International Journal of Discrete Mathematics
2019; 4(1): 45-51
http://www.sciencepublishinggroup.com/j/dmath
doi: 10.11648/j.dmath.20190401.17
ISSN: 2578-9244 (Print); ISSN: 2578-9252 (Online)

(Alpha, Beta)-Normal and Skew n-Normal Composite Multiplication Operator on Hilbert Spaces

Senthil[1, *], Nithya[2], Suryadevi[3], David Chandrakumar[3]

[1]Department of Economics and Statistics, Government of Tamilnadu, DRDA, Dindigul, India
[2]Department of Mathematics, Mother Teresa Women's University, Kodaikanal, India
[3]Department of Mathematics, Vickram College of Engineering, Enathi, India

Email address:
senthilsnc83@gmail.com (Senthil)
[*]Corresponding author

To cite this article:
Senthil, Nithya, Suryadevi, David Chandrakumar. (Alpha, Beta)-Normal and Skew n-Normal Composite Multiplication Operator on Hilbert Spaces. *International Journal of Discrete Mathematics*. Vol. 4, No. 1, 2019, pp. 45-51. doi: 10.11648/j.dmath.20190401.17

Received: February 15, 2019; **Accepted**: March 19, 2019; **Published**: May 6, 2019

Abstract: In this paper, the condition under which composite multiplication operators on Hilbert spaces become skew n-normal operators, (Alpha, Beta)-normal, parahyponormal and quasi-parahyponormal have been obtained in terms of radon-nikodym derivative.

Keywords: Composite Multiplication Operator, Conditional Expectation, Aluthge Transformation, Skew n-Normal Operator, Parahyponormal

1. Introduction

Let $(X,\sum,\mu)$ be a σ-finite measure space. Then a mapping T from X into X is said to be a measurable transformation if $T^{-1}(E)\in\sum$ for every $E\in\sum$. A measurable transformation T is said to be non-singular if $\mu(T^{-1}(E))=0$ whenever $\mu(E)=0$. If T is non-singular then the measure μT^{-1} defined as $\mu T^{-1}(E)=\mu(T^{-1}(E))$ for every E in $\sum$, is an absolutely continuous measure on $\sum$ with respect to μ. Since μ is a σ-finite measure, then by the Radon-Nikodym theorem, there exists a non-negative function f_0 in $L^1(\mu)$ such that $\mu T^{-1}(E)=\int_E f_0\,d\mu$ for every $E\in\sum$. The function f_0 is called the Radon-Nikodym derivative of μT^{-1} with respect to μ.

Every non- singular measurable transformation T from X into itself induces a linear transformation C_T on $L^p(\mu)$ defined as $C_T f = f\circ T$ for every f in $L^p(\mu)$. In case C_T is continuous from $L^p(\mu)$ into itself, then it is called a composition operator on $L^p(\mu)$ induced by T. We restrict our study of the composition operators on $L^2(\mu)$ which has Hilbert space structure. If u is an essentially bounded complex-valued measurable function on X, then the mapping M_u on $L^2(\mu)$ defined by $M_u f = u\cdot f$, is a continuous operator with range in $L^2(\mu)$. The operator M_u is known as the multiplication operator induced by u. A composite multiplication operator is linear transformation acting on a set of complex valued $\sum$ measurable functions f of the form

$$M_{u,T}(f)=C_T M_u(f)=(u\circ T)\ (f\circ T)$$

where u is a complex valued, $\sum$ measurable function. In case $u=1$ almost everywhere, $M_{u,T}$ becomes a composition operator, denoted by C_T.

In the study considered is the using conditional expectation of composite multiplication operator on L^2-spaces. For eac $f\in L^p(X,\sum,\mu)$, $1\le p\le\infty$, there exists an unique $T^{-1}(\sum)$-measurable function $E(f)$ such that

$$\int_A g\, f\, d\mu = \int_A g\, E(f)\, d\mu$$

for every $T^{-1}(\sum)$ -measurable function g, for which the left integral exists. The function $E(f)$ is called the conditional expectation of f with respect to the subalgebra $T^{-1}(\sum)$. As an operator of $L^p(\mu)$, E is the projection onto the closure of range of T and E is the identity on $L^p(\mu)$, $p \ge 1$ if and only if $T^{-1}(\sum) = \sum$. Detailed discussion of E is found in [1-4].

1.1. (Alpha, Beta)--Normal Operator [13]

An operator T is called (α, β) -normal operator if $\alpha^2 T^* T \le T T^* \le \beta^2 T^* T, (0 \le \alpha \le 1 \le \beta)$

1.2. Skew n-Normal Operator [12]

An operator T is called skew n-normal operator if $\left(T^n T^*\right) T = T \left(T^* T^n\right)$, for all natural number n.

1.3. p-Hyponormal Operator [15]

An operator T is called p-hyponormal operator if $\left(T^* T\right)^p \ge \left(T\, T^*\right)^p$, for $(0 \prec p \prec \infty)$

2. Related Work in the Field

The study of weighted composition operators on L^2 spaces was initiated by R. K. Singh and D. C. Kumar [5]. During the last thirty years, several authors have studied the properties of various classes of weighted composition operator. Boundedness of the composition operators in $L^p(\sum)$, $(1 \le p < \infty)$ spaces, where the measure spaces are σ-finite, appeared already in [6]. Also boundedness of weighted operators on $C(X, E)$ has been studied in [7]. Recently S. Senthil, P. Thangaraju and D. C. Kumar have proved several theorems on n-normal, n-quasi-normal, k-paranormal, and (n,k) paranormal of composite multiplication operators on L^2 spaces [8-11]. In this paper we investigate composite multiplication operators of (α, β) -normal operator and skew n-normal operator $L^2(\mu)$ -spaces.

3. Hyponormality for Composite Multiplication Operator

The results in the following proposition were proved in [12], as part of his doctoral dissertation.

3.1. Proposition [3]

Let $E = E(\cdot \setminus A)$ and let φ be a non-negative F measurable function.

Define the positive operator P_φ by $P_\varphi f = \varphi E(\varphi f)$.

Let $\hat{\varphi} = \dfrac{\varphi}{(E(\varphi^2))^{\frac{1}{4}}}$. Then $P_\varphi^{\frac{1}{2}} = P_{\hat{\varphi}}$.

Define the operator R_φ by $R_\varphi f = E(\varphi f)$. Then $\| R_\varphi \| = \left\| \sqrt{E(\varphi^2)} \right\|_\infty$.

In [3], has proved the following lemma, as noted for any non-negative function f,

$\sup port\, f \subset \sup port\ E(f^r)$ for any $r > 0$

3.2. Lemma [14]

Let α and β be non-negative functions, with $S = \sup port\, \alpha$. Then the following are equivalent:

For every $f \in L^2(\mu)$ $\int_X \alpha |f|^2 d\mu \ge \int_X |E(\beta f \setminus A)|^2 d\mu$

$\sup port\, \beta \subset S$ and $E\left(\dfrac{\beta^2}{\alpha} \chi_S \setminus A\right) \le 1$ almost everywhere.

3.3. Proposition

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then for $u \ge 0$

$(i)\ M^*_{u,T} M_{u,T} f = u^2 f_0\ f$

$(ii)\ M_{u,T} M^*_{u,T} f = (u^2 \circ T)\ (f_0 \circ T)\ \ E(f)$

$(iii)\ M^n_{u,T}(f) = (C_T M_u)^n(f) = u_n\ (f \circ T^n)$, $u_n = (u \circ T)\ (u \circ T^2)\ (u \circ T^3) .... (u \circ T^n)$

$(iv)\ M^*_{u,T} f = (u f_0)(E(f) \circ T^{-1})$

$(v)\ M^{*n}_{u,T} f = u f_0\ (E(u f_0) \circ T^{-(n-1)})\ (E(f) \circ T^{-n})$

where $E(u f_0) \circ T^{-(n-1)} = (E(u f_0) \circ T^{-1})\ (E(u f_0) \circ T^{-2}) .... (E(u f_0) \circ T^{-(n-1)})$

$(vi)\ \left| M_{u,T} \right| = u \sqrt{f_0}\, f$

$(vii) \left| M^*_{u,T} \right| = (\sqrt{u^2 f_0 \circ T}\ )\ (E(\sqrt{u^2 f_0 \circ T}\ \ f))$

with the notation from Herron's proposition,

$$\left| M^*_{u,T} \right| = P_{\sqrt{u^2 f_0 \circ T}} = P_v \text{, where } v = \frac{\sqrt{(u^2 f_0) \circ T}}{\left(E\left((u^2 f_0) \circ T\right)\right)^{\frac{1}{4}}}$$

Theorem 3.1

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$ with weight $u \geq 0$ and S be the support of f_0:

$|M_{u,T}| \geq |C|$ if and only if $u^2 \geq 1$

$|C| \geq \left| M^*_{u,T} \right|$ if and only if $\sup port\ v = \sup port \sqrt{f_0}$

where $v = \frac{\sqrt{(u^2 f_0) \circ T}}{\left(E\left((u^2 f_0) \circ T\right)\right)^{\frac{1}{4}}}$, $E\left(\frac{(u^2 f_0) \circ T \big/ \left(E(u^2 f_0) \circ T\right)^{\frac{1}{2}}}{\sqrt{f_0}} \chi_S \right) \leq 1$ almost everywhere.

Proof:

Since $|M_{u,T}|$ and $|C|$ are multiplication operators, we need only compare their symbols. After squaring, we get

$|M_{u,T}| \geq |C|$ if and only if $u\sqrt{f_0} \geq \sqrt{f_0}$

$|M_{u,T}| \geq |C|$ if and only if $u^2 f_0 \geq f_0$ Because $f_0 > 0$ almost everywhere,
we obtain (i).

As for this f, $|C| \geq \left| M^*_{u,T} \right|$

$$\forall\ f,\ \int \sqrt{f_0}\, |f|^2 \, d\mu \geq \left\langle \left| M^*_{u,T} \right| f, f \right\rangle$$

$$\left| M^*_{u,T} \right| = P_v \text{, where } v = \frac{\sqrt{(u^2 f_0) \circ T}}{\left(E\left((u^2 f_0) \circ T\right)\right)^{\frac{1}{4}}} = \int v E(v f)\ \overline{f}\ d\mu$$

However,

$$\int v E(v f)\ \overline{f}\ d\mu = \langle E(v f), v f \rangle = \| E(v f) \|^2 = \int | E(v\ f) |^2 \, d\mu$$

Since $\sup port\ v = \sup port \sqrt{f_0}$, the desired conclusion follows from lemma 3.2.

Theorem 3.5

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is p-hyponormal if and only if $uf_0 > 0, uf_0 \circ T > 0$
and $E\left(1 \big/ u^{2p} f_0 \right) \leq 1 \big/ u^{2p} f_0 \circ T$.

Proof:

Here $\left\langle \left(M^*_{u,T} M_{u,T} \right)^p f, f \right\rangle = \int_X u^{2p} f_0^{\,p} |f|^2 \, d\mu$

and $\left\langle \left(M_{u,T} M^*_{u,T} \right)^p f, f \right\rangle = \int_X \left| E\left((u^p f_0^{\frac{p}{2}}) \circ T\ f \right) \right|^2 d\mu$

This implies $\int_X u^{2p} f_0^{\,p} |f|^2 \, d\mu \geq \int_X \left| E\left((u^p f_0^{\frac{p}{2}}) \circ T\ f \right) \right|^2 d\mu$

Since by lemma 2.2, for every $f \in L^2(\mu)$

$\Rightarrow \quad \sigma((u^p f_0^{\frac{p}{2}})\circ T) \subset \sigma(u^p f_0^p)$ and $E\left[\dfrac{(u^{2p} f_0^p)\circ T}{u^{2p} f_0^p}\right] \le 1$

$\Rightarrow \quad E\left[\dfrac{1}{u^{2p} f_0^p}\right] \le \dfrac{1}{(u^{2p} f_0^p)\circ T}$ if $(u^{2p} f_0^p)\circ T > 0$ and $u^{2p} f_0^p > 0$.

4. Parahyponormal for Composite Multiplication Operator

Mahmound M. Kutkut [16] , has proved that an operator A is parahyponormal if and only if $(AA^*)^2 - 2C\,A^*A + C^2 \ge 0$ for all real C. In an analogous manner, we derive some characterization of parahyponormal and quasi-parahyponormal composite multiplication operator on L^2 -spaces.

Theorem 4.1

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is parahyponormal if and only if

$u^4 \circ T \cdot f_0^2 \circ T\; E(f) - 2Cu^2 f_0\, f + C^2 \ge 0$ almost everywhere, for all $C \ge 0$

Proof:

Suppose $M_{u,T}$ is parahyponormal. Then $(M_{u,T}\, M^*_{u,T})^2 - 2C\;\; M^*_{u,T} M_{u,T} + C^2 \ge 0$ for all $C \ge 0$.

This implies that

$\left\langle ((M_{u,T}\, M^*_{u,T})^2 - 2C\;\; M^*_{u,T} M_{u,T} + C^2) f,\; f \right\rangle \ge 0$ for all $f \in L^2(\mu)$

By the proposition 3.3 we get,

$\int_E \left\{ (u^2 \circ T \cdot f_0 \circ T \cdot E(f))^2 - 2C\, u^2\, f_0\, f + C^2 \right\} d\mu \ge 0$ for every $E \in \sum$.

$\Leftrightarrow \; u^4 \circ T \cdot f_0^2 \circ T\; E(f) - 2Cu^2 f_0\, f + C^2 \ge 0$ almost everywhere, for all $C \ge 0$

Theorem 4.2

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is M-parahyponormal if and only if

$m^2\, u^4 \circ T \cdot f_0^2 \circ T\; E(f) - 2Cu^2 f_0\, f + C^2 \ge 0$ almost everywhere, for all $C \ge 0$.

Proof:

Suppose $M_{u,T}$ is M-parahyponormal.

Then $m^2\,(M_{u,T}\, M^*_{u,T})^2 - 2C\;\; M^*_{u,T} M_{u,T} + C^2 \ge 0$ for all $C \ge 0$.

This implies that

$\left\langle (m^2 (M_{u,T}\, M^*_{u,T})^2 - 2C\;\; M^*_{u,T} M_{u,T} + C^2) f,\; f \right\rangle \ge 0$ for all $f \in L^2(\mu)$

By the proposition 3.3 we get,

$\int_E \left\{ m^2 (u^2 \circ T \cdot f_0 \circ T \cdot E(f))^2 - 2C\, u^2\, f_0\, f + C^2 \right\} d\mu \ge 0$ for every $E \in \sum$.

$\Leftrightarrow \; m^2\, u^4 \circ T \cdot f_0^2 \circ T\; E(f) - 2Cu^2 f_0\, f + C^2 \ge 0$ almost everywhere, for all $C \ge 0$

Theorem 4.3

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is M^*-parahyponormal if and only if

$m^2\, u^2 f_0\, E(u^2 f_0)\circ T^{-1} f - 2C\, u^2 \circ T \cdot f_0 \circ T\;\, E(f) + C^2 \ge 0$ almost everywhere, for all $C \ge 0$.

Proof:

Suppose $M_{u,T}$ is M^*-parahyponormal.

Then $m^2\, M^{*^2}_{u,T}\, M^2_{u,T} - 2C\;\; M_{u,T}\, M^*_{u,T} + C^2 \ge 0$ for all $C \ge 0$.

This implies that

$\left\langle (m^2\, M^{*^2}_{u,T}\, M^2_{u,T} - 2C\;\; M_{u,T}\, M^*_{u,T} + C^2) f,\, f \right\rangle \ge 0$ for all $f \in L^2(\mu)$

By the proposition 3.3 and $M^{*^2}_{u,T}\, M^2_{u,T}\, f = u^2 f_0\, E(u^2 f_0)\circ T^{-1} f\;\; u \ge 0$ we get,

$\int_E \left\{ m^2\, u^2 f_0\, E(u^2 f_0)\circ T^{-1} f - 2C\, u^2 \circ T \cdot f_0 \circ T\;\, E(f) + C^2 \right\} d\mu \ge 0$ for every $E \in \sum$.

$\Leftrightarrow\ m^2 u^2 f_0 E(u^2 f_0)\circ T^{-1} f - 2C\, u^2 \circ T \cdot f_0 \circ T\ E(f) + C^2 \geq 0$ almost everywhere, for all $C \geq 0$

5. Quasi-Parahyponormal for Composite Multiplication Operator

By result of [16] an operators A on H is quasi-parahyponormal if and only if $(A^2 A^{*^2})^2 - 2C(AA^*)^2 + C^2 \geq 0$ for all C. In an analogous manner, we derive the characterization of quasi-parahyponormal composite multiplication operator on L^2 -spaces.

Theorem 5.1

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is quasi-parahyponormal if and only if

$u^2 \circ T \cdot u^4 \circ T^2 \cdot f_0^2 \circ T^2 \cdot (E(uf_0))^2 \circ T \cdot E(f) - 2C u^4 \circ T \cdot f_0^2 \circ T \cdot E(f) + C^2 \geq 0$ almost everywhere, for all $C \geq 0$

Proof:

Suppose $M_{u,T}$ is quasi-parahyponormal.

Then $(M^2{}_{u,T} M^{*^2}{}_{u,T})^2 - 2C\,(M_{u,T} M^*{}_{u,T})^2 + C^2 \geq 0$ for all $C \geq 0$.

This implies that

$\left\langle ((M^2{}_{u,T} M^{*^2}{}_{u,T})^2 - 2C\,(M_{u,T} M^*{}_{u,T})^2 + C^2) f,\ f \right\rangle \geq 0$ for all $f \in L^2(\mu)$

By the proposition 3.3 and $M^2{}_{u,T} M^{*^2}{}_{u,T} f = u \circ T \cdot u^2 \circ T^2 \cdot f_0 \circ T^2 \cdot E(uf_0) \circ T \cdot E(f)$

$\int_E \left\{ (u \circ T \cdot u^2 \circ T^2 \cdot f_0 \circ T^2 \cdot E(uf_0) \circ T \cdot E(f))^2 - 2C(u^2 \circ T \cdot f_0 \circ T \cdot E(f))^2 + C^2 \right\} d\mu \geq 0$ for every $E \in \sum$

$\Leftrightarrow\ u^2 \circ T \cdot u^4 \circ T^2 \cdot f_0^2 \circ T^2 \cdot (E(uf_0))^2 \circ T \cdot E(f) - 2C u^4 \circ T \cdot f_0^2 \circ T \cdot E(f) + C^2 \geq 0$

almost everywhere, for all $C \geq 0$

Theorem 5.2

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is M-quasi-parahyponormal if and only if

$m^2 u^2 \circ T \cdot u^4 \circ T^2 \cdot f_0^2 \circ T^2 \cdot (E(uf_0))^2 \circ T \cdot E(f) - 2C u^4 \circ T \cdot f_0^2 \circ T \cdot E(f) + C^2 \geq 0$ almost everywhere, for all $C \geq 0$

Proof:

Suppose $M_{u,T}$ is M-quasi-parahyponormal.

Then $m^2\,(M^2{}_{u,T} M^{*^2}{}_{u,T})^2 - 2C\,(M_{u,T} M^*{}_{u,T})^2 + C^2 \geq 0$ for all $C \geq 0$.

This implies that

$\left\langle (m^2 (M^2{}_{u,T} M^{*^2}{}_{u,T})^2 - 2C\,(M_{u,T} M^*{}_{u,T})^2 + C^2) f,\ f \right\rangle \geq 0$ for all $f \in L^2(\mu)$

By the proposition 3.3 and $M^2{}_{u,T} M^{*^2}{}_{u,T} f = u \circ T \cdot u^2 \circ T^2 \cdot f_0 \circ T^2 \cdot E(uf_0) \circ T \cdot E(f)$

$\int_E \left\{ m^2 (u \circ T \cdot u^2 \circ T^2 \cdot f_0 \circ T^2 \cdot E(uf_0) \circ T \cdot E(f))^2 - 2C(u^2 \circ T \cdot f_0 \circ T \cdot E(f))^2 + C^2 \right\} d\mu \geq 0$ for every $E \in \sum$.

$\Leftrightarrow\ m^2 u^2 \circ T \cdot u^4 \circ T^2 \cdot f_0^2 \circ T^2 \cdot (E(uf_0))^2 \circ T \cdot E(f) - 2C u^4 \circ T \cdot f_0^2 \circ T \cdot E(f) + C^2 \geq 0$ almost everywhere, for all $C \geq 0$

6. Skew n-normal and (Alpha, Beta)-normal Composite Multiplication Operator

Theorem 6.1

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is skew n-normal if and only if

$u_n (u^2 \circ T^n)(f_0 \circ T^n) = (u^2 \circ T)(f_0 \circ T)\ (E(u_n) \circ T)$ almost everywhere.

Proof:

$M_{u,T}$ is skew n-normal. Then

$$(M^n{}_{u,T}\, M^*{}_{u,T}) M_{u,T}\, f = (M^n{}_{u,T}\, M^*{}_{u,T})\ (uf) \circ T$$

$$= M^n{}_{u,T}\ (u\, f_0\, E((uf) \circ T) \circ T^{-1})$$

$$= M^n{}_{u,T}\ u^2 f_0 f$$

$$= u_n (u^2 f_0 f) \circ T^n$$

$$= u_n (u^2 \circ T^n)(f_0 \circ T^n)(f \circ T^n)$$

Also,

$$M_{u,T}\ (M^*{}_{u,T} M^n{}_{u,T}) f = M_{u,T}\ M^*{}_{u,T} (u_n (f \circ T^n))$$

$$= M_{u,T}\, u\, f_0\, E(u_n (f \circ T^n)) \circ T^{-1}$$

$$= M_{u,T}\, u\, f_0\, E(u_n)(f \circ T^{n-1})$$

$$= u\ (u\, f_0\, E(u_n)(f \circ T^{n-1})) \circ T$$

$$= (u^2 \circ T)(f_0 \circ T)\ E(u_n) \circ T\ \ (f \circ T^n)$$

$\Leftrightarrow\ u_n (u^2 \circ T^n)(f_0 \circ T^n) = (u^2 \circ T)(f_0 \circ T)\ (E(u_n) \circ T)$ almost everywhere.

Theorem 6.2

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M^*{}_{u,T}$ is skew n-normal if and only if $u^2 f_0^2 = (u^2 \circ T^{-(n-1)})(f_0 \circ T^{-(n-1)})$ almost everywhere.

Proof:

$M^*{}_{u,T}$ is skew n-normal. Then

$$M^*{}_{u,T}(M_{u,T}\, M^{*n}{}_{u,T}) f = M^*{}_{u,T}\, M_{u,T} \Big(u\, f_0 \Big(E(u\, f_0) \circ T^{-(n-1)} \Big) \Big(E(f) \circ T^{-n} \Big) \Big)$$

$$= M^*{}_{u,T} \Big(u \Big(u\, f_0 \Big(E(u\, f_0) \circ T^{-(n-1)} \Big) \Big(E(f) \circ T^{-n} \Big) \Big) \Big) \circ T$$

$$= M^*{}_{u,T} \Big((u^2 \circ T)(f_0 \circ T)\ (E(u\, f_0) \circ T^{-(n+2)})(E(f) \circ T^{-(n-1)}) \Big)$$

$$= u\, f_0\, E\Big[\Big((u^2 \circ T)(f_0 \circ T)\ (E(u\, f_0) \circ T^{-(n+2)})(E(f) \circ T^{-(n-1)}) \Big) \Big] \circ T^{-1}$$

$$= u^3 f_0^2\ \ (E(u\, f_0) \circ T^{-(n-1)})\ (E(f) \circ T^{-n})$$

Also,

$$(M^{*n}{}_{u,T} M_{u,T}) M^*{}_{u,T}\, f = M^{*n}{}_{u,T} M_{u,T} \Big(u\, f_0\, E(f) \circ T^{-1} \Big)$$

$$= M^{*n}{}_{u,T} \Big((u^2 \circ T)\ (f_0 \circ T)\ E(f) \Big)$$

$$= u\, f_0\, E(u\, f_0) \circ T^{-(n-1)}\, (u^2 \circ T^{-(n-1)})\ (f_0 \circ T^{-(n-1)})\ E(f) \circ T^{-n}$$

$\Leftrightarrow\ u^2 f_0^2 = (u^2 \circ T^{-(n-1)})(f_0 \circ T^{-(n-1)})$ almost everywhere.

Theorem 6.2

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then $M_{u,T}$ is (α, β)-normal if and only if $\alpha^2 u^2\ f_0 f \le (u^2 f_0) \circ T\ E(f) \le \beta^2\ u^2\ f_0\ f$ almost everywhere.

Proof:

$M_{u,T}$ is (α, β)-normal. Thenit easy to check,

$$M^*_{u,T}\, M_{u,T}\, f = u^2\, f_0\, f\ ,\ M_{u,T}\, M^*_{u,T}\, f = (u^2 \circ T)\ (f_0 \circ T)\ \ E(f)$$

By definition,

$\alpha^2\, u^2\, f_0\, f \le (u^2 f_0) \circ T\ E(f) \le\ \beta^2\, u^2\, f_0\, f$ almost everywhere.

8. Conclusion

In the study of p-hyponormal operator, the Aluthge transform is a very useful tool. We investigate some basic properties of such operators and study the relation among skew n-normal operators, (α,β) -normal operator, parahyponormal and quasi-parahyponormal composite multiplication operators on $L^2(\mu)$ -space. In future try to generalize the composite multiplication operator on Poisson weighted sequence spaces.

Acknowledgements

We would like to thank the reviewers for carefully reading manuscript and for their constructive comments. I want to thank Professor Dr. R. David Chandrakumar, Department of Mathematics, Vickram College of Engineering for his support and encouragement during preparation of the paper.

References

[1] Campbell J & Jamison J, On some classes of weighted composition operators, Glasgow Math. J. vol. 32, pp. 82-94, (1990).

[2] Embry Wardrop M & Lambert A, Measurable transformations and centred composition operators, Proc. Royal Irish Acad, vol. 2(1), pp. 23-25 (2009).

[3] Herron J, Weighted conditional expectation operators on Hilbert spaces, UNC charlotte doctoral dissertation.

[4] Thomas Hoover, Alan Lambert and Joseph Quinn, The Markov process determined by a weighted composition operator, Studia Mathematica, vol. XXII (1982).

[5] Singh RK & Kumar DC, Weighted composition operators, Ph.D. thesis, Univ. of Jammu (1985).

[6] Singh RK, Composition operators induced by rational functions, Proc. Amer. Math. Soc., vol. 59, pp. 329-333(1976).

[7] Takagi H & Yokouchi K, Multiplication and Composition operators between two Hilbert spaces, Contem. Math., vol. 232, pp. 321-338 (1999).

[8] PanaiyappanS & Senthil kumar D, Parahyponormal and M-parahyponormal composition operators, Acta Ciencia Indica, vol. XXVIII (4) (2002).

[9] Senthil S, Thangaraju P & Kumar DC, n-normal and n-quasi-normal composite multiplication operator on Hilbert spaces, Journal of Scientific Research & Reports, vol. 8(4), pp. 1-9 (2015).

[10] Senthil S, Thangaraju P & KumarDC, k-*paranormal, k-quasi-*paranormal and (n,k)- quasi-*paranormal composite multiplication operator on Hilbert spaces, British Journal of mathematics and computer science, BJMCS20166 (2015).

[11] Senthil S, Thangaraju P & Kumar DC, Composite multiplication operator on Hilbert spaces of vector valued functions, International research Journal of Mathematical Sciences, vol. 4 (2), pp. 1(2015).

[12] Shaakir LK& Abdulwahid ES, Skew n-normal operators, Aust. J. of basic and appl. sci, Vol-8(16), pp. 340-344 (2014).

[13] Moslehian MS, On (Alpha, Beta)-normal operators in Hilbert spaces, IMAGE 39, Problem 39-4 (2007).

[14] Burnap C, Jung I &Lambert A, Separating partial normality classes with composition operators, J. Operator Theory Vol. 53(2), pp. 381–397 (2005).

[15] Fujii M & Nakatsu Y, On subclasses of hyponormal operators, Proc. Japan Acad. Ser. A. Math. Sci. vol. (51), pp. 243–246 (1975).

[16] Kutkut & Mahmoud M, On the class of parahyponormal operators, J. Math Sci, (Calcutta), vol. 4(2), pp. 73-88 (1993).