\\
&-P^* e_1(x,t)+P^* e_2(x,t)\\
&-[u_1(x,t-\tau)-u_3(x,t-\tau)]\\
&-P^*[u_1(x,t-\tau)-u_3(x,t-\tau)],\\
&w_3(x,t)=-\frac{\partial \varphi_3^*(x,t)}{\partial t}\\
&+\frac{\partial \varphi_3^*(x,t-\tau)}{\partial t}+be_3(x,t)\\
&-P^* e_3(x,t)+(b_s-b)v_3(x,t)\\
&-P^* w_3(x,t)+au_2(x,t-\tau)\\
&+[u_1(x,t-\tau)u_2(x,t-\tau)]\\
&+P^*[u_1(x,t-\tau)u_2(x,t-\tau)].
\end{aligned}\right\}\qquad (11)$$

and parameter adaptive laws

$$\left.\begin{aligned}
\frac{\partial a_s(x,t)}{\partial t}&=v_1(x,t)e_1(x,t),\\
\frac{\partial b_s(x,t)}{\partial t}&=v_3(x,t)e_3(x,t),\\
\frac{\partial c_s(x,t)}{\partial t}&=-v_2(x,t)e_2(x,t).
\end{aligned}\right\}\qquad (12)$$

Systems (5) and (6) can realize lag synchronization and the unknown confidents will be identified, i.e.; equation (8) and (9) will be achieved.

Proof Equation (10) can be converted to following form under the controller (11)

$$\left.\begin{aligned}
&\frac{\partial e_1(x,t)}{\partial t}=-ae_1(x,t)-\\
&(a_s-a)v_1(x,t),\\
&\frac{\partial e_2(x,t)}{\partial t}=-e_2(x,t)-\\
&(c_s-c)v_1(x,t),\\
&\frac{\partial e_3(x,t)}{\partial t}=-be_3(x,t)-\\
&(b_s-b)v_3(x,t).
\end{aligned}\right\}\qquad (13)$$

Consider a Lyapunov function as

$$\begin{aligned}V(x,t)=\frac{1}{2}\big(&e_1^2(x,t)+e_2^2(x,t)+e_3^2(x,t)+e_a^2(x,t)\\&+e_b^2(x,t)+e_c^2(x,t)\big).\end{aligned}$$

Where,

$$e_a(x,t)=a_s-a,$$

$$e_b(x,t)=b_s-b, e_c(x,t)=c_s-c.$$

Obviously, V is a positive definite function. Taking its time derivative along with the trajectories of equation (12) and (13) leads to

$$\begin{aligned}
\frac{\partial V(x,t)}{\partial t}&=e_1(x,t)\frac{\partial e_1(x,t)}{\partial t}+e_2(x,t)\frac{\partial e_2(x,t)}{\partial t}\\
&+e_3(x,t)\frac{\partial e_3(x,t)}{\partial t}+(a_s-a)\frac{\partial a_s(x,t)}{\partial t}\\
&+(b_s-b)\frac{\partial b_s(x,t)}{\partial t}+(c_s\\
&-c)\frac{\partial c_s(x,t)}{\partial t}=e_1(x,t)[-ae_1(x,t)-(a_s\\
&-a)v_1(x,t)+e_2(x,t)[-e_2(x,t)+(c_s\\
&-c)v_1(x,t)]
\end{aligned}$$

$$\begin{aligned}
&+e_3(x,t)[-be_3(x,t)-(b_s-b)v_3(x,t)]+(a_s\\
&-a)[e_1(x,t)v_1(x,t)]+(b_s\\
&-b)[v_3(x,t)e_3(x,t)]-(c_s\\
&-c)[v_2(x,t)e_2(x,t)]\\
&=-ae_1^2(x,t)-e_2^2(x,t)-be_3^2(x,t)\\
&=-e^T(x,t)Pe(x,t)\ \le 0,
\end{aligned}$$

Where $P = diag\{a, 1, b\}$. It is obvious that $\frac{\partial V(x,t)}{\partial t} = 0$, if and only if $e_i(x,t) = 0, i = 1,2,3$.

Namely the set

$$M = \{e_1(x,t) = 0, e_2(x,t) = 0, e_3(x,t) = 0, a_s = a, b_s = b, c_s = c\}$$

is the largest invariant set contained in $E = \left\{\frac{\partial V(x,t)}{\partial t} = 0\right\}$ for equation (13). So according to the LaSalle's invariance principle [22], starting with arbitrary initial values of equation (13), the trajectory converges asymptotically to the set M,

i.e.; $e_1(x,t) \to 0, e_2(x,t) \to 0, e_3(x,t) \to 0, a_s \to a, b_s \to b$, and $c_s \to c$ as $t \to \infty$.

This indicates that the lag synchronization of Lorenz chaotic system is achieved and the unknown parameters a_s, b_s and c_s can be successfully identified by using controller (11) and parameter adaptive laws (12). This completes the proof of the theorem, (Comp. [34-39].

4. Conclusion

This paper investigates the synchronization problem of coupled nonlinear diffusion systems. The lag synchronization of diffusion Lorenz chaotic system with uncertain coefficients is studied. The controller and coefficients adaptive laws are designed such that coefficient identification is realized and lag synchronization of the diffusion Lorenz system is achieved. The Lyapunov stability is also studied.

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