

Riemann's Hypothesis and Critical Line of Prime Numbers

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Abstract: The binary numeral system is applied to the proof of a hypothesis of Riemann to a number of prime numbers. On the ends of blocks of a step matrix of binary decomposition of prime numbers are located a reference point. Because of them there is a jump of an increment of a prime number. The critical line and formula for its description is shown, interpretation of mathematical constants of the equation is given. Increment of prime numbers appeared an evident indicator. Increment is a quantity of increase, addition something. If a number of prime numbers is called figuratively "Gauss-Riemann's ladder", increment can assimilate to the steps separated from the ladder. It is proved that the law on the critical line is observed at the second category of a binary numeral system. This model was steady and at other quantities of prime numbers. Uncommon zero settle down on the critical line, and trivial – to the left of it. There are also lines of reference points, primary increment and the line bending around at the left binary number. Comparison of ranks of different power is executed and proved that the critical line of Riemann is only on the second vertical of a number of prime numbers and a number of their increments.

Keywords: Prime Numbers, Complete Series, Increment, Critical Line, Root $1/2$

1. Introduction

In the course of work on the proof of correctness of a hypothesis of Riemann adhered to Gödel's strong theorem of incompleteness [18].

Incompleteness of the known *law of distribution of prime numbers* [19] is as follows:

- 1) as $n = 1, 2, 3, \dots$ zero isn't considered [19] (the truncated natural number sequence);
- 2) a traditional number of prime numbers $a(n) = 2, 3, 5, 7, \dots$ doesn't consider zero and unit;
- 3) the assumption that «the relation x to $\pi(x)$ upon transition from this degree of ten to the subsequent all the time increases approximately on 2.3» is obviously incorrect;
- 4) the statement that $\pi(x) \sim |x / \ln x|$, offered in 1896 Gauss, transfers prime numbers from a decimal numeral system to a numeral system with the basis $e = 2.71828\dots$;
- 5) power of prime numbers $\pi(x)$ among natural numbers x on orders are accepted in decimal arithmetic, and the relation $x / \ln x$.

2. Example of a Complete Series

The prime number is a natural number $N = \{0, 1, 2, 3, 4, \dots\}$, but only with a divider of 1 (division by itself is excess demand). From prime numbers $P = \{0, 1, 2, 3, 5, 7, \dots\}$ "Gauss - Riemann ladders" [7] separated "steps" with the parameter of an increment of prime numbers $p_j = P_{j+1} - P_j$, where $j = 0, 1, 2, 3, 4, \dots$ - an order rank. Refusal of a numeral system with the basis $e = 2.71828\dots$ led us to the translation of prime numbers from decimal in binary system.

We understood that mathematics, having been fond of factorization of prime numbers, forgot about advantage of decomposition of natural numbers. Thus they were fond of calculation of quantity of prime numbers in decimal categories of natural numbers. In the beginning it was necessary to spread out only prime numbers in a binary numeral system. Only after that it is possible to start the analysis of distribution 0 and 1 on verticals of categories of a binary numeral system.

In table 1 the example of transformation of a complete series from 501 prime numbers (in comparison with a Gaussian number of prime numbers includes 0 and 1 more) from decimal system in the binary is given.

Decomposition of prime numbers happens by known simple rules according to categories ranks $i = 0, 1, 2, 3, 4, \dots$ of system of binary notation. Then on binary numbers z (real

Parameters		Increment in binary system											
Order rank j	Prime number P _j	Incre-ment p _j	Envelope		Category-rank i _j ^p binary								
			border p' _j	category i _{jmax} ^p	6	5	4	3	2	1	0		
					Part of increment								
					32	16	8	4	2	1	2 ⁻¹		
7	13	4	4	3					1	0	0	1/2	
8	17	2	2	2						1	0	1/2	
9	19	4	4	3					1	0	0	1/2	
10	23	6	4	3					1	1	0	1/2	
11	29	2	2	2						1	0	1/2	
12	31	6	4	3					1	1	0	1/2	
13	37	4	4	3					1	0	0	1/2	
14	41	2	2	2						1	0	1/2	
15	43	4	4	3					1	0	0	1/2	
16	47	6	4	3					1	1	0	1/2	
17	53	6	4	3					1	1	0	1/2	
18	59	2	2	2						1	0	1/2	
19	61	6	4	3					1	1	0	1/2	
20	67	4	4	3					1	0	0	1/2	
...	
32	127	4	4	3					1	0	0	1/2	
33	131	6	4	3					1	1	0	1/2	
...	
55	251	6	4	3					1	1	0	1/2	
56	257	6	4	3					1	1	0	1/2	
...	
98	509	12	8	4					1	1	0	1/2	
99	521	2	2	2						1	0	1/2	
...	
173	1021	10	8	4					1	0	1	0	1/2
174	1031	2	2	2						1	0	1/2	
...	
310	2039	14	8	4					1	1	1	0	1/2
311	2053	10	8	4					1	0	1	0	1/2
...
499	3557	2	2	2						1	0	1/2	
500	3559	12	8	4					1	1	0	0	1/2

Note. By gaps are shown a reference point (between the beginning and the end of blocks) among prime numbers.

Symmetric geometrical patterns are shown in table 1, however we didn't carry out their analysis. It is visible that any prime number before itself is related 1/2 under a condition $i_j^p = 0$. But this fractional number $P_{ij} = 2^{i_j^p - 1} = 2^{(0-1)} = 2^{-1} = 1/2$ doesn't enter the sum of the composed.

The complex of mathematical expressions of [4, 5] parameters of a row has an appearance:

$$i_j = (1, m), j = (0, n), m = 6, n = 500, \tag{1}$$

$$P_{j+1} = P_j + p_j, p_j = P_{j+1} - P_j, \tag{2}$$

$$P_j = P'_j + P''_j, P'_j = 2^{i_{jmax} - 1}, P''_j = \sum_{i_j=1}^{i_{jmax} - 1} \xi_{ij} 2^{i_j - 1}, \xi_{ij} = 0 \vee 1, \tag{3}$$

$$p_j = p'_j + p''_j, p'_j = 2^{i_{jmax} - 1}, p''_j = \sum_{i_j=1}^{i_{jmax} - 1} \xi_{ij} 2^{i_j - 1}, \xi_{ij} = 0 \vee 1. \tag{4}$$

In table 1 we have two types of zero – trivial (empty cages) and non-trivial (0). The first for decomposition of prime numbers are located to the left of the broken line with figure 1.

And non-trivial zeros settle down in two columns with figures 1. More difficult with increment – bending around all units is wave, and it always concerns the critical line $i_j^p = 2$.

At a contact of the second vertical couples of prime numbers - Golston's twins are formed.

3. Binary Decomposition of Prime

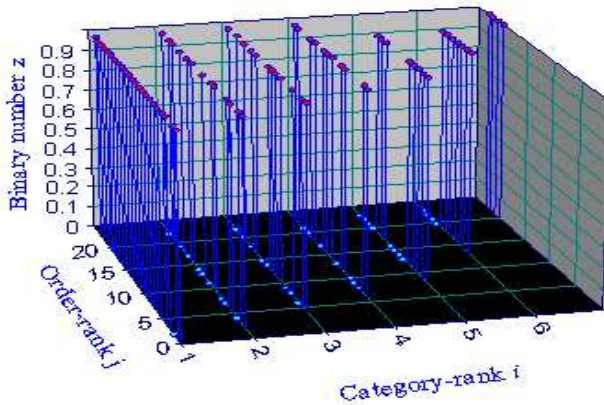
3.1. Mathematical «Landscape»

In movies "De Code" showed a three-dimensional picture of zeta-function of Riemann. Thus round non-trivial zero the slopes striving for infinity settle down.

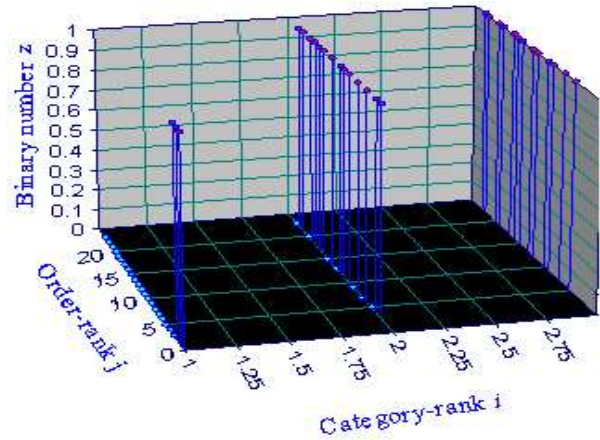
Thus, Riemann's zeta-function, because of attraction of complex numbers, can't give a real spatial picture of distribution of prime numbers. Transition from a decimal numeral system in the binary can give a spatial picture only. The deal in binary system about non-trivial zeros turns infinitely high "mountains" into ledges of the identical height, equal unit. Thus non-trivial zeros turn into points on the plane equal to zero. The landscape is given in figure 1 from 24 first prime numbers. At bigger quantity of prime numbers the landscape becomes not evident.

In figure 1 there is "ceiling" from figures 1, except "floor" from non-trivial zeros. Between them there is an unknown while factorial communication of generation of prime

numbers. Then the difficult surface Riemann's zeta-function, because of representation in complex numbers, will be transformed to "two-layer pie".



Prime numbers at the beginning of a number of distribution



Increment of prime numbers at the beginning of a number of distribution

Figure 1. Mathematical «landscape» of binary distribution of 24 first prime numbers.

Between 0 and 1 according to categories i there is a real function z of binary number.

3.2. Reference Points of Blocks of Binary Decomposition

They are available in table 1 on the left top corners of blocks of prime numbers. Upon transition from the block to the block there is a jump of increment of a prime number.

This jump was unclear still to mathematics. Reference points form block structure of distribution of prime numbers in any type of a row (Gaussian 2, 3, 5, 7, ... or full 0, 1, 2, 3,

5, ... according to table 1). Power of a number of prime numbers can quite be operated by means of reference points, they will be more reliable than decimal categories. Thus a reference point (the end of the previous block and the beginning of the subsequent block) are placed concerning a row $P_{ij} = 2^{(i_j^p - 1)}$.

From table 1 we will write out nodal points P_{R1} (an initial reference point of the block) and P_{R2} (a final reference point of the block) in table 2 and other parameters of reference points.

Table 2. Asymptotic reference points and nodal points of a complete series of 500 prime numbers.

i_j^p	1	2	3	4	5	6	7	8	9	10	11	12
j	0	2	4	6	8	13	20	33	56	99	174	311
P_{ij}	1	2	4	8	16	32	64	128	256	512	1024	2048
P_{R1}	0	2	5	11	17	37	67	131	257	521	1031	2053
P_{R2}	1	3	7	13	31	61	127	251	509	1021	2039	4093
$P_{R1} - P_{ij}$	-1	0	1	3	1	5	3	3	1	9	7	5
$P_{i+1j} - P_{R2}$	1	1	1	3	1	3	1	5	3	3	9	3

Measurement of power of a number of prime numbers on reference points is much more economic $\pi(x)$, that is in comparison with quantity of prime numbers in decimal categories.

3.3. Influence of Prime Numbers According to the First Category of Binary Notation

From table 1 it is visible that at the zero category of a binary numeral system $i_j^p = 0$ is always $z_0 = 1/2$.

If the truncated number of prime numbers begins with 1, in a column $i_j^p = 1$ (fig. 2) there is only one non-trivial zero theoretically on an extent $j = (0, \infty)$.

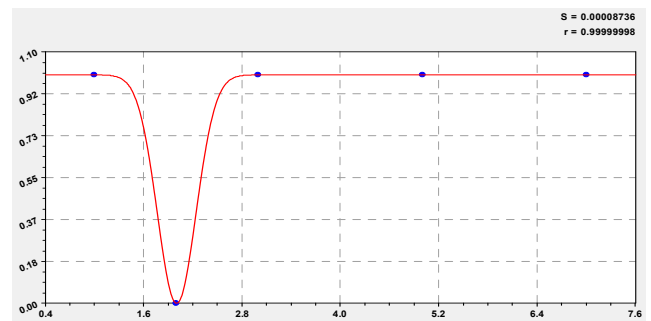


Figure 2. The schedule of a formula (5) of distribution of binary number of the truncated number of prime numbers: S - dispersion; r - correlation coefficient.

Under law of Gauss of «normal» distribution implicitly given by us

$$z_{1j} = 1 - \exp(-10.11900(P_{j=1,2,\dots} - 2)^2) \quad (5)$$

in figure 2 we have a hollow in the form of a hole trap which is a certain barrier transition from a prime number 1 to a prime number 3 through figure 2.

Table 3. First column of a binary matrix

Order-rank j	Prime number P _j	Binary number (at i _j ^p =1)
0	0	0
1	1	1
2	2	0
3	3	1

$$z_{1j} = 1 - \exp(-9.894419P_{j=0,1,2,\dots}^{0.847179}) - \exp(-10.331409(P_{j=0,1,2,\dots} - 2)^2) \quad (6)$$

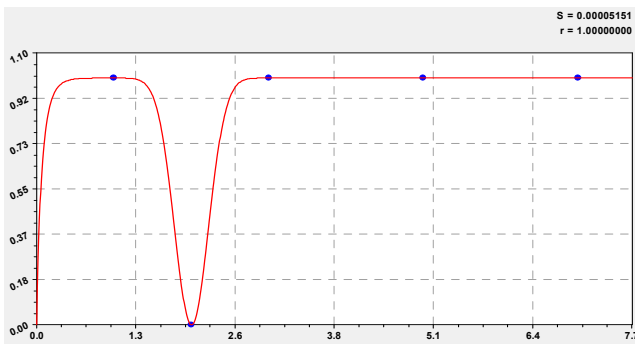


Figure 3. Beginning of the first vertical of a binary matrix of a complete series of prime numbers.

The first component of a formula (6) in the form of 1 specifies that theoretically all lines of the first vertical have to be equal 1. Therefore crisis at zero value of a prime number ranging from 0 to 1 gets out under the law of exponential death. Together two members of the equation (6) form the law of distribution of Weibull $y = a - b \exp(-cx^d)$ under conditions $a = 1$ and $b = 1$. Then between prime numbers 1 and 3 there comes the second crisis, but already under Gauss-Laplace's well-known law, that is under the so-called normal law distribution.

The maximum of recession is the share of prime numbers 0 and 2, that is of two even numbers.

Then prime numbers 0, 1 and 2 are critical, and a noncritical row begins with 3, thus under a condition $j \geq 3$ always there will be an equality $z_1 = 1$.

3.4. Critical Line (Second Vertical)

On Riemann's critical line there is a formula

$$z_{2j} = 1/2 - 0.707107 \cos(\pi P_j / 2 - 0.78540) = \frac{1}{2} - 0.707107 \cos\left(\frac{\pi}{2} P_j - \frac{\pi}{4}\right) \quad (7)$$

Then it was revealed that $0.707107 \rightarrow \sqrt{2}/2$, therefore we

Order-rank j	Prime number P _j	Binary number (at i _j ^p =1)
4	5	1
5	7	1

Further we will give regularities on the first vertical of influence of six prime numbers (they in table 3 are in one position from 0 to 9 decimal numeral systems) for a complete series of prime numbers. From data of table 1 we will write out these numbers.

The beginning of a complete series of prime numbers $P = \{0, 1, 2, 3, 5, 7, \dots\}$ contains two uncommon zero and four units. After identification of two steady laws the statistical model of binary number in the form of a formula was received (fig. 3)

will write down a formula (7) in a look

$$z_{2j} = \frac{1}{2} - \frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{2} P_j - \frac{\pi}{4}\right) \quad (8)$$

Function of a cosine is more convenient for identification. However, after replacement of trigonometric function of a cosine by sine function, from (8) we will receive a formula

$$z_{2j} = \frac{1}{2} - \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{2} P_j\right) \quad (9)$$

4. Mathematical Constants

We executed (fig. 4) the proof «Riemann's well-known hypothesis that the real part of a root always is equal in accuracy 1/2» [19]. This real part is always present at all formulas of distribution of binary number on verticals of binary decomposition of a number of prime numbers of any power. Frequency of fluctuation is equal $\pi/2$, and shift at trigonometric function of a cosine is equal $\pi/4$.

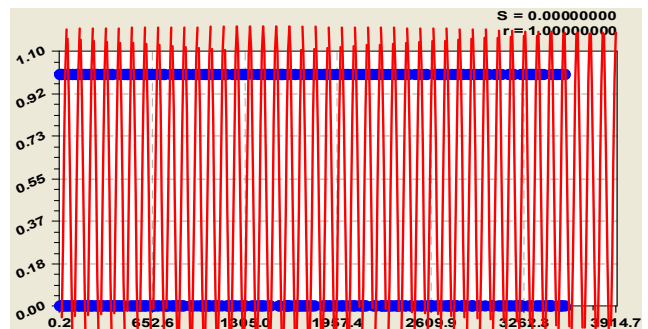


Figure 4. Schedule of a formula (7) of distribution of binary number.

At a formula (9) there are four mathematical constants:

- 1) number 1 [8] - one of the basic concepts in the theory of groups — unit;
- 2) fractional number 1/2 – a rational root of zeta-function of Riemann, that is a basis of the proof of a hypothesis of Riemann by real part of a root [19];

3) irrational number $\sqrt{2}$ - a constant of Pythagoras [16];
 4) $\pi = 3,14159\dots$ Archimedes's number (space number) [16].

The rational root of Riemann in numerator has figure 1, the major figure which is ahead of «metal proportions» [25] in a statement [17] on the following formula

$$\Phi_m = \frac{m + \sqrt{4 + m^2}}{2}, \quad (10)$$

where Φ_m - positive root of a quadratic equation $x^2 - mx - 1 = 0$, m - Fibonacci's [25] numbers, and we accept not $m > 0$, and a condition $m \geq 0$.

If to accept in (10) $m = 0, 1, 2, 3, 4, \dots$, we will receive the following mathematical constants for which at $m > 0$ Vera Shpinadel [25] thought up special names:

$m = 0$, $\Phi_0 = 1$ - the only positive number that is equal to its inverse;

$m = 1$, $\Phi_1 = (1 + \sqrt{5}) / 2$ - gold proportion;

$m = 2$, $\Phi_2 = 1 + \sqrt{2}$ - silver proportion;

$m = 3$, $\Phi_3 = (3 + \sqrt{13}) / 2$ - bronze proportion;

$m = 4$, $\Phi_4 = 2 + \sqrt{5}$ - copper proportion, etc.

In mathematics it is accepted not to carry unit neither to simple, nor to composite numbers. It breaks uniqueness of

decomposition, important for the theory of numbers, on multipliers.

Henri Lebesgue in 1899 was the last of professional mathematicians who considered 1 as a prime number. Karl Sagan included 1 in the list of prime numbers in the book «Contact» which has left in 1985. Other mathematics simply switched off 1 of a row.

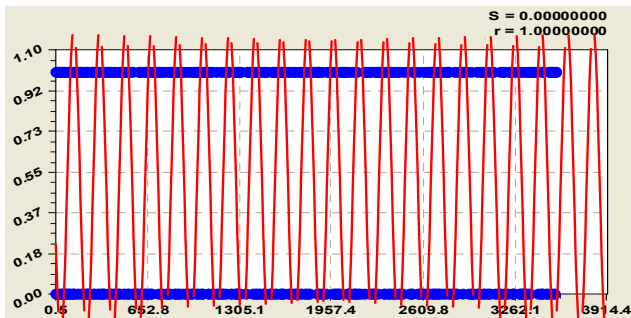
We consider that uniqueness of decomposition on multipliers can't be referred to prime numbers. They are displayed not on multipliers, and on composed in a binary numeral system. So the numbers 0 and 1 is high time to include the full range of prime numbers.

5. Influence of Prime Numbers According to Other Categories

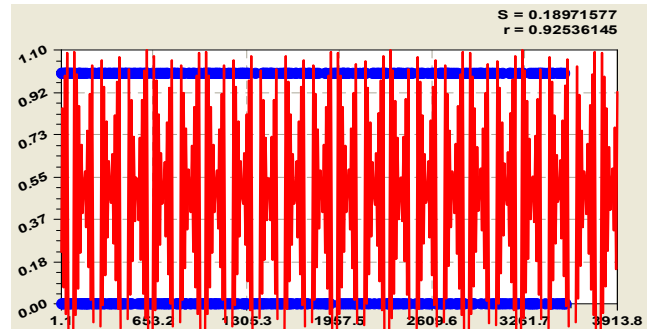
Then the model was received

$$z_{3j} = \frac{1}{2} - \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{4} P_j\right). \quad (11)$$

Mathematical constants remain here (fig. 5). Only the share of number of Archimedes (space number) changes. Thus dispersion is equal to zero, and the coefficient of correlation is equal 1.



Statistical model (11) at the third category



Statistical model (12) at the fourth category

Figure 5. Schedules of distribution of binary number of the making prime numbers.

Montgomery and Dyson applied statistical physical methods of the analysis of distributions in relation to a number of prime numbers and determined the average frequency of emergence of zero. But, it appears, this average frequency through binary transformation of prime numbers turns out functionally connected with number of space $\pi = 3.14159\dots$

And $i_j^p = 2$ with and further in a denominator under value π there is a number of decimal submission of categories of a binary numeral system in the form of numbers 2, 4, 8, 16, 32, etc.

For example, from the remains to the maximum value 0,25 for the fourth category of a binary numeral system from data of table 1 the statistical model was received (fig. 5)

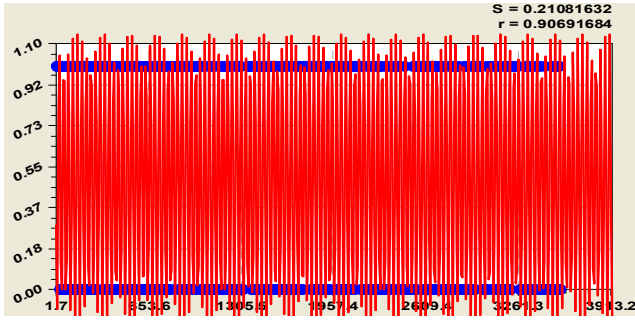
$$z_{4j} = \frac{1}{2} - 0.648348 \sin\left(\frac{\pi}{8} P_j\right). \quad (12)$$

On this fourth vertical dispersion big, and correlation coefficient less than 1. With further increase in power of a complete series of prime numbers correlation, apparently, has to increase, and number 0,648348 has to aspire to $\sqrt{2} / 2$.

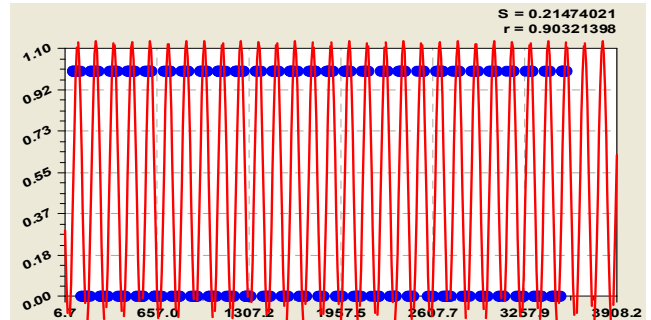
For the fifth and sixth categories (fig. 6) regularities were received:

$$z_{5j} = \frac{1}{2} - 0.643132 \sin\left(\frac{\pi}{16} P_j\right); \quad (13)$$

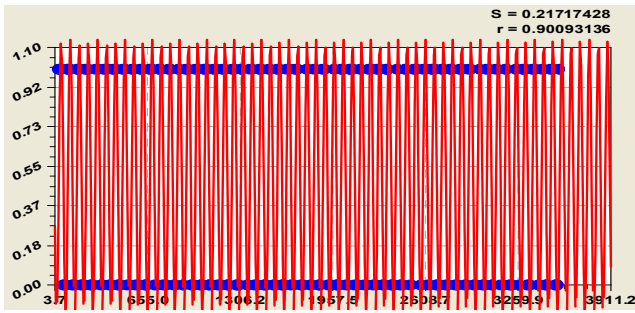
$$z_{6j} = \frac{1}{2} - 0.638209 \sin\left(\frac{\pi}{32} P_j\right). \quad (14)$$



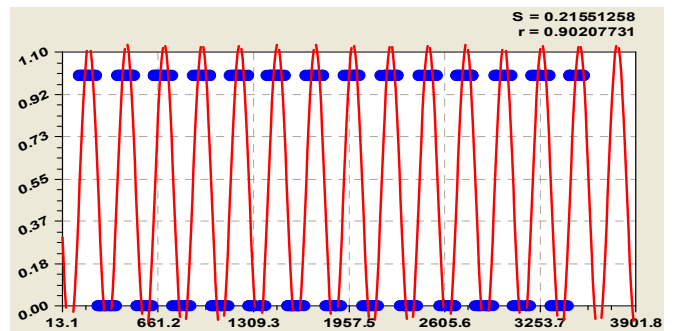
Statistical model (13) at the fifth category



Statistical model (15) at the seventh category



Statistical model (14) at the sixth category



Statistical model (16) at the eighth category

Figure 6. Schedules of distribution of binary number of the making prime.

Figure 7. Schedules of distribution of binary number of the making prime.

It is noticeable that with increase in the category of a binary numeral system the remains (an absolute error of statistical model) increases. It is visible on schedules on decrease in value of coefficient of correlation. Thus formula parameter before a sine also moves away from value $\sqrt{2}/2$, that is from a mathematical constant in the form of a half of a constant of Pythagoras.

In 1972 of Montgomery proved oscillatory character of an arrangement of zero on the critical line. From formulas (9) and others it is visible that they (as well as figures 0 and 1) really fluctuate along verticals of binary decomposition. As we will show further, they fluctuate and across verticals table 1. We explain aspiration of prime numbers, and also their figures 0 transformed in a binary numeral system and 1 on verticals, to run up from each other because of the unknown force arising in a progression $P'_j = 2^{j_{\max} - 1}$.

These forces form block structure of binary decomposition. And non-trivial zeros run up in the plane (i, j) on a formula (3) composed P''_j at incidence $\xi_{ij} = 0 \vee 1$. Thus the border of trivial zeros is removed to the left as it is visible from an arrangement of empty cages in table 1.

For the seventh and eighth categories (fig. 7) formulas of a similar design are received:

$$z_{7j} = \frac{1}{2} - 0.633145 \sin\left(\frac{\pi}{64} P_j\right); \quad (15)$$

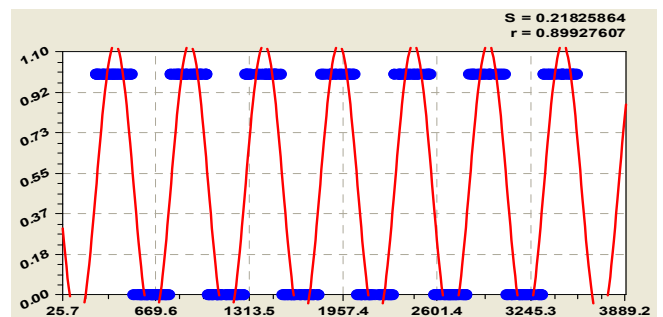
$$z_{8j} = \frac{1}{2} - 0.636929 \sin\left(\frac{\pi}{128} P_j\right). \quad (16)$$

For the ninth and tenth categories (fig. 8) similar regularities were received:

$$z_{9j} = \frac{1}{2} - 0.638599 \sin\left(\frac{\pi}{256} P_j\right); \quad (17)$$

$$z_{10j} = \frac{1}{2} - 0.636726 \sin\left(\frac{\pi}{512} P_j\right). \quad (18)$$

And for the 11th penultimate category of system of binary notation (fig. 9) it was received (at this $z_{12j} = 1$) an empirical formula similar on a design



Statistical model (17) at the ninth category

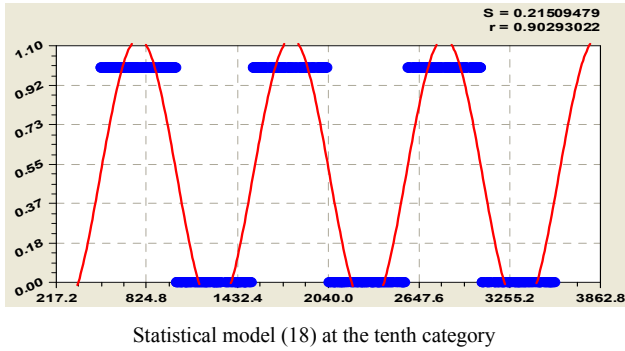


Figure 8. Schedules of distribution of binary number of making prime.

$$z_{11j} = \frac{1}{2} - 0.633526 \sin\left(\frac{\pi}{1024} P_j\right). \quad (19)$$

There is among mathematicians a statement: in distribution of prime numbers there is no geometry (though they considered only quantity of prime numbers in categories of a decimal numeral system). However patterns on a binary matrix in table 1 and on the remains after a formula (17) in figure 9 disprove this statement.

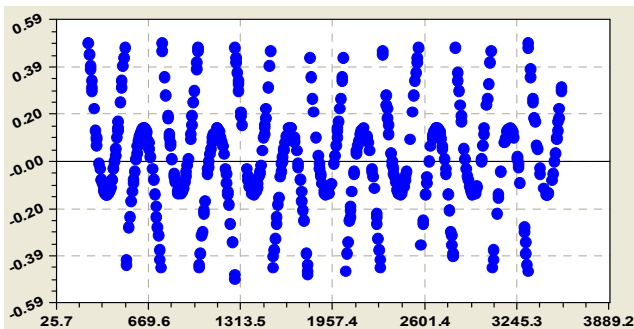
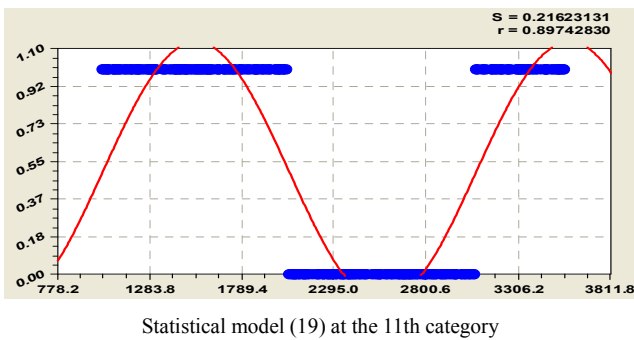


Figure 9. Schedules of distribution of binary number of making prime.

Thus, in all revealed formulas the rational root 1/2 remains. In fact, it also is the proof of a hypothesis of Riemann about the valid root.

Thus, our approach of identification to a number of prime numbers as to statistical selection, unambiguous on accuracy (mathematical object), gives on verticals in binary decomposition of 500+1 prime numbers the correlations of regularity decreasing on coefficient. Thus the formula (9) showing distribution 0 and 1 along a number of prime

numbers becomes the most adequate, and with increase of the category of binary decomposition of prime numbers the coefficient of correlation of the revealed regularities decreases. As it will be shown at generalization of formulas of all verticals by our example, oscillatory indignation is observed.

6. The Generalized Formula of Binary Number

From all previous examples of modeling on table verticals 1 formula for all categories and ranks of prime numbers has an appearance

$$z_{i_j^p} = \frac{1}{2} - a_z \cdot \sin\left(\frac{\pi}{2^{\wedge}(i_j^p - 1)} P_j\right), \quad (20)$$

where a_z - the parameter of statistical model before function of a sine.

In table 4 the actual values of the parameter \hat{a}_z , received from the previous formulas on verticals from table 1, for a row from 501 prime numbers are given.

Table 4. Parameter of the generalized formula (20)

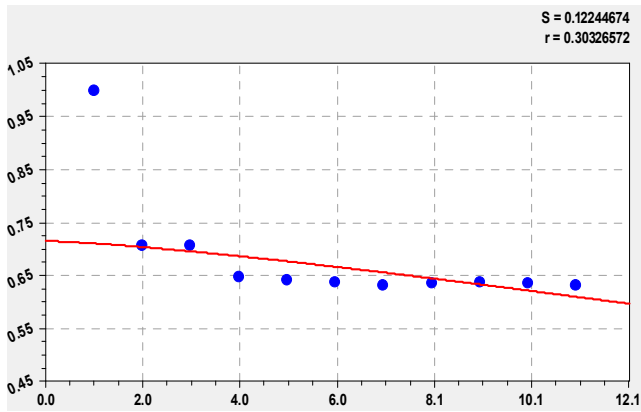
Category	Parameter	Calculated values on model (21)		
		a_z	$\epsilon = \hat{a}_z - a_z$	$\Delta = 100\epsilon / \hat{a}_z$
0	0.5	0.500004	-3.51628e-006	-0.0007
1	1*	0.999857	0.000143339	0.0143
2	0.707107	0.707114	-7.38474e-006	-0.0010
3	0.707107	0.707157	-5.0444e-005	-0.0071
4	0.648348	0.648317	3.13728e-005	0.0048
5	0.643132	0.643001	0.000131208	0.0204
6	0.638209	0.638223	-1.42384e-005	-0.0022
7	0.633145	0.633337	-0.000191533	-0.0303
8	0.636929	0.637091	-0.000161691	-0.0254
9	0.638599	0.638632	-3.31112e-005	-0.0052
10	0.636726	0.636617	0.000108822	0.0171
11	0.633526	0.633645	-0.000119097	-0.0188
12	1	-	-	-

Notes: *Without a barrier hollow on the second vertical; ϵ - the remains or absolute error of model (21), Δ - a relative error of model (21), and the maximum value Δ_{max} is marked out.

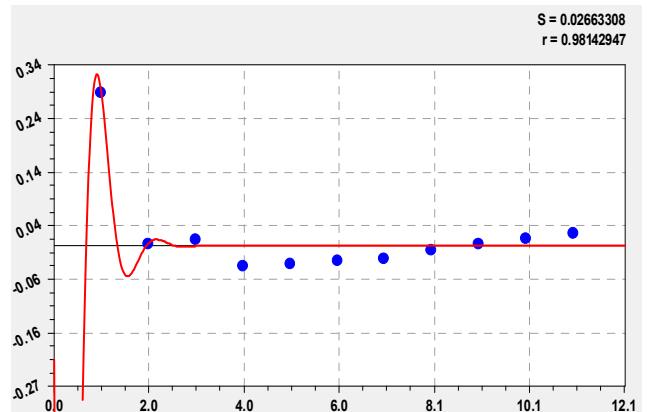
It is obvious that the left border consisting of 1 in table 1, with increase of power of a number of prime numbers will move to the left together with new blocks of binary decomposition. Therefore for statistical modeling we exclude the last line from table 4: she strives for infinity at $j \rightarrow \infty$. On any block from the penultimate category i_j^p value of parameter a_z will aspire to 1. Then, apparently, on the remained ranks there will be a fluctuation a_z .

This hypothesis of existence of fluctuations of binary number is proved by the statistical model (fig. 10) which parameters are given in table 5 in the form of a compact matrix notation with the indication of adequacy of each component in

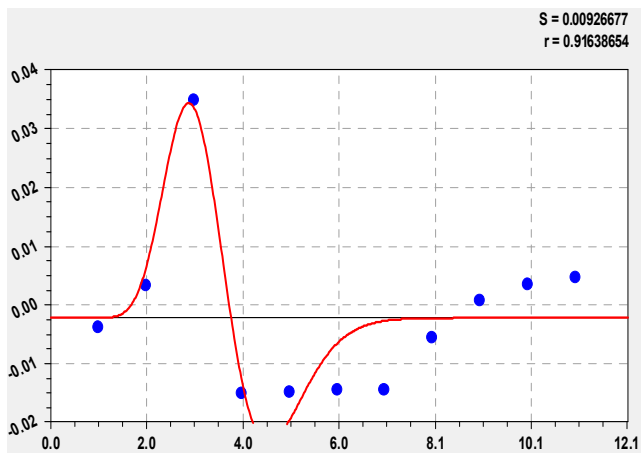
the form of correlation coefficient.



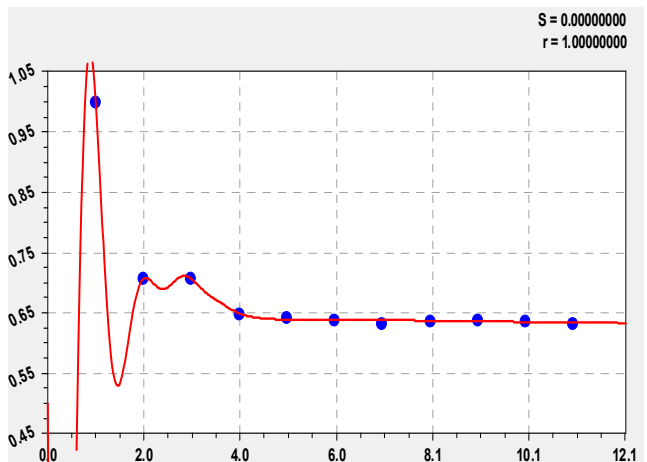
trend (death or recession law)



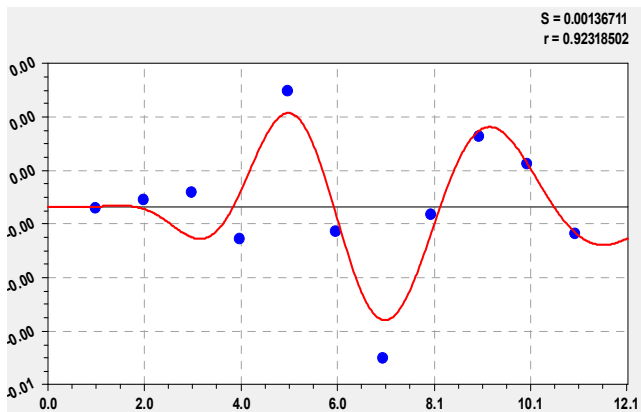
first lonely wave of oscillatory indignation



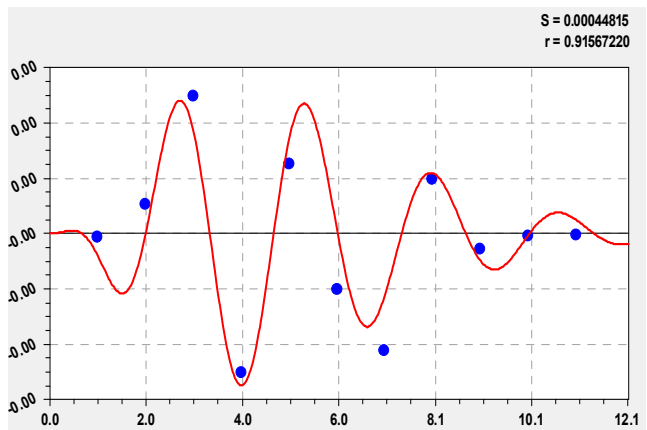
second lonely wave of oscillatory indignation



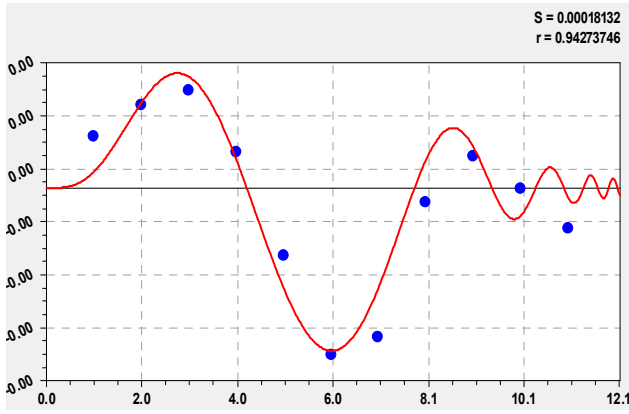
sum of the first three members of model



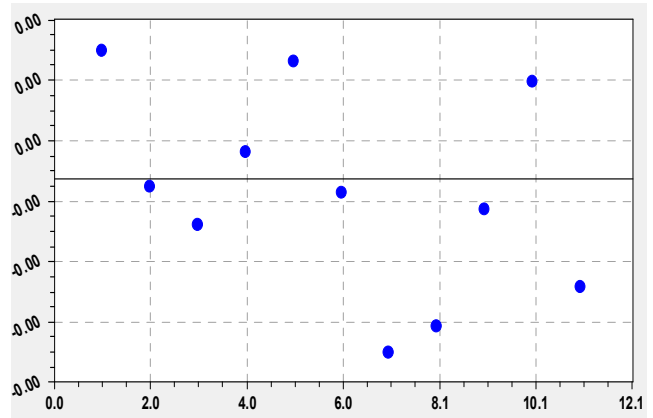
third lonely wave of oscillatory indignation



fourth wave of a finite-dimensional wavelet



fifth wave finite-dimensional wavelet



the remains after six members of the generalized model

Figure 10. Schedules of members of statistical model (21) changes of parameter a_z .

The first three members who were integrated by computing opportunities of the program CurveExpert-1.40 environment, gave the coefficient of correlation which is very close to 1. But on the small remains three additional asymmetric wavelet

signals were identified. Therefore $\Delta_{max} = 0.0303\%$.

The model a_z in parameters from table 5 will register in the form of mathematical expression

$$a_z = \sum_{k=1}^m a_{zk}, \quad x = i_j^p,$$

$$a_{zk} = a_{1k} x^{a_{2k}} \exp(-a_{3k} x^{a_{4k}}) \cos(\pi x / (a_{5k} + a_{6k} x^{a_{7k}}) - a_{8k}), \quad (21)$$

where k - component number, $a_1...a_8$ - model parameters (21).

wave functions. In these fluctuations amplitude and a half-cycle are variables.

Even at the small remains correlation coefficient at three last wavelet signals higher than 0.9. This fact specifies that prime numbers, even at the power of row of 501 piece, have

The last fifth wave has fading character. Therefore it is possible to assume that there is such border among after which waves of indignation will be little significant.

Table 5. Parameters of the general equation of change a_z .

Number k	Asymmetric wavelet $a_{zk} = a_{1k} x^{a_{2k}} \exp(-a_{3k} x^{a_{4k}}) \cos(\pi x / (a_{5k} + a_{6k} x^{a_{7k}}) - a_{8k})$								Correlation coefficient r
	amplitude (half) of fluctuation				fluctuation half-cycle		shift		
	a_{1k}	a_{2k}	a_{3k}	a_{4k}	a_{5k}	a_{6k}	a_{7k}	a_{8k}	
1	0.64713	0	0.030208	0.78597	0	0	0	0	
2	0.024703	0	2.25488	1	0.69442	-0.051466	1	1.52830	1.0000
3	0.024703	18.37957	6.10439	1.00007	6.30158	0.53543	1.11615	-0.028948	
4	3.09780e-5	5.22950	0.71670	1.03549	1.95577	0.0046193	1.56250	1.37043	0.9232
5	0.00067898	2.61171	0.68415	1	1.33906	-5.76923e-5	1	-0.014297	0.9157
6	0.00015541	3.89987	0.88843	1.01866	9.43661	-0.29541	1.32730	0.21280	0.9427

The proof of such hypothesis requires statistical modeling on more powerful finite-dimensional ranks of prime.

7. Increment of Prime Numbers

7.1. The Concept of Increment

This new indicator was evident and at the same time mathematically simpler for studying of a number of prime numbers. Increment changes according to the same categories of system of binary notation therefore further we accept designation $i = i_j^p = i_j^p$. For an example we accept a Gaussian number of prime numbers 2, 3, 5, ... with increment at $i \geq 1$.

Increment is a quantity of increase, addition something.

If a traditional number of prime numbers $a(n) = \{2, 3, 5, 7, 11, 13, 17, \dots\}$ long ago figuratively call "Gauss-Riemann ladder", increment can quite assimilate to the steps separated from a bearing farm of the basis of this ladder. The long and high ladder physically quite may contain two parts – separately a design from steps and separately a basis farm.

The algorithm of building of a Gaussian number of prime numbers as it is widely known, has an appearance

$$a(n+1) = a(n) + p(n), \quad (22)$$

where $p(n)$ - increment of a prime number, n - an order (serial number) of a prime number.

A number of prime numbers is set initially, it is defined by check on simplicity by an indivisibility condition on numbers, except as on 1 and itself (the last condition superfluous).

Therefore increment is always calculated by way of subtraction on expression

$$p(n) = a(n+1) - a(n) \tag{23}$$

7.2. Increment of 500 Prime Numbers

In table 6 increment fragments from a number of prime numbers $a(n) = \{2, 3, 5, \dots, 3571\}$ are given. Thus by italics from table 1 the first two lines are transferred.

Table 6. Number of increment of prime numbers in the 10th and binary.

Order n	Prime number a(n)	Increment p(n)	Category i of binary system					
			6	5	4	3	2	1
			Part of an increment			$p_i(n) = 2^{i-1}$		
			32	16	8	4	2	1
-	0	1						1
-	1	1						1
1	2	1						1
2	3	2					1	0
3	5	2	trivial zeros				1	0
4	7	4				1	0	0
5	11	2					1	0
6	13	4				1	0	0
7	17	2					1	0
8	19	4				1	0	0
9	23	6				1	1	0
10	29	2					1	0
11	31	6				1	1	0
12	37	4				1	0	0
13	41	2					1	0
14	43	4				1	0	0
15	47	6				1	1	0
16	53	6				1	1	0
17	59	2					1	0
18	61	6				1	1	0
19	67	4				1	0	0
20	71	2					1	0
21	73	6				1	1	0
22	79	4				1	0	0
23	83	6				1	1	0
24	89	8			1	0	0	0
25	97	4				1	0	0
26	101	2					1	0
27	103	4				1	0	0
28	107	2					1	0
29	109	4				1	0	0
30	113	14			1	1	1	0
31	127	4				1	0	0
32	131	6				1	1	0
33	137	2					1	0
...
496	3541	6				1	1	0
497	3547	10			1	0	1	0
498	3557	2					1	0
499	3559	12			1	1	0	0

Among 500 prime numbers there was one maximum increment $p(217) = 34$ with a code 100010 in a binary numeral system for a prime number $a(217) = 1327$.

Radical difference of a number of increment from the number of prime numbers is that in increment (too number – an abstract measure of quantity) only one column $1_2 = 2$ of the category of binary numbers is completely filled on a vertical and critical according to Riemann, and the first category has only zeros for a set $a(n) > 2$ (tab. 6). Other difference – increment has always even numbers, since a prime number 3, and prime numbers always the odd.

Full filling of the second vertical $i=2$ will proceed indefinitely therefore it is possible to take for granted the fact emergence $p(n) = 2$ (increment of numbers of Golston or simple numbers-twins) at any power $a(n)$.

Then from table 6 considerably that non-trivial zeros are on only two verticals at $i=1$ and $i=2$. They appear on the third and the subsequent verticals only at $p(n) = 8, 10, 16, 18, \dots$ with an exception of couples $p'(n) = 12, 14, 28, 32, \dots$ (their non-trivial zeros appear only on the first and second verticals). Therefore blocks become rather small on length along verticals. In this regard trivial zeros at the left along all second vertical approach directly to it and form simple numbers-twins (Golston's number).

From a noncritical prime number 3 on the first vertical $i=1$ always will be figure 0, and the second vertical, in comparison with a number of prime numbers, in a series of increments will be Riemann's critical line. We also will prove it further statistical modeling of wave function of change of binary number z .

If to compare patterns in tables 1 and 6, the geometry of increment is simpler in comparison with geometry of a number of prime numbers. In table 6 the diamond-shaped structure which in different places of a number of prime numbers is broken is noticeable. These violations, apparently, have also any substantial sense. Therefore studying of topology of patterns becomes actual.

7.3. Mathematical Landscape of Increment

All pay attention to uncommon zero on the critical line. They were counted already by some trillions. But still mathematicians have no confidence in them not a triviality. From data of table 6 on increments it is visible, on Riemann's critical line $i=2$, that is on the second vertical of binary decomposition of increment of prime numbers, there is a natural alternation of non-trivial zero and 1, in trivial zeros start appearing already from the third vertical. The proof of that non-trivial zeros can be and not on Riemann's critical line, it is visible also from table 6: for example, the prime number 89 at increment 8, for the first time has an non-trivial zero on the third vertical $i=3$.

For creation of a mathematical landscape of increment (fig. 1) we will accept in an example $i = i_2 = 1, 2, 3, 4, 5$ and we will exclude those lines in which on five verticals there is at least one trivial zero. It recognizes that empty cages (trivial cages in table 6) aren't figures and therefore the program

CurveExpert-1.40 environment doesn't perceive them.

Then minimum, considered in figure 11, increment it was equal 16.

Indicator is the binary number $z = z_2$ in the field of real numbers (0;1). The mathematical landscape of increment in figure 11 was more difficult than a landscape in figure 1.

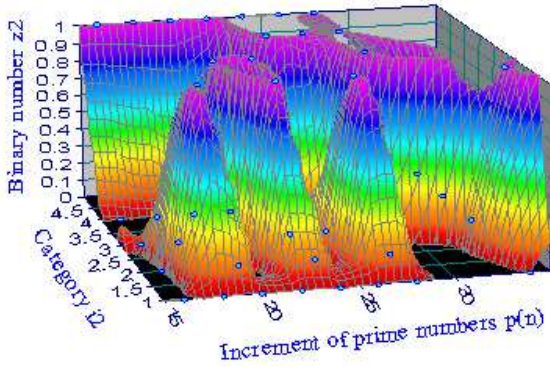
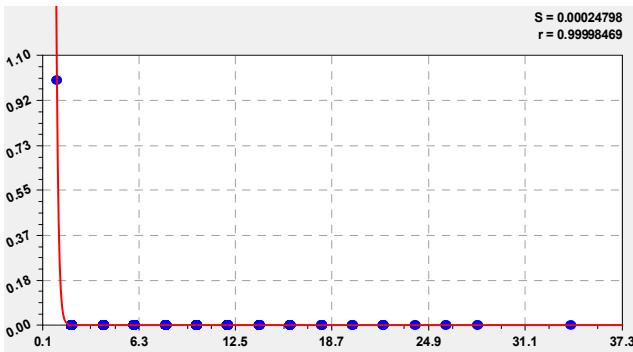


Figure 11. Increment landscape among from 500 prime numbers.

Blocks of binary decomposition of increment have the difficult line in comparison with in steps extending to the left borders of blocks of a number of prime numbers (because of emergence of simple numbers-twins and approach of the left



the statistical model (24) of the Gaussian a number of prime

border from 1 to the second vertical or Riemann's critical line).

7.4. Influence of Increment on the First Category

Bernhard Riemann in 1859 by results of the analysis of zeta-function claimed that non-trivial zeros are on one line. Nowadays believe that it as the critical line crosses a mathematical landscape of zeta-function.

However from data of table 6 it is visible that for the new parameter of a row – increment of prime numbers – such only line is. It is a vertical $i = 2$. We will show that other verticals for making prime numbers slowly come nearer to the critical line under a condition $j \rightarrow \infty$. Thereby once again we will confirm Hardy's proof that among there is an infinite set of non-trivial zeros which part can not lie on Riemann's critical line (see a prime number 89 at increment 8 in table 6).

For the first category $i = 1$ (fig. 12) on non-trivial zeros to verticals we have:

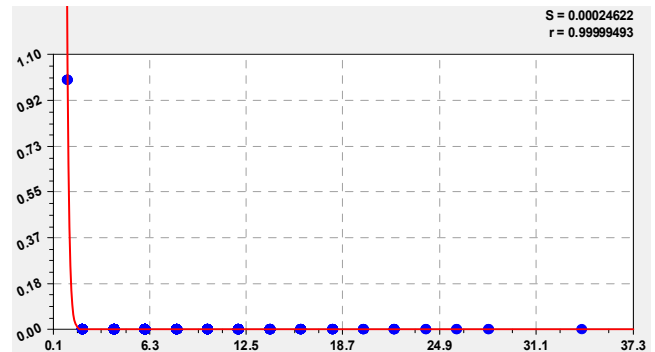
- for a Gaussian row from 499 prime numbers

$$z_{1j} = 1772.5075 \exp(-7.48032p_j)$$

- Laplace's law (in the physicist - Mandelbrot); (24)

- for a complete series 500+1 prime numbers

$$z_{1j} = 1781.3383 \exp(-7.48515p_j) . \tag{25}$$



model (25) of a complete series of prime numbers

Figure 12. Schedules of distribution of binary number at increment of prime numbers on the first category of binary system.

Influence of increment on binary number depends on type of a number of prime numbers a little. However it is clear that at theoretical zero increment we receive the huge binary number 1781 much more exceeding 1. This fact indicates abnormal influence of zero increment.

7.5. Increment Influence on the Second Category

On the second vertical, as can be seen from table 6, in both rows increment (a Gaussian row and a complete series of prime numbers) in the beginning are trivial zeros (empty cages). Therefore mathematical processing is possible only since a line $a(n) = 3$.

After statistical modeling we receive the fundamental law increment (fig. 13) for any number of prime numbers in the form of the equation

$$z_{2j} = \frac{1}{2} - \frac{1}{2} \cos\left(\frac{1}{2} \pi p_j\right) . \tag{26}$$

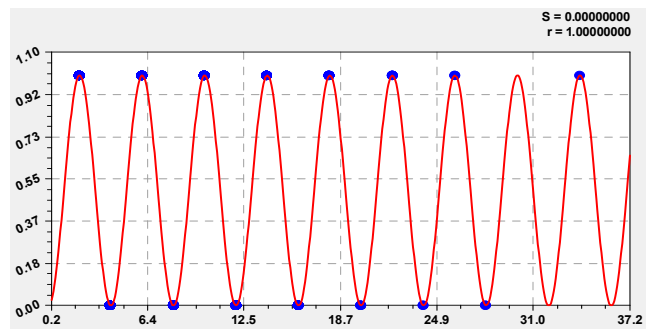


Figure 13. The schedule of distribution of binary number depending on increment of 500+1 prime numbers on model (26) on Riemann's critical line (on the second vertical of table 6).

Riemann's critical line $i=2$ received an unambiguous formula of influence of increment on binary number, and with trigonometric function of a cosine without wave shift.

Here the mathematical constant $\sqrt{2}$ was excluded. In (26) there were three mathematical constants: 1) number 1; 2) $1/2$ – rational root on Riemann's hypothesis (this root $1/2$ in a formula (26) meets three times); 3) $\pi=3.14159\dots$ Archimedes's number (space number). Then it turns out that because of existence π formula (26) shows essence of properties at space, and without time.

7.6. Influence of Increment on Other Categories

For the third category $i=3$ of system of binary notation according to table 6 we will exclude from cage verticals with trivial zero. Then at simple numbers-twins 97 first numbers will be excluded and will remain 402 lines.

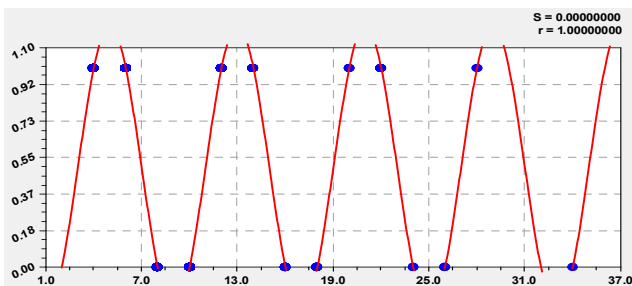
After identification (fig. 14) of 402 values regularity of a look was received

$$z_{3j} = \frac{1}{2} - \frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{4} p_j - \frac{\pi}{4}\right), \tag{27}$$

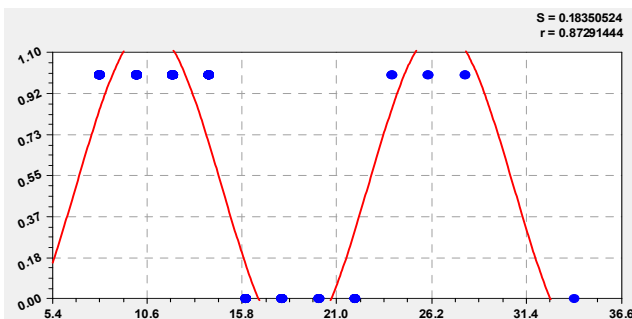
which doesn't coincide with a formula (11) only on fluctuation shift ($\pi/4$ instead of $\pi/2$).

For the category $i=4$ (fig. 14) on 183 values the trigonometric formula is received

$$z_{4j} = \frac{1}{2} - \frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{8} p_j - 1,05021\right). \tag{28}$$



statistical model (27) at the third category of an increment



statistical model (28) at the fourth category

Figure 14. Schedules of distribution of binary number from influence increment of prime numbers.

Still the bigger deviation from the critical line occurs on the fifth category. From 37 points on the fifth vertical only one

receives 0, and other 36 increment matter 1. Therefore for increase of adequacy of models on the fourth and the subsequent verticals it is necessary to increase the power of a number of prime numbers.

7.7. Riemann's Critical Line

We will in more detail consider the second vertical. The first three lines from table 6 drop out of a set. After that with $a(n)=3$, and on any length at a number of prime numbers, the binary number is equal in the first column $i=1$ to zero. Then each increment value from right to left begins from zero and comes to the end with unit. And behind unit in the form of the wave broken line only trivial zeros settle down.

All non-trivial zeros are located in any line between 1 (at the left) and 0 (on the right a column $i=1$). Then Riemann's critical line is unambiguously located vertically in a column $i=2$. Further we will make the analysis of increment on a number of noncritical prime numbers $a(n)=3,5,\dots$.

8. Features Increment

8.1. Influence of the Category i

In the Excel environment we will summarize table 6 on columns with $a(n)=3$) and we will receive quantity of units $\sum z$ according to categories of binary system (tab. 7).

Table 7. Influence of the category of binary system (498 lines).

i	p_i	$\sum z$	Share 1	$\sum(z=0)$	Share 0	$2^{i-1} \sum z$	$\frac{\sum z}{\sum \sum z}$
1	1	0	0	498	1	0	0
2	2	298	0.5984	200	0.4016	596	0.3855
3	4	285	0.5723	213	0.4277	1140	0.3687
4	8	153	0.3072	345	0.6928	1224	0.1979
5	16	36	0.0723	462	0.927	576	0.0466
6	32	1	0.0020	497	0.9980	32	0.0013
Total	773	-	-	2215	-	3568	-

Models it is better to give in relative sizes - shares that allows to compare ranks of prime numbers, different in power.

After identification of the biotechnical law [4-7] the following statistical regularities were obtained:

- shares of units in (fig. 15) lines of a binary matrix of increment of prime numbers

$$v(1) = \sum z / 498 = 0.61623(i-1)^{0,28783} \times \exp(-0.029314(i-1)^{3,22295}); \tag{29}$$

- shares of zero in (fig. 15) lines of a binary matrix of increment of prime numbers

$$v(0) = (498 - \sum z) / 498 = 1 - 0.61623(i-1)^{0,28783} \times \exp(-0.029314(i-1)^{3,22295}). \tag{30}$$

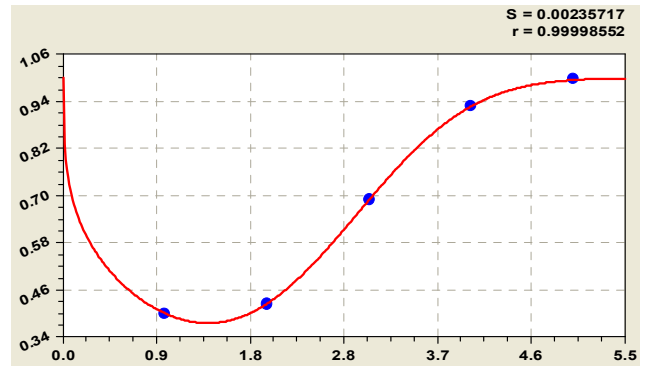
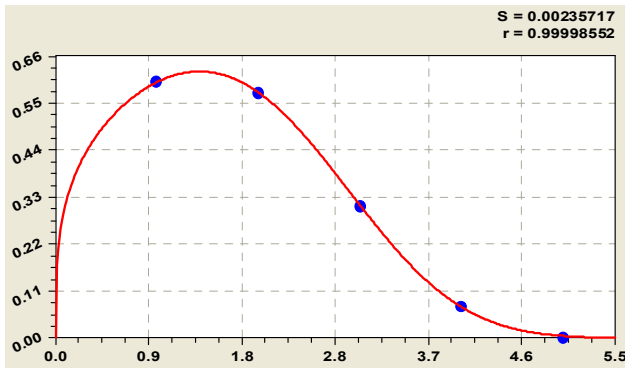


Figure 15. Schedules of distribution of binary number from influence increment of prime numbers.

In favor of calculations of quantity of units there are two distinctive features:

- 1) the number of zero (trivial and non-trivial) are almost three times more than units (tab. 7);
- 2) on a design the formula (29) is simpler in comparison with expression (30).

Apparently, parameter 0.61623 with growth of a row $n \rightarrow \infty$ will come nearer to a golden ratio 0.618 Then on the critical line are $\phi^{-1} = 0.618\dots$ units and 0.6182 zeros.

The contribution of the sum of units on columns (fig. 16) to the sum (in table 7 it is equal 773) will be equal

$$\sum z / \sum \sum z = 0,39902(i-1)^{0,32247} \exp(-0.034914(i-1)^{3,09819}). \tag{31}$$

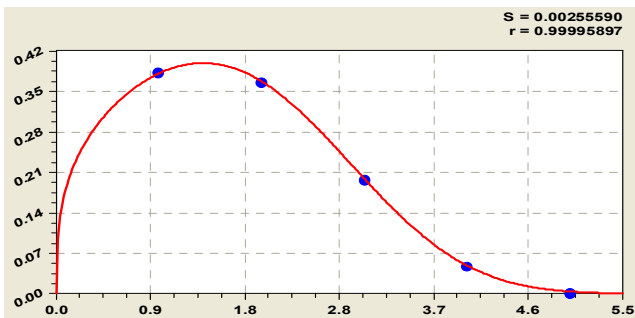


Figure 16. Schedule of the contribution to the sum 1 on the columns of table 7.

On the critical line $i = 2$ the contribution of the sum 1 will come nearer to a square of a golden ratio. Earlier, on growth of a prime number through an influence formula (10) m - Fibonacci's [25] numbers, we indirectly received functional communication with a gold proportion and others mathematical constants. Thus found out that number 1 - one of the basic concepts in the theory of groups - is a fundamental mathematical constant. Therefore, dividing 1 on 2, we receive fractional number $1/2$ - a rational root of zeta-function of Riemann [19].

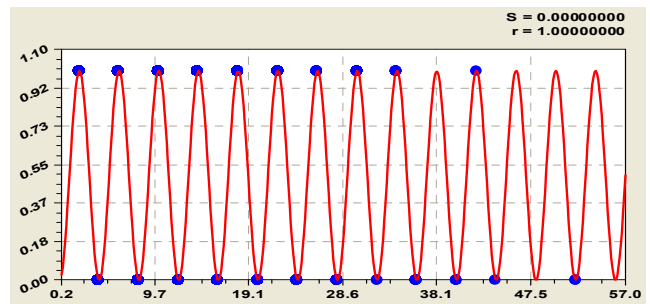
And by consideration separately steps of a ladder of Gauss-Riemann in formulas (29) and (30) we accurately received that on Riemann's critical line (the second vertical) increment are units $\phi^{-1} = 0.618\dots$ and 0.6182 zero. On a formula (10) under a condition $m = 1$ we receive - $\phi = \Phi_1 = (1 + \sqrt{5}) / 2$ a gold proportion - or number 1.618

The beginning of coordinates accurately is defined in a point $(Z = 0, P_z = 0)$. It is a singularity point because by existing definition of a prime number (property division of on itself) there is a division of a prime number on itself, that is $0 / 0$. Thus division only on 1 turns this point into zero.

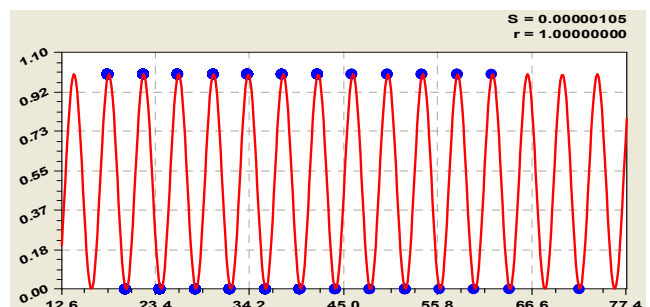
8.2. Verification of the Law (26)

On the critical line $i = 2$ the specified model of the law is steady and at other quantities of prime numbers (fig. 17).

The design of model (26) doesn't change. With increase in power of a row to 3000 the quantity of points increases in the schedule (fig. 17). For check the subset (1704 lines) of increment $p(n) \geq 18$ from 100 000 prime numbers was taken. From this it follows that in any selection our law (26) critical lines is observed.



Increment of prime numbers among from 3000 members



Increment of more than 16 in a number of 100 000

Figure 17. Schedules of the law of distribution of binary numbers 0 and 1.

8.3. Minimum Selection of Prime Numbers

Method of cutting off of the last lines of table 6 (tab. 8), we will define the minimum selection where the steady law of the critical line still works.

Table 8. Minimum number of prime numbers.

Order n	Prime number a(n)	Increment p(n)	Category _i of number							
			6	5	4	3	2	1		
			Part of increment							
			32	16	8	4	2	1		
2	3	2	trivial zeros						1	0
3	5	2							1	0
4	7	4							1	0

The minimum number of noncritical prime numbers is formed by only three members on whom the equation (5) with the rational parameters specified in table 6 was received.

The error of approach $0.5 \rightarrow 1/2$ is negligible.

The schedule of the simple equation on a design is shown in figure 18.

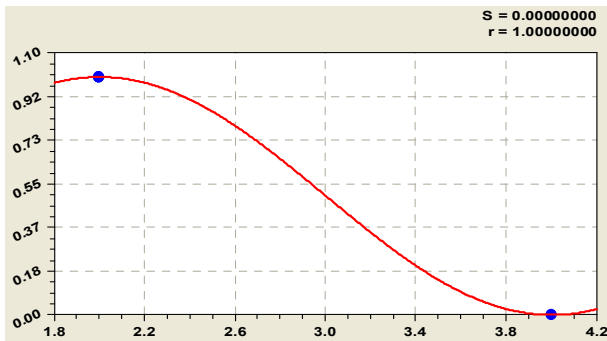


Figure 18. The schedule for three prime numbers

At other categories $i_2 > 2$ (tab. 9) the increasing quantity (power) of prime numbers is necessary.

Table 9. Influence of increment of a prime number on binary number on the second to categories of binary system.

Prime number r a(n)	Incre ment p(n)	Parameters (5)			Correlation coefficient r	Fault ε
		a ₁	a ₂	a ₃		
3	2	1	1			-9.989e-10
5	2	2	2	2	1	-9.989e-10
7	4					9.989e-10

Between increment and its component there is a regularity of transition of numbers from a decimal numeral system in the binary.

8.4. Reference Points

The first left units form the asymptotic line more to the left of which there are only trivial zero.

We will consider reference points in a row from 500 prime numbers.

Reference point form blocks. In the massif from 500 points their there is some (tab. 10), only five.

Table 10. Reference point of increment of 500 prime numbers.

Order n	Prime number a(n)	Increment p(n)	Category _i binary							
			6	5	4	3	2	1		
			Part of increment							
			32	16	8	4	2	1		
2	3	2							1	0
4	7	4							1	0
24	89	8							1	0
9	523	18							1	0
217	1327	34	1	0	0	0	0	1	0	

Proceeding from a condition that at the beginning of a number (tab. 2) of increment is equal to unit, the formula was received (fig. 19)

$$p_R(n) = \exp(0.80738a_R(n)^{0.20489}), \quad (32)$$

where the index R designates a reference point of prime number. Application of reference points is much more compact than the relation $x / \pi(x)$.

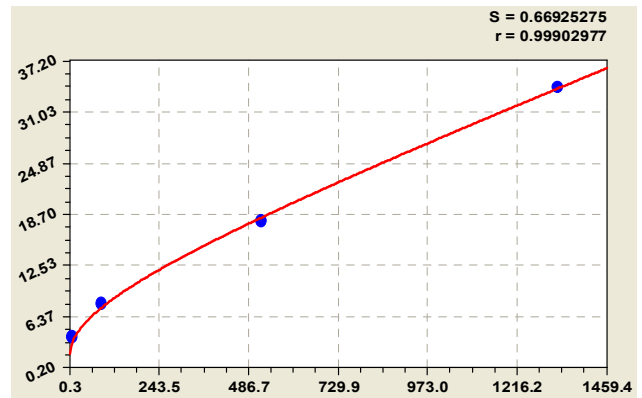


Figure 19. Schedule increment function reference point.

8.5. Primary Increment

It is the third indicator (the first – the critical line 1/2), giving a picture of growth of increment of prime numbers.

Parameter $p_p(n)$ for a row from 100 000 prime numbers is specified in table 11. Primary increment is irregular, for example, increment 14 appears after 8 and before values 10 and 12.

Table 11. The primary increment in series of 100 000.

Prime number a(n)	Increment p(n)	Category _i binary							
		6	5	4	3	2	1		
		Part of increment							
			32	16	8	4	2	1	
3	2							1	0
7	4							1	0
23	6							1	1
89	8							1	0
113	14							1	1
139	10							1	0
199	12							1	0
523	18							1	0
887	20							1	0
1129	22							1	0
1327	34	1	0	0	0	0	1	0	
1669	24							1	0

Prime number a(n)	Increment p(n)	Category i binary					
		6	5	4	3	2	1
		Part of increment					
		32	16	8	4	2	1
1831	16		1	0	0	0	0
2477	26		1	1	0	1	0
2971	28		1	1	1	0	0
4297	30		1	1	1	1	0
5591	32	1	0	0	0	0	0
9551	36	1	0	0	1	0	0
15683	44	1	0	1	1	0	0
16141	42	1	0	1	0	1	0
19333	40	1	0	1	0	0	0
19609	52	1	1	0	1	0	0
28229	48	1	1	0	0	0	0
30593	38	1	0	0	1	1	0
34061	62	1	1	1	1	1	0
35617	54	1	1	0	1	1	0

Various font allocated triangles (geometry patterns) with the parties (at $i_2 = 1$ – non-trivial zeros). Then harmonious geometrical structures define algorithm of building of increment and even a prime number.

The increment line changes with initial constant "two", and fluctuation, on a trend will be farther

$$p_p(n) = 2 + 2.09287p(n)^{2.09287} \exp(-0.31341p(n)^{1.6442}). \quad (33)$$

For a condition $n \rightarrow \infty$ always will be $p_{\min}(n) = 2$.

8.6. Envelope Line

Increments more to the left of the asymptotic line have trivial zero. Therefore the wave envelope line which in different places concerns the critical line is considered $i_2 = 2$. It is the fourth parameter of a row. We will divide increment into two parts $p(n) = p'(n) + p''(n)$. On the envelope line by line are $p'(n) = 2^{i_{2\max} - 1}$ in the table (fig. 20). And in blocks $0 \leq p''(n) = 2^{i_{2\max} - 1} - 1$.

The trend with unit from a formula with three fluctuations has an appearance

$$p'(n) = 1 + 0.59470p(n)^{1.06436} + \dots \quad (34)$$

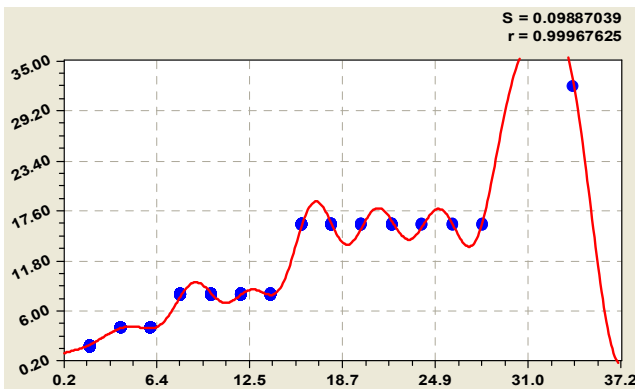


Figure 20. Schedule increment function reference point.

At $n \rightarrow \infty$ in a formula (34) will always be at the beginning of 1.

9. Conclusions

The well-known hypothesis of Riemann is proved. Transformation of a number of prime numbers was for this purpose executed from a decimal numeral system in the binary. Four new criteria are received. There were geometrical patterns. Became visible "on the floor" non-trivial zeros and appeared instead of the steep "hills" zeta-function units "on the ceiling" of the distribution of 0 and 1.

The critical line of Riemann is located vertically in a column $i_2 = 2$ of a binary matrix of increment of a prime number. Not all non-trivial zeros settle down on it. There are also lines of reference points, primary increment (gives patterns of symmetry) and envelope.

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