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On Some Lag Synchronization and Higher Order Parabolic Systems

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Abstract: Chaos synchronization is a topic of great interest, due to its observation in a huge variety of phenomena of different nature. We study synchronization of two chaotic oscillators in a Master- Slave configuration. The two dynamic systems are coupled via a directed feedback that randomly switches among a finite set of given constant function at a prescribed time rate. And we use Lyapunov stability theory. This paper discussed the using of lag synchronization approach, and provided the equilibrium solutions of a new class of higher order parabolic partial differential equations to be applicable for Lorenz chaotic system in order to minimize the dynamical error of large Lorenz chaotic system

Keywords: Higher Order Parabolic Partial Differential Equations, Lag Synchronization, Adaptive Technique, Lorenz Chaotic System

1. Introduction

Chaotic systems provide a rich mechanism for signal design and generation, with potential applications to communications and signal processing, because chaotic signals are typically broad band, noise like, and difficult to predict they can be used in various contexts for masking in formation bearing wave forms.

The early work on chaos synchronization was reported by Pecora and Carroll [1], in the derive-response systems. Chaos synchronizationis an interesting nonlinear phenomenon have been attracted extremely attention due its possible implementation in many workable engineering domains, and dynamical systems. For examples secure connection [2], information processing [3], and so on. There are many reported schemes for chaos synchronization including non linearfeed back approach [4, 5], adaptive technique [6], back stepping method [7], impulsive control method [8], etc.

Recently different types of synchronization were studied including: complete synchronization [9], generalized synchronization [10], phase synchronization [11], lag synchronization [12], dislocated synchronization [13]. In this paper, we generalize some results about the lag synchronization of Lorenz chaotic system with uncertain parameters by considering higher order parabolic equations as follows:

This work aims to minimize the synchronization error in the dynamical systems by finding suitable controller, and coefficients for these systems. See [14-16]

2. Parabolic synchronization

Consider the following chaotic system (1), and (2).

$$\frac{\partial u_1(x,t)}{\partial t} = \sum_{|q|\le 2M} a_q(x) D^q u_1(x,t) \tag{1}$$

$$+\alpha[u_2(x,t) - u_1(x,t)]$$

$$\frac{\partial u_2(x,t)}{\partial t} = \sum_{|q|\le 2M} b_q(x) D^q u_2(x,t)$$

$$+[u_1(x,t)u_2(x,t) - \beta u_2(x,t)]$$

$$\frac{\partial u_3(x,t)}{\partial t} = \sum_{|q|\le 2M} C_q(x) D^q u_3(x,t)$$

$$+[\delta u_1(x,t) - u_1(x,t)u_3(x,t) - u_3(x,t)$$

Where:

$$U_i\,(x,0) = \varphi_i\,(x), i = 1,2,3$$

$\varphi_1, \varphi_2, \varphi_3$ are bounded continuous functions on R^n and $x = (x_1, x_2, \ldots x_n) \in R^n$, R^n is the

n − dimensional Euclidean space,

$$D^q = D_1^{q_1} \ldots D_n^{q_n}, D_j = \frac{\partial}{\partial x_j}$$

(q $= (q_1, \ldots, q_n)$ is an n − dimensional

multi − index, $|q| = q_1 + q_2 + \cdots + q_n$

and a_q, b_q, c_q are continuous bounded function defined on R^n for all $q \le 2M$.

It is assumed that the operators:

$$\sum_{|q|\le 2M} a_q(x) D^q, \sum_{|q|\le 2M} b_q(x) D^q, \sum_{|q|\le 2M} c_q(x) D^q$$

are strictly parabolic.. This means that:

$$\sum_{|q|\le 2M} a_q(x)\, y^q \ge \delta |y|^{2M}, \sum_{|q|\le 2M} b_q(x)\, y^q \ge \delta |y|^{2M},$$

$\sum_{|q|\le 2M} c_q(x)\, y^q \ge \delta |y|^{2M}\; for\ all\ x \in R^n$, where

$$y = (y_1, \ldots, y_n), |y|^2 = y_1^2 + \cdots + y_i^2,$$

where δ is a positive constant [17].

Let us consider the following second chaotic system:

$$\frac{\partial v_1(x,t)}{\partial t} = \sum_{|q|\le 2M} a_q(x) D^q\, v_1(x,t) \qquad (2)$$

$$+\alpha_s[v_2(x,t) - v_1(x,t)] + w_1(x,t)$$

$$\frac{\partial v_2(x,t)}{\partial t} = \sum_{|q|\le 2M} b_q(x) D^q\, v_2(x,t)$$

$$+[v_1(x,t)v_2(x,t) + \beta_s v_2(x,t)] + w_2(x,t)$$

$$\frac{\partial v_3\,(x,t)}{\partial t} = \sum_{|q|\le 2M} C_q(x) D^q\, v_3(x,t) + +[\delta_s v_1(x,t) - v(x,t)v_3(x,t) - v_3(x,t)] + w_3(x,t)$$

Where α_s, β_s, and δ_s of (2) are unknown coefficients, we have:

$$u_1(x,t) = \int_{R^n} G_1(\,x,y,t)\, u_1(y)\, dy + \alpha \int_0^t \int_{R^n} G_1(\,x,y,t-\theta)\, [u_2(y,\theta) - u_1(y,\theta)] dy d\theta,$$

$$u_2(x,t) = \int_{R^n} G_2(\,x,y,t)\, u_2(y)\, dy + \int_0^t \int_{R^n} G_2(\,x,y,t-\theta)\, [u_1(y,\theta) u_2(y,\theta) - \beta u_2(y,\theta)] dy d\theta$$

$$u_3(x,t) = \int_{R^n} G_3(\,x,y,t)\, u_3(y)\, dy + \int_0^t \int_{R^n} G_3(\,x,y,t-\theta)\, [\delta u_1(y,\theta) - u_1(y,\theta) u_3(y,\theta) - u_3(y,\theta)] dy d\theta$$

Where $G_i(\,x,y,t), i = 1,2,3, and\ G_1, G_2, G_3$ is the fundamental solutions ofthe following problems: see [18-23]

$$\frac{\partial V_i\,(x,t)}{\partial t} = L_i(\,x,t,D) V_i$$

Where: $L_1(\,x,t,D) = \sum a_q(x) D^q$,, $L_2(\,x,t,D) = m \sum b_q(x) D^q$, and

$$L_3(\,x,t,D) = \sum c_q(x) D^q$$

Let: $u_i^*(\,x,t) = \int_{R^n} G_i(\,x,y,t)\, u_i(y)\, dy$

It is easy to see that:

$$u_1(x,t) = u_1^*(\,x,t) + \alpha \int_0^t \int_{R^n} G_1(\,x,y,t-\theta)[\,u_2(y,\theta) - u_1(y,\theta)\, dy d\theta,$$

$$u_2(x,t) = u_2^*(\,x,t) + \int_0^t \int_{R^n} G_2(\,x,y,t-\theta)[\,u_1(y,\theta)\, u_2(y,\theta) - \beta u_2(y,\theta)] dy d\theta,$$

$$u_3(x,t) = u_3^*(\,x,t) + \alpha \int_0^t \int_{R^n} G_3(\,x,y,t-\theta)[\,\delta u_1(y,\theta) - u_1(y,\theta) u_3(y,\theta) - u_3(y,\theta)] dy d\theta,$$

Now we have:

$$\frac{\partial u_1(x,t)}{\partial t} = \frac{\partial}{\partial t} u_1^*(\,x,t) + \alpha\, [u_2(\,x,t) - u_1(\,x,t) \;\; \alpha \int_0^t \int_{R^n} \frac{\partial}{\partial t} G_1(\,x,y,t-\theta).[\,u_2(y,\theta) - u_1(y,\theta)\, dy d\theta$$

$$\frac{\partial u_2(x,t)}{\partial t} = \frac{\partial}{\partial t} u_2^*(\,x,t) + [u_1(\,x,t) u_2(\,x,t) - \beta u_2(\,x,t)] + \int_0^t \int_{R^n} \frac{\partial}{\partial t} G_2(\,x,y,t-\theta)[\,u_1(y,\theta)\, u_2(y,\theta) - \beta u_2(y,\theta)] dy d\theta$$

$$\frac{\partial u_3(x,t)}{\partial t} = \frac{\partial}{\partial t} u_3^*(\,x,t) + [\delta\, u_1(x,t) - u_1(x,t) u_3(x,t) - u_3(x,t) + \int_0^t \int_{R^n} \frac{\partial}{\partial t} G_3(\,x,y,t-\theta)[\,\delta u_1(y,\theta) - -u_1(y,\theta) u_3(y,\theta) - u_3(y,\theta)]\, dy d\theta.$$

Let:

$$f_1(x,t) = \int_0^L \int_{R^n} G_1(x,y,t-\theta) f_1(x,t)\, dx dt$$

$$f_2(x,t) = \int_0^L \int_{R^n} G_2(x,y,t-\theta) f_2(x,t)\, dx dt$$

$$f_3(x,t) = \int_0^L \int_{R^n} G_3(x,y,t-\theta) f_3(x,t)\, dx dt$$

Now considering the following chaotic
Parabolic system (1), and (2) see [24-38]

$$\frac{\partial u_1(x,t)}{\partial t} = \frac{\partial u_1^*(x,t)}{\partial t} + \alpha\,[u_2(x,t) - u_1(x,t)] + \frac{\partial f_1(x,t)}{\partial t},$$

$$\frac{\partial u_2(x,t)}{\partial t} = \frac{\partial u_2^*(x,t)}{\partial t} + [u_1(x,t)u_2(x,t) - \beta u_2(x,t)] + \frac{\partial f_2(x,t)}{\partial t},$$

$$\frac{\partial u_3(x,t)}{\partial t} = \frac{\partial u_3^*(x,t)}{\partial t} + [\delta\, u_1(x,t) - u_1(x,t)u_3(x,t) - u_3(x,t) + \frac{\partial f_3(x,t)}{\partial t}.$$

and

$$\frac{\partial v_1(x,t)}{\partial t} = \frac{\partial v_1^*(x,t)}{\partial t} + \alpha_s[v_2(x,t) - v_1(x,t)] + \frac{\partial g_1(x,t)}{\partial t} + w_1(x,t)$$

$$\frac{\partial v_2(x,t)}{\partial t} = \frac{\partial v_2^*(x,t)}{\partial t} + [v_1(x,t)v_2(x,t) - \beta_s v_2(x,t)] + \frac{\partial g_2(x,t)}{\partial t} + w_2(x,t),$$

$$\frac{\partial v_3(x,t)}{\partial t} = \frac{\partial v_3^*(x,t)}{\partial t} + [\delta_s v_1(y,t) - v_1(y,t)v_3(y,t) - v_3(x,t) + \frac{\partial g_3(x,t)}{\partial t} + w_3(x,t)$$

Where:

$$\frac{\partial f_1(x,t)}{\partial t} = \alpha \int_0^t \int_{R^n} \frac{\partial}{\partial t} \partial G_1(x,y,t-\theta).[u_2(y,\theta) - u_1(y,\theta)]\, dy d\theta,$$

$$\frac{\partial f_2(x,t)}{\partial t} = \int_0^t \int_{R^n} \frac{\partial}{\partial t} \partial G_2(x,y,t-\theta)\, [u_1(y,\theta)\, u_2(y,\theta) - \beta\, u_2(y,\theta)]\, dy d\theta,$$

$$\frac{\partial f_3(x,t)}{\partial t} = \int_0^t \int_{R^n} \frac{\partial}{\partial t} \partial G_3(x,y,t-\theta)\, [\delta u_1(y,\theta) - u_1(y,\theta) u_3(y,\theta) - u_3(y,\theta)]\, dy d\theta,$$

$$\frac{\partial g_1(x,t)}{\partial t} = \alpha_s \int_0^t \int_{R^n} \frac{\partial}{\partial t} G_1(x,y,t-\theta).[v_2(y,\theta) - v_1(y,\theta)]\, dy d\theta,$$

$$\frac{\partial g_2(x,t)}{\partial t} = \int_0^t \int_{R^n} \frac{\partial}{\partial t} G_2(x,y,t-\theta)\, [v_1(y,\theta)\, v_2(y,\theta) - \beta_s\, v_2(y,\theta)]\, dy d\theta,$$

$$\frac{\partial g_3(x,t)}{\partial t} = \int_0^t \int_{R^n} \frac{\partial}{\partial t} G_3(x,y,t-\theta)\, [\delta_s v_1(y,\theta) - v_1(y,\theta) v_3(y,\theta) - v_3(y,\theta)]\, dy d\theta.$$

Where α_s, β_s, and δ_s are unknown coefficients.
Let:

$$e_1(x,t) = v_1(x,t) - u_1(x,t-\tau) \quad (3)$$

$$e_2(x,t) = v_2(x,t) - u_2(x,t-\tau)$$

$$e_2(x,t) = v_3(x,t) - u_3(x,t-\tau)$$

Where $t > 0$ is the time delay of the errors dynamical system. Now the goal of the parameters identified, and lag synchronization is to find an appropriate controller $W(x,t)$, and parameter adaptive laws of α_s, β_s, and δ_s such that the synchronization errors

$$e_1(x,t) \to 0,\ e_2(x,t) \to 0,\ e_3(x,t) \to 0 \quad (4)$$

as $t \to 0$, and the unknown coefficients satifiy the conditions:

$$\lim_{t\to\infty} \alpha_s = \alpha, \lim_{t\to\infty} \beta_s = \beta, \lim_{t\to\infty} \delta_s = \delta \quad (5)$$

Remark 1: When $\tau > 0$, the lag synchronization will appears, when $\tau < 0$, the anticipated synchronization will appear. Generally complete synchronization will appear

when, $\tau = 0$.

Remark 2: For the anticipated synchronization, and complete synchronization. The discussions
are similar to the method given in this paper.

3. Adaptive Lag Synchronization of Lorenz Chaotic System

$$\frac{\partial e_1(x,t)}{\partial t} = \frac{\partial v_1(x,t)}{\partial t} - \frac{\partial u_1(x,t-\tau)}{\partial t}$$

$$\frac{\partial e_1(x,t)}{\partial t} = \frac{\partial v_1^*(x,t)}{\partial t} + \alpha_s v_2(x,t) - \alpha_s v_1(x,t) + \frac{\partial g_1(x,t)}{\partial t} + w_1(x,t)$$

$$\frac{\partial u_1^*(x,t-\tau)}{\partial t} - \alpha\, u_2(x,t-\tau) + \alpha\, u_1(x,t-\tau) - \frac{\partial f_1(x,t)}{\partial t}$$

$$\frac{\partial e_1(x,t)}{\partial t} = \frac{\partial e_1^*(x,t)}{\partial t} + \alpha_s v_2(x,t) - \alpha_s v_1(x,t) + \frac{\partial g_1(x,t)}{\partial t} + w_1(x,t) - \alpha\, u_2(x,t-\tau) + \alpha u_1(x,t-\tau) - \frac{\partial f_1(x,t)}{\partial t}$$

$$\frac{\partial e_1(x,t)}{\partial t} = \frac{\partial e_1^*(x,t)}{\partial t} + \alpha_s v_2(x,t) - \alpha_s v_1(x,t) + \frac{\partial g_1(x,t)}{\partial t} + w_1(x,t) - \alpha\, u_2(x,t-\tau) + \alpha u_1(x,t-\tau) - \alpha v_1(x,t-\tau) + \alpha v_1(x,t) - \frac{\partial f_1(x,t)}{\partial t}$$

$$\dot{e}_1(x,t) = \frac{\partial e_1^*(x,t)}{\partial t} - \alpha\, e_1(x,t) + \alpha_s v_2(x,t)$$

$$-(\alpha_s-\alpha)v_1(x,t)+\frac{\partial g_1(x,t)}{\partial t}+$$
$$w_1(x,t)-\alpha\,u_2(x,t-\tau)-\frac{\partial f_1(x,t)}{\partial t}$$

$$\frac{\partial e_2(x,t)}{\partial t}=\frac{\partial v_2(x,t)}{\partial t}-\frac{\partial u_2(x,t-\tau)}{\partial t}$$

$$\frac{\partial e_2(x,t)}{\partial t}=\frac{\partial v_2^*(x,t)}{\partial t}+v_1(x,t)v_2(x,t)-\beta_s v_2(x,t)+\frac{\partial g_2(x,t)}{\partial t}+w_2(x,t)-$$
$$-\frac{\partial u_2^*(x,t-\tau)}{\partial t}-u_1(x,t-\tau)u_2(x,t-\tau)+\beta\,u_2(x,t-\tau)-\frac{\partial f_2(x,t)}{\partial t}$$

$$\frac{\partial e_2(x,t)}{\partial t}=\frac{\partial e_2^*(x,t)}{\partial t}+v_1(x,t)v_2(x,t)$$
$$-\beta_s v_2(x,t)+\frac{\partial g_2(x,t)}{\partial t}+w_2(x,t)-u_1(x,t-\tau)u_2(x,t-\tau)+\beta u_2(x,t-\tau)$$
$$\frac{\partial f_2(x,t)}{\partial t}$$

$$\frac{\partial e_2(x,t)}{\partial t}=\frac{\partial e_2^*(x,t)}{\partial t}+v_1(x,t)v_2(x,t)$$
$$-\beta_s v_2(x,t)+\frac{\partial g_2(x,t)}{\partial t}+w_2(x,t)+\beta\,v_2(x,t)-\beta\,v_2(x,t)$$
$$-u_1(x,t-\tau)u_2(x,t-\tau)+\beta\,u_2(x,t-\tau)-\frac{\partial f_2(x,t)}{\partial t}$$

$$\frac{\partial e_2(x,t)}{\partial t}=\frac{\partial e_2^*(x,t)}{\partial t}+v_1(x,t)v_2(x,t)$$
$$-(\beta_s-\beta)v_2(x,t)+\frac{\partial g_2(x,t)}{\partial t}$$
$$+w_2(x,t)-\beta[v_2(x,t)-u_2(x,t-\tau)]-u_1(x,t-\tau)u_2(x,t-\tau)-\frac{\partial f_2(x,t)}{\partial t}$$

$$\dot{e}_2(x,t)=\frac{\partial e_2^*(x,t)}{\partial t}+v_1(x,t)v_2(x,t)$$
$$-(\beta_s-\beta)v_2(x,t)+\frac{\partial g_2(x,t)}{\partial t}+w_2(x,t)-\beta e_2(x,t)-u_1(x,t-\tau)u_2(x,t-\tau)-\frac{\partial f_2(x,t)}{\partial t}$$

$$\frac{\partial e_3(x,t)}{\partial t}=\frac{\partial v_3(x,t)}{\partial t}-\frac{\partial u_3(x,t-\tau)}{\partial t}$$

$$\frac{\partial e_3(x,t)}{\partial t}=\frac{\partial v_3^*(x,t)}{\partial t}+\delta_s v_1(x,t)-v_1(x,t)$$
$$v_3(x,t)-v_3(x,t)+\frac{\partial g_3(x,t)}{\partial t}+w_3(x,t)$$
$$-\frac{\partial u_3^*(x,t-\tau)}{\partial t}-\delta u_1(x,t-\tau)+u_1(x,t-\tau)u_3(x,t-\tau)+u_3(x,t-\tau)-\frac{\partial f_3(x,t)}{\partial t}$$

$$\frac{\partial e_3(x,t)}{\partial t}=\frac{\partial e_3^*(x,t)}{\partial t}+\delta_s v_1(x,t)-v_1(x,t)v_3(x,t)-v_3(x,t)$$
$$+\frac{\partial g_3(x,t)}{\partial t}+w_3(x,t)+\delta u_1(x,t-\tau)+u_1(x,t-\tau)u_3(x,t-\tau)+u_3(x,t-\tau)-\frac{\partial f_3(x,t)}{\partial t}-\delta v_1(x,t)+\delta u_1(x,t)+$$

$$\frac{\partial e_3(x,t)}{\partial t}=\frac{\partial e_3^*(x,t)}{\partial t}+(\delta_s-\delta)\,v_1(x,t)\,v_3(x,t)-$$
$$[\,v_1(x,t)\,v_3(x,t)\,[$$
$$-v_3(x,t)-u_3(x,t)+\frac{\partial g_3(x,t)}{\partial t}+w_3(x,t)-$$
$$-\delta\,u_1(x,t-\tau)+u_3(x,t-\tau)-\frac{-\partial f_3(x,t)}{\partial t}+\delta v_1(x,t)$$

$$\dot{e}_3(x,t)=\frac{\partial e_3^*(x,t)}{\partial t}-(\delta_s-\delta)v_1(x,t)-v_1(x,t)\,v_3(x,t)-$$
$$e_3(x,t)+\frac{\partial g_3(x,t)}{\partial t}+w_3(x,t)-\delta\,u_1(x,t-\tau)+$$
$$u_1(x,t-\tau)\,u_3(x,t-\tau)-\frac{\partial f_3(x,t)}{\partial t}+\delta v_1(x,t)$$

We have the error dynamical system.

$$\dot{e}_1(x,t)=\frac{\partial e_1^*(x,t)}{\partial t}-\alpha\,e_1(x,t)+\alpha_s v_2(x,t)$$
$$-(\alpha_s-\alpha)v_1(x,t)+\frac{\partial g_1(x,t)}{\partial t}+$$
$$w_1(x,t)-\alpha\,u_2(x,t-\tau)-\frac{\partial f_1(x,t)}{\partial t} \tag{6}$$
$$\dot{e}_2(x,t)=\frac{\partial e_2^*(x,t)}{\partial t}+v_1(x,t)v_2(x,t)$$
$$-(\beta_s-\beta)v_2(x,t)+\frac{\partial g_2(x,t)}{\partial t}+w_2(x,t)-\beta e_2(x,t)$$
$$-u_1(x,t-\tau)u_2(x,t-\tau)-\frac{\partial f_2(x,t)}{\partial t}$$
$$\dot{e}_3(x,t)=\frac{\partial e_3^*(x,t)}{\partial t}+(\delta_s-\delta)\,v_1(x,t)-v_1(x,t)\,v_3(x,t)-$$
$$e_3(x,t)+\frac{\partial g_3(x,t)}{\partial t}+w_3(x,t)-\delta\,u_1(x,t-\tau)+$$
$$u_1(x,t-\tau)\,u_3(x,t-\tau)-\frac{\partial f_3(x,t)}{\partial t}+\delta v_1(x,t)$$

It is clear that lag synchronization of system (1), and (2) appears if the dynamical errors (6) have stable equilibrium point, and converge to zero (e (x, t) = 0)

Where: $e(x,t)=[e_1(x,t),e_2(x,t),e_3(x,t)]^T$

Then the following theorem was obtained:

Assuming that Lorenz chaotic system (2) take:

$$w_1(x,t)=-\frac{\partial e_1^*(x,t)}{\partial t}-\alpha_s v_2(x,t)-\frac{\partial g_1(x,t)}{\partial t}+\alpha\,u_2(x,t-\tau)+\frac{\partial f_1(x,t)}{\partial t}$$

$$w_2(x,t)=-\frac{\partial e_2^*(x,t)}{\partial t}-v_1(x,t)v_2(x,t)-\frac{\partial g_2(x,t)}{\partial t}+u_1(x,t-\tau)u_2(x,t-\tau)+\frac{\partial f_2(x,t)}{\partial t}$$

$$w_3(x,t)=-\frac{\partial e_3^*(x,t)}{\partial t}+v_1(x,t)v_3(x,t)-\frac{\partial g_3(x,t)}{\partial t} \tag{7}$$
$$+\delta u_1(x,t-\tau)-u_1(x,t-\tau)u_3(x,t-\tau)+\frac{\partial f_3(x,t)}{\partial t}-\delta v_1(x,t)$$

And parameter adaptive laws.

$$\dot{\alpha_s} = v_1(x,t))e_1(x,t) \quad (8)$$

$$\dot{\beta_s} = v_2(x,t))e_2(x,t)$$

$$\dot{\delta_s} = -v_1(x,t))e_3(x,t)$$

Both the systems (1, and (2) could realize lag synchronization, and the unknown coefficients will be identified, i. e. equations (4), and (5) will be achieved.

Proof:

Equation (6) can be converted to the following form under the controller (7).

$$\dot{e}_1(x,t) = -\alpha\, e_1(x,t) – (\delta_s - \delta)\, v_1(x,t) \quad (9)$$

$$\dot{e}_2(x,t) = -\beta\, e_2(x,t) – (\beta_s - \beta)\, v_2(x,t)$$

$$\dot{e}_3(x,t) = -\, e_3(x,t) + (\delta_s - \delta)\, v_1(x,t)$$

Consider A Lypanov function as:

$$v = \frac{1}{2}\,[\, e_1^2(x,t) + e_2^2(x,t) + e_3^2(x,t) +$$

$$(\alpha_s - \alpha)^2 + (\beta_s - \beta)^2 + (\delta_s - \delta)^2\,]$$

It is clear that, V is a positive definite function. Taking its time derivative along with the trajectories of equation (8), and (9) leads to

$$\dot{V} = e_1(x,t)\dot{e_1}(x,t) + e_2(x,t)\dot{e_2}(x,t)e_3(x,t)\dot{e_3}(x,t) +$$

$$+ (\alpha_s - \alpha)\dot{\alpha_s} + (\beta_s - \beta)\dot{\beta_s} + (\delta_s - \delta)\dot{\delta_s}\,]$$

$$\dot{V} = e_1(x,t)\,[-\alpha e_1(x,t) - (\alpha_s - \dot{\alpha})\, v_1(x,t)] + e_2(x,t)$$

$$[-\beta e_2(x,t) - (\beta_s - \beta)\, v_2(x,t)] +$$

$$e_3(x,t)\,[-e_3(x,t) - (\dot{\delta}_s - \delta)\, v_1(x,t)]$$

$$+ (\alpha_s - \alpha)$$

$$[\, v_1(x,t)\, e_1(x,t)] + (\beta_s - \beta)[\, v_2(x,t)\, e_2(x,t)] +$$

$$(\delta_s - \delta)[-\, v_1(x,t) e_3(x,t))]$$

$$\dot{V} = -\,\alpha e_1{}^2(x,t) - \beta e_2{}^2(x,t) - e_3{}^2(x,t)$$

$$= -e^t P_e \le 0$$

Where P = diag $\{\alpha, 1, \beta\}$. It is clear that

$\dot{V} = 0$ if and only if $e_i(x,t) = 0, \forall$ i = 1, 2, 3

Namely the set:

$$Q = \begin{Bmatrix} e_1(x,t) = 0, e_2(x,t) = 0, e_3(x,t) = 0, \\ \alpha_s = \alpha,\ \beta_s = \beta,\ \delta_s = \delta \\ \end{Bmatrix}$$

Is the largest invariant set contained in E = $\{\dot{v} = 0\}$

From equation (9), so according to the La Salles

Invariance principle [39]. Starting with arbitrary initial values of equation (9), the trajectory converges asymptotically to the set Q, i.e.

$$e_1(x,t) \to 0,\ e_2(x,t) \to 0,\ e_3(x,t) \to 0 \text{ as}$$

$$(\alpha_s \to \alpha,\ (\beta_s \to \beta), \text{and } (\delta_s \to \delta)$$

As $t \to \infty$

This indicates that the lag synchronization of Lorenz chaotic system is achieved, and the unknown coefficients: $(\alpha_s, \beta_s, \delta_s)$

Can be successfully identified by using controller (7), and the parameter adaptive law (8). (Comp [40–44]

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