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Statistical Analysis of Strength of W/S Test of Normality Against Non-normal Distribution Using Monte Carlo Simulation

Ukponmwan H. Nosakhare, Ajibade F. Bright

Department of General Studies, Mathematics and Computer Science Unit, Petroleum Training Institute, Warri, Nigeria

Email address:
Nosay2k@gmail.com (U. H. Nosakhare), equalright_bright@yahoo.com (A. F. Bright)

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Abstract: Among the test of normality in existence is the W/S which has standard table as the interval for critical region with both lower and upper bound. The test is suitable for sample size ranging from 3 as displayed in the W/S Critical table. But the sensitivity of the test can be determined by computation of power of the test which would show how sensitive the test is to non-normal distribution. The paper addressed the sensitivity of the test using some selected distributions which are from asymmetric and symmetric in nature. Monte Carlo Simulation technique was used with 100 replicates for sample sizes of 5 to 100 with regular interval of 5. Distributions considered include; Weibull, Chi-Square, t and Cauchy distributions. The result shows inconsistency of the test as it has weak power for distribution used except Cauchy distribution. The findings shows that the test should be used with caution has it has weak or low power which could lead to statistical error, thereby call for proper modification of the test to improve its power.

Keywords: Range, Standard Deviation, Descriptive Statistics, Simulation, Power-of-Test

1. Introduction

Gaussian distribution is also known as normal distribution which has some characteristics and widely used in the field of Statistics. For instance, most of the parametric test statistics have assumption of normality which implies before the usage of such test statistic, the data must be normally distributed otherwise, the computation becomes invalid.

Generally, parametric and nonparametric tests in Statistics can be distinguished as one having some basic assumptions and the other been robust and distribution free. Based on this fact, Analysis of Variance is different from Kruskal-Wallis test, although, ANOVA has three assumptions which include normality of the set of data while Kruskal-Wallis does not dependent on the distribution of the data. Also, t-test either paired or unpaired requires that the data must be normally distributed (Eze, 2002). The error term in the regression analysis must be tested for normality as the error term must be normally distributed. These are the proofs of necessity of normality test in the field of Statistics.

The proliferation of normality tests may confuse the user on the appropriate test to use for better and most accurate result. From the findings of researchers, including Ryan (1990), sample size play key role in the choice of normality test as most of the test cannot be used for small sample sizes. The report of Ryan on a test statistic for the test of normality, in which the researcher after some years of proposing the technique reported that the test cannot be used for small sample sizes. This report shows the test should be used with caution as the test is sample size sensitive.

In a related research, Shapiro and Wilk formulated a technique for the test of normality and later modified the test statistic which involves Shapiro and Francia. In a like manner, the popular technique of test of normality called JaqueBera which was proposed by Jaque and Bera was later modified for better result due to the inconsistency of the method. Also, Kolmogorov-Smirnov which was used by most of the researchers as test of the distribution of a set of data was later modified and called Lilliefors test in the year 1983. [Douglas and Edith, 2002; Jarque, Carlos and Bera, 1980; 1981; 1987].

The critical look at the frequently used normality tests revealed that W/S test is seldom used by researchers despite its simplicity as the test is void of rigorous mathematical computation unlike the frequently used normality tests. This led to the interest in the investigation on the sensitivity of the test for proper recommendation of its usage by researchers in the test of normality.

The primary aim of the research is to investigate the sensitivity of the W/S normality test to non-normal distribution for the possibility of reducing type II error in every research that involves test of normality.

Hypothesis to be Tested

Null Hypothesis: the data is normally distributed

Alternative Hypothesis: the data is not normally distributed.

2. Method of Data Analysis

W/S Statistic is given by;

$$q = \frac{w}{s} \tag{1}$$

Where "w" is the range of the set of data and "s" is the standard deviation of the data. The critical region of the test was accessed from http://www.watpon.com/table/normality.pdf. Using the significant value of 5%, the corresponding values were compared with the calculated value and the decision rule for the test is as follows;

Accept the null hypothesis if the calculated value falls within the interval of the critical value, otherwise, the null hypothesis is rejected.

Error rate: Since the actual distribution of the set of data used are known, the error rate was computed as the ratio of wrong conclusion to the number of replicates (100). Power of the test was computed by subtracting the error rate from 1 as the error rate is the type II error, that is, the probability of accepting the wrong null hypothesis (β).

Power of Test is;

$$(1-\beta) \tag{2}$$

3. Data Analysis

Equation 1 was applied to the simulated data from the four selected distributions and the result is as follows:

Table 1. *Simulated Data for Sample Size of 5 [Weibull (3)].*

S/N	Sample 1	Sample 2	Sample 3	…	Sample 100
1	3.16431	2.448008	3.07158	…	3.18657
2	2.629742	2.80808	2.336241	…	2.243208
3	2.834131	2.856368	3.236005	…	2.531471
4	2.564815	2.74527	3.140439	…	3.16234
5	2.673078	2.881323	2.57478	…	3.012885

For sample 1, the range (q) of the data is computed using the expression

$$Range = (Maximum\ Observation - Minimum\ Observation)$$
$$= 3.16431 - 2.564815 = 0.599495$$

The computation was done for all the simulated data and the result is presented in table 2 as shown below;

Table 2. *Calculated Values of W/S Test and the Corresponding Critical Values.*

Range	Std. Deviation	q=r/s	Table value for Alpha Value of 5%	Remark
0.60	0.24016	2.496224	2.15 - 2.753	Accept Null Hypothesis
0.43	0.17545	2.469736	2.15 - 2.753	Accept Null Hypothesis
0.90	0.39363	2.285830	2.15 - 2.753	Accept Null Hypothesis
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.94	.41966	2.247938	2.15 - 2.753	Accept Null Hypothesis

The calculated values of the test statistic are compared with the range of critical values and the remark, which is the decision, is recorded in the last column of table 2. The error rate, which is the number of time the test statistic gives erroneous conclusion, is 91 out of 100 repeated trials which is 0.10%. Therefore, the error rate of the test using Weibull distribution is 0.91%.

Similar computation was done for all the distributions of interest and the varying sample sizes and the result is summarised in the table below;

Table 3. *Error Rate of W/S Test Statistic.*

	Asymmetry		Symmetry	
Sample Size	*Weibull 7(3)*	*Chi-Sqr.(4)*	*t(8)*	*Cauchy(7,3)*
5	0.91	0.86	0.91	0.92
10	0.88	0.97	0.89	0.71
15	0.91	0.91	0.80	0.48
20	0.82	0.94	0.79	0.25
25	0.88	0.81	0.76	0.17
30	0.87	0.85	0.85	0.14
35	0.92	0.85	0.77	0.08

	Asymmetry		Symmetry	
Sample Size	***Weibull 7(3)***	***Chi-Sqr.(4)***	***t(8)***	***Cauchy(7,3)***
40	0.91	0.82	0.78	0.04
45	0.89	0.90	0.65	0.04
50	0.88	0.78	0.76	0.01
55	0.86	0.78	0.79	0.01
60	0.76	0.79	0.70	0.01
65	0.78	0.81	0.71	0.01
70	0.84	0.83	0.73	0.02
75	0.75	0.76	0.66	0.00
80	0.79	0.77	0.69	0.01
85	0.73	0.76	0.65	0.00
90	0.77	0.78	0.64	0.00
95	0.78	0.79	0.60	0.00
100	0.74	0.82	0.62	0.00

Table 3 shows the error rate of the W/S normality technique. The error rate was computed as the ratio of the number of erroneous conclusion divided by the number of the replicate. From the result, it can be seen vividly that the test is highly inconsistent as the error rate is considerably high irrespective of the sample size and the distribution of the set of data. For the symmetric distribution, using Cauchy distribution, the error rate of the test is considerably low for large sample size but the behaviour of the test seems different for t-distribution which belongs to the same category as Cauchy distribution which could be tagged as the inconsistency of the test. Power of the test can be computed for non-normal distribution by subtracting the error rate from 1. Using equation 2, the power of W/S test is shown below

Table 4. *Power of W/S Test Statistic.*

	Asymmetry		Symmetry	
Sample Size	***Weibull 7(3)***	***Chi-Sqr.(4)***	***t(8)***	***Cauchy(7,3)***
5	0.09	0.14	0.09	0.08
10	0.12	0.03	0.11	0.29
15	0.09	0.09	0.2	0.52
20	0.18	0.06	0.21	0.75
25	0.12	0.19	0.24	0.83
30	0.13	0.15	0.15	0.86
35	0.08	0.15	0.23	0.92
40	0.09	0.18	0.22	0.96
45	0.11	0.1	0.35	0.96
50	0.12	0.22	0.24	0.99
55	0.14	0.22	0.21	0.99
60	0.24	0.21	0.3	0.99
65	0.22	0.19	0.29	0.99
70	0.16	0.17	0.27	0.98
75	0.25	0.24	0.34	1
80	0.21	0.23	0.31	0.99
85	0.27	0.24	0.35	1
90	0.23	0.22	0.36	1
95	0.22	0.21	0.4	1
100	0.26	0.18	0.38	1

The power of W/S test is considerably weak as the values are less than 80% except for Cauchy distribution. The result shows inconsistency of the test statistic as a set of data may belong to any unknown distribution and the ability of the test statistic to perfectly detect that the data is not from Gaussian distribution makes the test sensitive and adequate for the test of normality.

Graphical Representation of Power of W/S Test

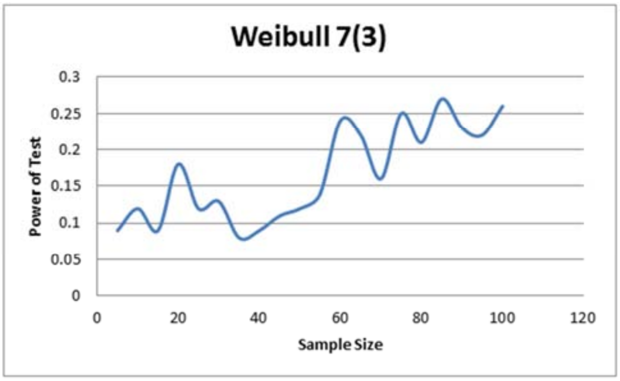

Figure 1. *Power of W/S Test using Weibull distribution.*

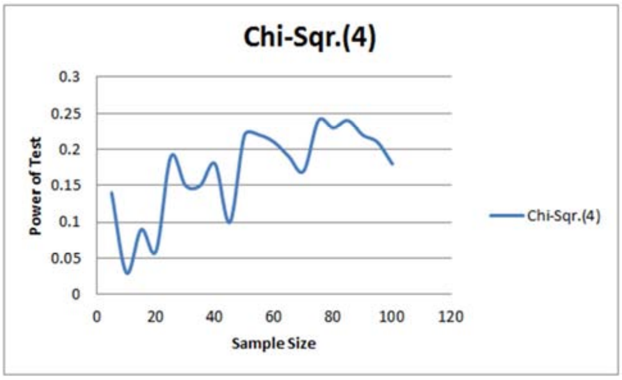

Figure 2. *Power of W/S Test using Chi-Square distribution.*

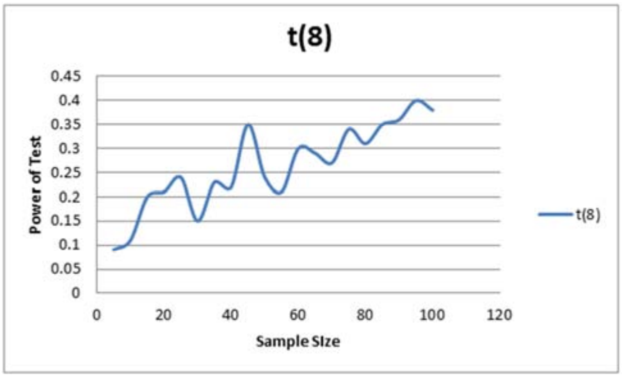

Figure 3. *Power of W/S Test using t distribution.*

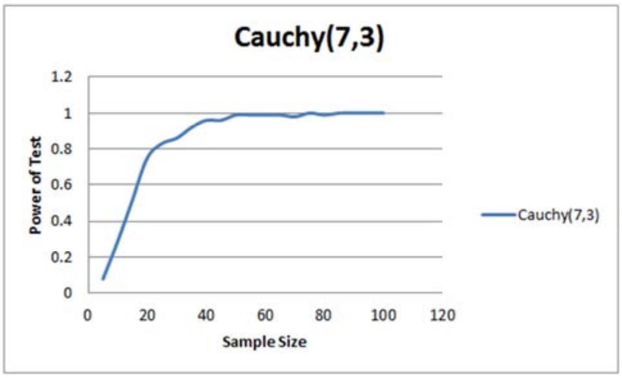

Figure 4. *Power of W/S Test using Cauchy distribution.*

Using Weibull distribution, the highest power of the test is approximately 0.3. Using Chi-Square distribution, the highest power of the test is approximately 0.25. Using t distribution, the highest power of the test is approximately 0.4. Using Cauchy distribution, the highest power of the test is 1.0. The significant fluctuation in the power of the test irrespective of the class of the distribution used led to the conclusion of inconsistency of the test. Having low power is an indication that using the test, there is greater chance of committing type I or type II error.

4. Conclusion

W/S statistic is one of the test statistics for test of normality and one of its advantages is that is require less computation just range and standard deviation unlike most of other test statistics for the test of normality. Critical look of the test of normality frequently used by researchers revealed that researchers hardly use the test despite the easy computation which leads to the investigation of its power when the distribution of a set of data is known. Using Monte Carlo simulation technique, data were simulated from symmetric and asymmetric distributions for both small and large sample sizes bearing in mind the central limit theory.

Sample sizes considered include 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95 and 100. 100 replicate was used to actually observe the rate of deviation of the technique from accurate conclusion and the error rates were noted.

The computation of the power of W/S reveals that the test is inconsistent as it has weak power for three of the four distributions used. Therefore, there is need for proper modification of the test to be able to increase its power and compete favourably with the other known techniques of test of normality.

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