The Model Inversion of Leaf Area Index of Vegetation by Means of Electromagenetic Wave Radiative Transfer Model

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Abstract: This paper puts forward a novel approach for model inversion of leaf area index (LAI) of vegetation based on the integrated arithmetic of data assimilation and genetic-particle swarm algorithm (DAGS). The article expounds the design principle of electromagnetic wave radiative transfer model (ERTM) for vegetation canopies. On this basis, this study constructs the inversion model of LAI based on DAGS. Furthermore, this experiment realizes the model inversion of LAI with the aid of Remote Sensing (RS) multi-spectral data and biophysical component data of vegetation canopies, which are provided by the multispectral RS observation data set (MOD15A2). The bullet points of the text are summarized as follows. (1) The contribution proposes DAGS for LAI inversion. (2) The article discusses ERTM model for electromagnetic wave radiative transfer mechanism of vegetation canopies. (3) This text achieves LAI inversion with the help of RS multi-spectral data and biophysical component data of vegetation canopies supplied by MOD15A2. The experimental results demonstrate the validity and reliability of the model inversion of LAI by making use of DAGS. The proposed algorithm exploits a novel algorithmic pathway for the model inversion of LAI by means of RS multi-spectral data and biophysical component data of vegetation canopies.

Keywords: LAI, Model Inversion, Biophysics Component Parameters, DAGS, ERTM

1. Introduction

Leaf area index (LAI) is an important biophysical parameter of vegetation canopies, and it is generally utilized to characterize the space-time distribution regularities of the earth's surface vegetation. LAI is also a critical parameter which is utilized to structure the parameter retrieval model of vegetation configuration by means of Remote Sensing (RS) multi-spectral data. Studies show that many physiological processes of vegetation ecosystem are closely related to LAI of vegetation. As one of the key structural canopy characteristics and biophysical values, LAI values influence the vegetation ecology parameters, such as photosynthesis, chlorophyll generation, nutrient cycle, carbon cycle, respiration, net primary production, evaporation and transpiration, waste decomposition, intercept and capture of precipitation, energy balance, and other ecological parameters. Furthermore, LAI is also an extremely important evaluating indicator for annual net primary productivity (NPP) of terrestrial vegetation, which is the net amount of carbon fixed by plant photosynthesis, so LAI is an important index in global climate ecology change. Therefore, it appears very important to obtain information about LAI of vegetation as well as its distribution situation in the local area to the region, and even the global scale in the end. Making use of LAI data obtained by the model inversion, the distribution regularities of vegetation ecosystem in the regional scale can be intensively lucubrated, and its research outcome can be further utilized to provide the basic data for the identification and classification of vegetation type, and for the regionalization and planning of the regional vegetation with the aid of RS image (Dawson, et al 2010). LAI is defined as the area of the leaf projected on per square meter of the ground surface, measured in m² per m². Since LAI is closely related to the biophysical process of vegetation, so LAI is one of very important input parameters for the
ecological process simulation of vegetation (Gao, et al 2008). It plays extremely significant effect to accurately estimate the value of LAI in the research of the carbon cycle and energy circulation of vegetation, as well as in deliberation of the environmental impact assessment. LAI is also a very critical input parameter of the land vegetation ecosystem research, and the research of its retrieval approach is still a hot-issue of RS application study all along. From the point of methodological view, the extraction method of physical model (Gemmell 2008). Because the retrieval method based on the statistical model is insufficient in physical basis, so its reliability and universality are exceedingly terrible. Although the lookup table methods (LUT) and the nonparametric methods (NPM) are developed for the collection of LAI, but they are nonetheless essentially for the retrieval means of the physical model. Despite the fact that the nonparametric retrieval approach based on the physical model eliminates the drawbacks, but the retrieval result of the model is generally non-unique due to the morbidity of the inversion process (Gong, et al 2009). This paper discusses the algorithmic principle of LAI inversion with the physical model, and then, the article expounds the principle and the method of LAI inversion with DAGS. On the basis of the physical model of LAI inversion, this manuscript builds the electromagnetic wave radiative transfer model (ERTM) of vegetation canopies, and then, LAI of vegetation is retrieved with DAGS. The experimental results demonstrate the validity and reliability of LAI inversion with DAGS. The proposed algorithm opens up a novel algorithmic pathway for LAI inversion of vegetation by making use of RS multi-spectral data and biophysical component data of vegetation canopies. Because traditional method for the model inversion of LAI is a statistical regression algorithm based on vegetation index, and it is very easily affected by various extrinsic factors, such as background value of soil elements, etc., so the algorithm is deficient in transportability. Therefore, this research pays particular emphasis on structuring ERTM that is used to describe the radiative transfer process of the electromagnetic wave within vegetation leaves. However, the model inversion of LAI based on ERTM is an opposite problem, and it possesses the uncertainty, that is, it dissatisfies the existence of the solution, the uniqueness of the solution, and the continuous dependence which the solution acts on the observed data (Goward, et al 2008). This is caused by two main reasons below: (1) a different model parameter can produce almost the same reflective spectral. For example, the spectral reflectance, which vegetation leaves are sparse and their leaves are horizontal distribution, is similar to the spectral reflectance which vegetation leaves are dense and their leaves are vertical distribution (Jacquemoud, et al 2006); (2) the reflectance simulated with the model as well as the observed reflectance there exists the uncertainty, respectively. The uncertainty in the reflectance of the model simulation comes predominantly from simplification of the scattering effect of vegetation canopies, and it derives principally from the idealization of the Lambert scattering supposition of vegetation canopies. However, the uncertainty of the observed reflectance comes primarily from RS transducer noise, and it derives chiefly from the noise produced by RS data pretreatment. Since the uncertainty of an opposing problem, so this will lead the solution of the model inversion to produce jumping in the parameter space. This means the solved solution is likely to be unevenly distributed in the whole parameter space, rather than converging at near the actual solution. For this purpose, PROSAIL model is used to simulate the hyper-spectral reflectance of vegetation leaves (Jacquemoud, et al 2009). In this case, LAI is supposed retaining at near an initial value for the inversion of numerical value of the reflectance model used for vegetation canopies. The research result shows that the less the value of LAI, the greater the leaf structure parameter N of vegetation, likewise the less the LAD (mean leaf inclined angle distributions) of vegetation canopies. The above conclusion explains, for the foliar reflectance of vegetation, that the influence of LAI and LAD is contrary to that of the foliar structure parameter N. Further research discovers that the combination of different parameters can correspond to almost similar spectra (Karami, et al 2012). Therefore, the morbid problem of the model inversion is still always existent, and so far it is still the bottle-neck issue of LAI model inversion of (Kimes 2007). In order to resolve the above problem, this article builds ERTM of vegetation canopies, and puts forward DAGS for the model inversion of LAI. Experimental result demonstrates the validity and stability of LAI inversion by employing DAGS. In this way, the uncertainty of LAI inversion is correspondingly reduced. Moreover, the retrieval precision of LAI is evidently improved (Kimes et al 2008).

2. Algorithmic Principle of Data Assimilation Arithmetic (DAA)

2.1. Data Assimilation Arithmetic (DAA)

Algorithmic principle of DAA is summarized as follows. (1) According to an internal related model within the observed data, a predicted model of the unobserved data is built, and then it is regarded as an initial-estimate model of data. (2) An initialization processing is implemented for an updated initial-estimate model of data. (3) Next data are predicted with the model, and then a new forecasting model is built with the observed data. (4) A new forecasting model is regarded as an initial-estimate model of next updated data, and then the resulting data are predicted with the model. (5) If all data are predicted, and then the algorithm is over. Otherwise the algorithm returns to step (1).

The above cycling course is explained below: inserting observed data → updating initial-estimate model → initialization processing → model predicting → inserting observed data → updating initial-estimate model →
initialization processing → model forecasting,…, end. The beginning of each cycle in data assimilation arithmetic, the forecasting model is updated with the new observed data (Kuusk 2007). The update is principally divided into two modes, that is, one-dimensional assimilation and two-dimensional assimilation. The one-dimensional assimilation is the data obtained with the observation directly replaces the model predicted value that is most close to the observed data in accordance with the minimum distance algorithm. Such update mode is similarly defined as direct inserting of the observed data or direct updating of the forecast value. The two-dimensional assimilation is explained in the following. The forecasting model is regarded as an initial-estimate model, and the observed data are analyzed statistically, and then a forecast model of data is built according to the interrelation within data. Such update mode is likewise defined as indirect inserting of the observed data or indirect updating of the predicted value (Kuusk 2006).

2.2. Objective Function of the Data Assimilation System

The objective function $F\left(y^j(\tau)\right)$ of the data assimilation system is defined in the following.

$$F\left(y^j(\tau)\right)=\frac{1}{2}\left[y^j(\tau)-y^j(\tau)^T\right]C^{-1}\left[y^j(\tau)-y^j(\tau)^T\right] + \frac{1}{2}\sum_{j=1}^{M}\left[G_j(L(s_j))-x_j^j\right]^T D_j^{-1}\left[G_j(L(s_j))-x_j^j\right]$$

(1)

Where, $F\left(y^j(\tau)\right)$ is the objective function; $\tau_j$ denotes moment $j$; $y^j(\tau)$ is the initial value of the state vector, and it is also a column matrix of the assimilated variable, and it denotes the initial state of the assimilation period $j$; $y^j(\tau)$ is the ambient field; $C$ is the covariance matrix of the error of the ambient field; $x^j$ is the observed value of the moment $j$, and it is a multi-dimensional vector of the observed data; $L$ is the model operator; $s_j$ is the $j$th observed parameter of $L$; $G_j$ is the observed operator; $D_j$ is the covariance matrix of the error of the observation field.

3. Algorithmic Principle of DAGS

3.1. Data Assimilation Algorithm of Genetic-Particle Swarm

Suppose that $V_m$ is $m$-dimensional search space of the objective ($m$ is number of assimilation variable), then there exists a population $\xi = \{\xi_1, \xi_2, \cdots, \xi_n\}$ that consists of $n$ particles, and the position of the particle $j$ is $\xi_j = \{\xi_{j1}, \xi_{j2}, \cdots, \xi_{jm}\}$ indicates the $j$th solution of the algorithm. Each particle searches a new algorithmic solution by continually adjusting its own position $\xi_j$ (Liang, et al 2008). The optimal solution (fitness degree of particle) that is searched by the particle $j$ is marked as $q_{js}$, and the optimal solution (colony fitness degree) that is searched in the $k$ iteration by whole particle swarm is marked as $q_{ks}$ (Manevski, et al 2012). The velocity of the particle $j$ in the particle swarm is marked as $U_j = \{u_{j1}, u_{j2}, \cdots, u_{jm}\}$. When both the two optimal solution $q_{js}$ and $q_{ks}$ are found out, then the particle updates its own velocity with formula (2). Afterwards, the particle adjusts its own position with formula (3) (Mustafa, et al 2012).

$$u_{js}(\tau+1) = w u_{js}(\tau) + \alpha \text{rand()}(q_{js} - \xi_{js}(\tau)) + \beta \text{rand()}(q_{ks} - \xi_{js}(\tau))$$

(2)

$$\xi_{js}(\tau+1) = \xi_{js}(\tau) + u_{js}(\tau+1)$$

(3)

Where, $u_{js}(\tau)$ is the $s$-dimensional velocity of particle $j$ in the $\tau$ iteration; $u_{js}(\tau+1)$ is the $s$-dimensional velocity of particle $j$ in the $\tau+1$ iteration; $\xi_{js}(\tau+1)$ is the $s$-dimensional position of particle $j$ in the $\tau+1$ iteration; $w$ is an inertia weighted constant; $\alpha$ and $\beta$ are an accelerated parameter ascertained by the iteration, respectively; function rand()$(0, 1)$ shows a random number between $(0, 1)$.

3.2. The Genetic-Iterative Optimization Algorithm of DAGS

As previously discussed, the genetic-iterative optimization algorithm of DAGS for the model inversion of LAI is defined as follows.

(1) Both the initial position and initial velocity of each particle in the particle population are respectively initialized in the search space.

(2) Both the update velocity and update position of each particle are respectively calculated with formula (2) and formula (3).

(3) Particle fitness degree $q_{js}$ of the particle $j$ is calculated $(j = 1, 2, \cdots, n)$, and colony fitness degree $q_{ks}$ of the particle $j$ is also calculated $(k = 1, 2, \cdots, l)$.

(4) For the particle $j$ $(j = 1, 2, \cdots, n)$, if its current fitness degree is better than the fitness degree $q_{js}$ of its best position which it has been gone through, then the value of its current fitness degree is assigned to $q_{js}$.

(5) For the particle $j$ , both its particle fitness degree $q_{js}$ $(j = 1, 2, \cdots, n)$ and its colony fitness degree $q_{ks}$ $(k = 1, 2, \cdots, l)$ are respectively compared with the current best $q_{js}$ and $q_{ks}$. If there exist better $q_{js}$ and $q_{ks}$, then both the current $q_{js}$ and $q_{ks}$ are respectively updated with better $q_{js}$ and $q_{ks}$.
(6) The value of the fitness degree of each particle is ranked ordering from the greater value through the smaller one. For posterior particles in the sequence, the following interlacing operation and variation operation are implemented, and an original particle is substituted with a newly generated particle.

(7) The interlacing operation is performed. The particles of the parent population are randomly mated processing. It is random to determine the crossing position for each pair of particles of the parent population. Then the new particles of the descendant population are generated by the interlacing operation.

(8) The variation operation is carried out. A number \( j \in \{1,2,\cdots,m\} \) is selected randomly and discretionarily, and then the \( j \) th variable \( \xi_j \) of the particle \( \xi \), whose variation is going to be generated, performs the variation operation with formula (4).

\[
\xi_j' = \xi_j + \text{rand}'(\xi_j - \xi_j)
\]

Where, function \( \text{rand}'() \in (-1,1) \) indicates a random number among \((-1,1)\); \( \xi_j \) is the value of fitness degree of the particle \( j \) before the variation; \( \xi_j \) is the value of colony fitness degree of the particle \( j \) before the variation. Both the update velocity and update position of each particle are respectively calculated with formula (2) and formula (3). If the update velocity and update position of each particle are all optimal, then the algorithm is over. Otherwise the operation returns to step (2) and continues to iterate (Myneni, et al 2007).

4. LAI Inversion Algorithm

4.1. The Radiative Transfer Model of Vegetation Canopies

Based on the above discussion, the radiative transfer model of vegetation canopies is defined as follows

\[
\rho = \rho_c + \rho_s + \rho_m
\]

Where,

\[
\rho_c = \int_0^L \frac{G_{L}(r',r,z)}{\mu} \mu(z)Q(r',r,z)dz 
\]

\[
\rho_s = \rho_{SOIL}(r',r)\exp\left[-L_0\left(\frac{S_{L}'}{\mu} + \frac{S_L}{\mu}\right)\right] \times \left[1 + \exp\left(\frac{-\alpha}{2L_2\sqrt{\mu'\mu}}\right)\right]^{-1}. 
\]

\[
\rho_m = \chi - \frac{\omega S_L S_L'}{G_{L}'\mu + 2S_L S_L' + S_L'\mu'}(\chi - \rho_{SOIL}) \exp(-2L_0) 
\]

\[
\chi = \frac{2S_L S_L'}{(1 - 1 - \omega)} \exp(-2L_0) 
\]

\[
\omega = \gamma + \tau + \left(1 - \kappa \right)^2 \left(1 + \kappa \right)^2 
\]

\[
\rho_{SOIL} = \frac{1}{\pi} \int_{z} \rho_{SOIL}(r',r,\omega) \mu(\omega) d\omega
\]

In formulas (5)-(11), \( \rho \) represents reflectance of vegetation canopies; \( \rho_c \) denotes the component of direct reflectance of vegetation canopies; \( \rho_m \) means the component of direct reflectance of soil; \( \rho_s \) indicates the multiple-scattering reflectance component of vegetation-soil; \( G_{L}(r',r,z) \) shows the scattering phase-function of vegetation canopies; \( H \) means the depth of vegetation; \( \mu' \) and \( \mu \) are the cosine value of the zenith angle of the sun and the zenith angle of observation, respectively; \( \mu_L(z) \) is the volume density of the leaf area in the vegetation depth \( z \); \( r' \) and \( r \) are the solar azimuth and the direction angle of observation, respectively; \( Q(r',r,z) \) is the void ratio between the solar direction and the observed direction in the vegetation depth \( z \); \( R_{SOIL} \) expresses full-reflectance of soil; \( \rho_{SOIL}(r',r,\omega) \) is the bidirectional reflectance of soil; \( L_0 \) is \( LAI \) of the whole vegetation canopies; \( S_L \) and \( S_L' \) are the leaf area of the direction of the solar zenith angle and the observed zenith angle, respectively; \( \alpha \) is the contained angle between the solar direction and the observed direction; \( L_2 \) is a parameter that indicates the ratio between the geometrical characteristic scale of vegetation canopies and the vegetation depth; \( \gamma \), \( \tau \), and \( \kappa \) are the reflection coefficient of vegetation canopies, transmission coefficient of vegetation canopies, and Fresnel refraction coefficient of the horny layer of vegetation canopies, respectively; \( \omega \) means the full-reflectance of vegetation canopies.

Soil reflectance \( R_{SOIL} \) is a function of the reflected light wave length \( \lambda \), and \( R_{SOIL} \) is calculated from the following formula (Niemann, et al 2012).

\[
R_{SOIL}(\lambda) = w_1\gamma_1(\lambda) + w_2\gamma_2(\lambda) + w_3\gamma_3(\lambda) + w_4\gamma_4(\lambda)
\]

Where, \( \lambda \) is the wavelength; \( \gamma_1(\lambda) \), \( \gamma_2(\lambda) \), \( \gamma_3(\lambda) \) and
\( \gamma_{k}(\lambda) \) represent the basic function, respectively; \( w_1, w_2, w_3 \) and \( w_4 \) denote the weighted coefficient, respectively.

Since parameter \( R_{SOIL}(\lambda) \) is as soil reflectance of the nadir direction, so it is generally thoughtless of the directional information. The bidirectional distribution function of soil reflectance \( \rho_i(\lambda, \theta_1, \theta_2, \phi) \) can be expressed as the following formula.

\[
\rho_i(\lambda, \theta_1, \theta_2, \phi) = \frac{R_{SOIL}(\lambda)}{f_0(\theta_1)} \left[ f_0(\theta_1) + f_1(\theta_1) \theta_2 \cos \phi + f_2(\theta_1) \theta_2^2 \right]
\] (13)

Where, \( \rho_i(\lambda, \theta_1, \theta_2, \phi) \) is the bidirectional distribution function of soil reflectance; \( \theta_1, \theta_2 \) and \( \phi \) mean the solar zenith angle, the observed zenith angle, and the observed azimuth angle, respectively. The modified function of radiation of the angle \( \theta_1 \) in formula (13) is shown below.

\[
\begin{align*}
    f_0(\theta_1) &= 3.642 \theta_1^2 - 14.62 \\
    f_1(\theta_1) &= 6.58 \theta_1^2 - 2.46 \\
    f_2(\theta_1) &= 8.642 \theta_1^2 - 3.68
\end{align*}
\] (14)

4.2. Structuring the Cost Function for LAI Inversion of Vegetation Canopies

The cost function needed in LAI inversion should be first built. The cost function sets up the standard by which the model-simulation values of the retrieved parameters and RS observed values are arriving at an agreement. The cost function \( J(x) \) of LAI inversion in this experiment is established in the following.

\[
J(x) = \sum_{i=1}^{k} \sum_{j=1}^{L} \left[ \rho_i'(x) - \rho_0'(x) \right]^2 + \sum_{j=1}^{L} \rho(x_j)
\] (15)

Where, \( J(x) \) is the retrieval cost function of the retrieved parameter \( x \); \( w_i \) means the weighted coefficient of the band \( i \); \( \rho_i'(x) \) indicates the model-simulation relectivity of the band \( i \); \( k \) shows the simulated band number; \( \rho_0'(x) \) denotes the observed reflectivity of the band \( i \); \( \rho(x_j) \) expresses the reflectance of the retrieved parameter \( x \) in the band \( j \); \( L \) represents the observed band number.

4.3. Building the Objective Function for LAI Inversion of Vegetation Canopies

Supposing the observed data \( v_o \) and the parameter \( m \) of LAI inversion subjecting to Gaussian normal distributions, and then the objective function \( T(m) \) of LAI inversion for vegetation canopies is defined in the following.

\[
T(x) = \frac{1}{2} \left[ (v_o - v_p)^T B_{ij}^{-1} (v_o - v_p) + (x - x_p)^T B_{ij}^{-1} (x - x_p) \right]
\] (16)

Where, \( T(x) \) is the objective function of LAI inversion for vegetation canopies; \( v_p \) means the calculated value of the parameter; \( v_o \) indicates the observed value of the parameter; \( x \) shows the retrieved parameter; \( x_p \) expresses the prior estimation of the parameter; \( B_{ij} \) is the covariance matrix of the observed data; \( B_{ij} \) is the covariance matrix of the prior estimation of the parameter.

4.4. LAI Inversion of Vegetation Canopies Based on DAGS

This experiment constructs the following retrieval model for LAI model inversion of vegetation canopies based on the DAGS, and the inversion model \( R(x) \) is defined as follows.

\[
R(x) = (x - x_B)^T B^{-1} (x - x_B) + \sum_{i=1}^{L} \left[ \rho_i(x_i) - \rho_0(x_i) \right]^T Q^{-1} \left[ \rho_i(x_i) - \rho_0(x_i) \right]
\] (17)

Where, \( R(x) \) is the retrieved vector of LAI; \( i \) means the retrieval moment; \( x \) is the vector of the retrieved parameter at different retrieval moments, and it is described as the expanded vector; \( x_i \) is the \( i \) th component of \( x \), and it means the retrieved parameter at the \( i \) th retrieval moments; \( x_B \) is the priori knowledge of the retrieved parameters; \( B \) is the covariance matrix of a priori error of the retrieved parameters; \( \rho_i(x) \) is the value of the simulated reflectance; \( \rho_0(x) \) is the value of the observed reflectance; \( Q \) is the error covariance matrix of \( \rho_i(x) \) and \( \rho_0(x) \).

4.5. Data Processing for LAI Inversion of Vegetation Canopies

After the minimum objective function is determined, the uncertainty for LAI inversion of vegetation canopies can be calculated with following formula.

\[
\{ \bar{x}_i \pm \sigma_i \} \cap \{ \min x_i, \max x_i \}
\] (18)

Where, \( \bar{x}_i \) and \( \sigma_i \) are normal mean and standard deviation of the \( i \) th retrieved parameter \( x_i \), respectively. In the above formula, when \( \bar{x}_i \geq \sigma_i \), then it assigns negative sign “-”; and when \( \bar{x}_i < \sigma_i \), then it assigns positive sign “+”; \( \min x_i \) and \( \max x_i \) are the lower bound and upper bound of the \( i \) th retrieved parameter \( x_i \), respectively. The predicted values and uncertainty of each relevant parameter for five vegetation types, such as broad-leaved forest, shrub forest, pasture, dense crops, and sparse crops, are calculated with the above formula, and the calculated results are shown in Table 1 to Table 3. In Table 1 to Table 3, \( N \) means \( LISP \) (leaf internal structure parameter); \( C_{ab} \) denotes \( LCC \) (leaf chlorophyll \( a+b \) content); \( C_{ab} \) shows \( LMC \) (leaf moisture content); \( C_{ab} \) indicates \( LDMC \) (leaf dry matter content); \( S \) expresses \( LAHR \) (leaf area/crop height ratio); \( S \) indicates \( SRI \) (soil reflectance index) indicates soil reflectance index.
The significance of other parameters refers to the introduction of the text. Since soil reflectance in the vegetation canopy changes very little, so the parameter of soil reflectance, which is generated with the soil reflectance model, changes very slight among the vegetation types as well. Therefore, the predicted values and uncertainties of LMC are appropriately adjacent and coincident with the five vegetation types (See Table 1 to Table 3).

### Table 1. Predicted value of the sensitivity and component parameters for different vegetation types.

<table>
<thead>
<tr>
<th>Vegetation type</th>
<th>LAI</th>
<th>LAD</th>
<th>N</th>
<th>Cm</th>
<th>Cw</th>
<th>Cm</th>
<th>S1</th>
<th>SRI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broad-leaved forest</td>
<td>0.72</td>
<td>65</td>
<td>1.56</td>
<td>48</td>
<td>0.26</td>
<td>0.12</td>
<td>0.06</td>
<td>0.62</td>
</tr>
<tr>
<td>Shrub forest</td>
<td>0.84</td>
<td>54</td>
<td>1.65</td>
<td>46</td>
<td>0.78</td>
<td>0.43</td>
<td>0.03</td>
<td>0.76</td>
</tr>
<tr>
<td>Pasture</td>
<td>1.64</td>
<td>58</td>
<td>1.72</td>
<td>48</td>
<td>0.24</td>
<td>0.15</td>
<td>0.04</td>
<td>0.66</td>
</tr>
<tr>
<td>Dense crops</td>
<td>1.82</td>
<td>56</td>
<td>1.84</td>
<td>52</td>
<td>0.34</td>
<td>0.18</td>
<td>0.05</td>
<td>0.86</td>
</tr>
<tr>
<td>Sparse crops</td>
<td>2.42</td>
<td>62</td>
<td>2.42</td>
<td>50</td>
<td>0.36</td>
<td>0.14</td>
<td>0.04</td>
<td>0.78</td>
</tr>
</tbody>
</table>

### Table 2. Predicted values and the uncertainties of the component parameters of the partial leaves for different vegetation types.

<table>
<thead>
<tr>
<th>Predicted value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetation type</td>
<td>N</td>
</tr>
<tr>
<td>Broad-leaved forest</td>
<td>1.58</td>
</tr>
<tr>
<td>Shrub forest</td>
<td>1.64</td>
</tr>
<tr>
<td>Pasture</td>
<td>1.74</td>
</tr>
<tr>
<td>Dense crops</td>
<td>1.84</td>
</tr>
<tr>
<td>Sparse crops</td>
<td>1.36</td>
</tr>
</tbody>
</table>

### Table 3. Predicted values and the uncertainties of the biophysical component parameters of the partial leaves for different vegetation types.

<table>
<thead>
<tr>
<th>Predicted value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetation type</td>
<td>Cm</td>
</tr>
<tr>
<td>Broad-leaved forest</td>
<td>0.12</td>
</tr>
<tr>
<td>Shrub forest</td>
<td>0.43</td>
</tr>
<tr>
<td>Pasture</td>
<td>0.15</td>
</tr>
<tr>
<td>Dense crops</td>
<td>0.18</td>
</tr>
<tr>
<td>Sparse crops</td>
<td>0.14</td>
</tr>
</tbody>
</table>

### 5. The Experiments of LAI Inversion

#### 5.1. Data Set Description

The proposed ERTM supposes the vegetation canopies to be uniformly distributed. In addition, the radiative transfer of ascending and descending of the light wave is evenly radiation process within the vegetation canopy. While the model is working, the required input parameter includes a series of the observed data enumerated as follows: reflectance \(\rho(\lambda)\) of vegetation canopies, transmittance \(\tau(\lambda)\) of vegetation canopies, LAI (leaf area index), LAD (mean leaf inclined angle distributions), soil reflectance \(\rho_s(\lambda)\), as well as horizontal visibility \(V\) that is used to calculate the scattering component of the solar radiation. Furthermore, the spectral direction of vegetation radiation is simulated with a series of the observed parameters, such as the solar zenith angle \(\theta_s(\cdot)\), the solar azimuth \(\phi_s(\cdot)\), the observed zenith angle \(\theta_v(\cdot)\), the observed azimuth \(\phi_v(\cdot)\), and other parameters.

For the range selection of the input parameters for LAI model inversion, this study consults biophysical parameter data of vegetation canopies, which are provided by the multispectral RS observation data set (MOD15A2). The experiment employs multi-spectral RS data of vegetation canopies as well as component parameter data of biophysics of vegetation canopies to implement the model inversion of LAI.

This data set contains over 80 species of multi-spectral RS data of woody plants and herbaceous plants, including over 80 species of multi-spectral RS data of leaf samples. These data reflect multi-spectral characteristics of vegetation canopies as well as the species diversity of vegetation. These data consist of a series of component parameters of vegetation leaves, such as internal structure, chlorophyll content, moisture content, and other component content of vegetation canopies. Therefore, the multi-spectral characteristics of vegetation canopies are provided with broad representativeness of various vegetation types (Nilson 2009). By making use of over 80 spectral data of vegetation leaves, the inversion of LAI is implemented in the experiment. Spectrums of vegetation canopies are regarded as the hemispherical reflectance and transmittance, and their wavelength ranges cover 550nm-2800nm, and their sampling intervals are 2nm as well. In order to reduce the errors, the mean value of the spectral data of four leaves in each category is adopted to represent the spectral data of the same category. For the sake of decreasing the noise level, the sampling intervals of the spectrums of vegetation canopies are 6nm.

#### 5.2. Analysis of the Sensitivity and Uncertainty of Parameters for LAI Inversion

The model inversion of LAI in this experiment is achieved with the aforementioned DAGS as well as the algorithm given in formula (14)-(18). First of all, the cost function given by formula (15) is minimized with the iteration optimization algorithm. Then the minimum objective
function given by formula (16) is ascertained with the algorithm of the iteration optimization. Afterwards, LAI inversion is implemented with the algorithm given by formula (17). Finally, the uncertainty of the retrieval result is calculated with the algorithm given by formula (18). The uncertainties of the parameters that have not participated in the inversion are invariable. The most uncertain and most sensitive parameters are selected from the next group of the retrieved parameters according to the aforementioned DAGS. After that, the inversion of the second stage begins to come into effect, until the parameter value of the inversion is no longer changing (North 2006).

5.2.1. Determining Sensitive Parameters for Different Vegetation Types

Experimental results indicate, during the inversion model being performed, that the quantities of the retrieved parameters will be increased in an exponential way along with the size of the data sets that are generated by the retrieved parameters. In this case, more resources of a computer will be occupied, and a longer time will be consumed as well. Therefore, it is necessary for the model inversion of LAI to properly decrease the quantities of the retrieved parameters under the conditions which the retrieval precision is significantly ensured. In order to more accurately apply the established simulation model of vegetation leaves, it is requisite for LAI inversion to ascertain the change step-length and the mobility scale of the needed parameters during the retrieval model being operated. After input parameters for LAI inversion are determined in accordance with a priori knowledge, the corresponding reflectance of vegetation canopies can be achieved by the retrieval model of vegetation canopies. If changing one parameter of them, and keeping other parameters unchanged, then another group of the reflectance of vegetation canopies can be again obtained as well. By adjusting the change step-length and the mobility scale of input parameters, it can make the mobility scale of input parameters become smaller as much as possible. In this way, this will be able to accurately simulate the actual spectra of vegetation canopies as much as possible. Thus, the simulated spectra and the actual spectra of vegetation canopies can be consistently matched as much as possible. In order to determine the influence extent of input parameters for the retrieval model, this paper introduces the concept of the sensitivity. By analysis of the sensibility of input parameters for the retrieval model, the change step-length and the mobility scale of input parameters are accurately determined. The analysis process of the parameter sensibility is summarized as follows. When the parameter change-range $\Delta \sigma$ is given, a change situation for simulated value $\Delta \varepsilon$ of the model output is investigated near a reference value $\sigma_0$. The analytical algorithm of the parameter sensibility is defined as follows.

$$\Delta \varepsilon = \frac{1}{M} \left[ \sum_{i=1}^{M} (\sigma_0 + \Delta \sigma_i) \right] - \frac{1}{M} \left[ \sum_{i=1}^{M} (\sigma_0 - \Delta \sigma_i) \right]$$ (19)

Where, $\Delta \varepsilon$ is the simulated value of the model output; $\sigma_0$ is the $i$th reference value of $\sigma_0$; $\Delta \sigma_i$ is the change-range of the $i$th parameter; $M$ is number of input parameter.

In accordance with the above formula, the sensitivity of input parameter on each RS band is defined as follows.

$$V_j = \frac{1}{\Delta \sigma} \left\{ \frac{1}{N} \sum_{j=1}^{N} \left[ \gamma_j (\sigma_0 + \Delta \sigma) - \gamma_j (\sigma_0) \right] \right\}^{\frac{1}{2}} - \frac{1}{\Delta \sigma} \left\{ \frac{1}{N} \sum_{j=1}^{N} \left[ \tau_j (\sigma_0 + \Delta \sigma) - \tau_j (\sigma_0) \right] \right\}^{\frac{1}{2}}$$ (20)

Where, $V_j$ denotes the sensitivity of the input parameter on band $j$; $N$ is number of RS bands; $\gamma_j(\sigma_0)$ is the reflectance vector of vegetation canopies at reference point vector $\sigma_0$; $\tau_j(\sigma_0)$ is the transmissivity vector of vegetation canopies at reference point vector $\sigma_0$.

When the sensitivity of each input parameter is calculated with the above formula, the parameter should be moved left to $\Delta \sigma$ from the reference point $\sigma_0$, and other parameters are fixed at the reference point $\sigma_0$. The determining process of sensitive parameters for different vegetation types is explained as follows. The spectral distribution range and the mean spectrum as well as their identification and classification results of different vegetation types is regarded as a priori knowledge of LAI inversion of (See Figure 1). Biophysical parameter data of vegetation canopies are consulted for the range selection of input parameters used for the retrieval model. Due to the difference of vegetation type and leaf structure, and the limitation of the quantities of the training samples, the measured values of biophysics of the fresh leaves in the data set are only regarded as the reference values of input parameters in the experiment. In this way, the step length of input parameters is appropriately reduced (Privette, et al 2006). Figure 2 is the distribution diagrams of the wavelength-sensitivity curve for biophysical component of vegetation canopies, which are produced by the above algorithm. The meaning of each symbol in Figure 2 is explained as follows: LMC means leaf moisture content of vegetation; LCC indicates leaf chlorophyll a+b content of vegetation; LDMC expresses leaf dry matter content of vegetation; LISP shows leaf internal structure parameter of vegetation. It can be seen from figure 2 that both the sensitivity curves of LDMC and LISP appear a minor fluctuant in the entire wavelength interval, and that the sensitivity curves of LMC present a major undulant in the entire wavelength interval. It can be further seen from figure 2 that the wavelength range of sensitivity of LCC to spectral is among 1400-800nm. The sensitivity of the other three biophysical components is respectively much less than that of LCC in the interval, and the chlorophyll a+b content is no longer sensitive to the spectral in the interval of wavelength $\lambda > 800$nm. However, the sensitivity of LMC is increased rapidly in the interval of the wavelength. Because the
maximum wavelength of the spectral of vegetation leaves is 2800nm, so the sensibility of LMC can be likewise neglected. By the analysis of the distribution regularities of the wavelength-sensitivity curves for biophysical component of vegetation canopies, the sensitive input parameter is assigned with the major weighted coefficient according to the priori knowledge, and the insensitive input parameter is assigned with the minor weighted coefficient in accordance with the priori knowledge. However, the input parameter that can be ignored is subsequently assigned with the constant value according to a priori knowledge. In this way, the quantities of the retrieved parameters can be greatly decreased. Accordingly, during the inversion model is working, fewer resources of the computer will be occupied, and the calculating time will be correspondingly greatly shortened.

![Figure 1. Average spectral reflectance of different vegetation types.](image)

![Figure 2. Distribution diagram of the wavelength-sensitivity curve for biophysical component of vegetation canopies.](image)

In order to reduce the uncertainty of LAI inversion, it is necessary to restrict the solution space of the parameters with prior knowledge. The sensitivity of the retrieved parameter can be calculated using the above formula, and the sensitive parameters will be selected for LAI inversion. By the calculation of the sensitivity of the retrieved parameter, it is a well known fact that the reflectance of vegetation canopies is varied with the sensibility of the different parameters. For example, the reflectance of vegetation canopies is sensitive to parameter LAI, to LAD, and LISP, etc. However, the reflectance of vegetation canopies is little sensitive to parameter LCC, to LMC, to LDMC, and LAHR, etc. Meanwhile, corresponding sensitive parameters of different vegetation type as well as their value range there exists a large difference as well. Therefore, by giving a prior knowledge for restricting the solution space, the retrieval precision of the parameters can be significantly improved during the parameter inversion is implemented. For this reason, first of all, the spectral distribution ranges and the mean spectral values of different vegetation types are statistically calculated by the consequences of the identification and classification of RS image (See Figure 1). Then the sensitivity of distinctive vegetation type is analyzed so that the sensitive parameters of diverse vegetation type are accurately determined. According to the aforementioned analytic results of the sensibility for vegetation parameters, the sensitive parameters that are selected from the different vegetation types as well as their value ranges of the uncertainties are calculated with the above algorithm (See Table 1 to Table 3).

5.2.2. The Model Inversion of LAI

Making use of the sampled multi-spectral data of vegetation canopies, LAI is retrieved with formula (5)-(20), and then values of LAI are optimized and calculated with the aforementioned DAGS in the experiment. Figure 3 shows the comparison of the measured LAI values with the retrieved LAI values of the model inversion. Where, the correlation coefficient is $R = 0.86$, the root-mean-square error is $RMSE = 0.24$. It can be seen from Figure 3, from the perspective of probability and statistics analysis, that when $LAI$ value <3, then the retrieved $LAI$ value with the model is appropriately adjacent and coincident with the measured one, and there is a little difference between them, and that when $LAI$ value >3, then the retrieved $LAI$ value with the model is generally less than the measured one. The experimental results indicate that when $LAI$ value >3, then the reflectance of vegetation canopies is no longer sensitive to $LAI$. The above conclusion is strictly coincident with the research result of the predecessors (Piwowar 2011).

Besides, the priori knowledge, such as spectral distribution range and the value of the mean spectral of different vegetation type, as well as the identification and classification results of the spectral, is set up as the predicted value of the prior knowledge for LAI inversion, and on this basis, LAI inversion is implemented perfectly. Changing regularity of the percentage of the absolute value of the difference between the retrieval result and its actual value for its true value along with the noise is as shown in Figure 4. In Figure 4, the horizontal ordinate is the percentage $\xi$ of the noise level, and the vertical ordinate is the percentage $\eta$ of deviation of LAI inversion. It can be seen from Figure 4, under the condition of the percentage $\xi \in (3, 70)$ of the noise level, that the percentage
of the retrieved deviation of \( LAI \) is shown at \( \eta \in (-15, 4) \). In addition, it can be also seen from Figure 4 that when \( \zeta \to 3 \), \( \eta \to 2 \), then \( \eta \) is converged to the point \( P(3, 2) \), and that when \( \zeta \to 70 \), \( \eta \to -15 \), then \( \eta \) is converged to the point \( Q(70, -15) \) as well. When the percentage of the noise level \( \zeta \) is located in the interval \((3, 70)\), the maximum value and minimum value of the percentage \( \eta \) of deviation of \( LAI \) inversion are \( \text{Max} \; \eta = 4 \), \( \text{Min} \; \eta = -15 \), respectively (See Figure 4). As a consequence, it is not difficult to conclude that the retrieval precision of \( LAI \) can be significantly improved by employing multi-spectral RS data of vegetation canopies in the framework of DAGS. By making use of the proposed algorithm to perform the model inversion of \( LAI \), the mean value of the retrieved result is appropriately adjacent and coincident with the truth-value, that is, the deviation of the inversion is in a permitted range.

**Figure 3.** Comparison of the measured \( LAI \) values with the retrieved \( LAI \) values.

**Figure 4.** The relative deviation of \( LAI \) retrieval results for different vegetation types.

### 6. Analyses and Discussions

As stated above, if \( LAI \) value \( >3 \), then the reflectance of vegetation canopies is no longer responsive to \( LAI \). The conclusion is strictly coincident with the research result of the predecessors. Furthermore, research results show that \( LAD \) (mean leaf inclined angle distributions) of vegetation canopies is closely related to \( LAI \). If the value range of \( LAD \) is greater than the actual range, then it will cause the retrieved value of \( LAI \) getting a little greater. Otherwise, if the setting range of \( LAD \) is less than the actual range, then it will cause the retrieved value of \( LAI \) getting a little less. Meanwhile, the value of the determined parameter basically represents the mean value of the parameters of vegetation type, and it cannot completely represent all value of vegetation parameters of the vegetation type. Therefore, when the \( LAI \) whose \( LAI \) value is greater than 3 is retrieved, it is obviously important to accurately determine \( LAD \) of vegetation canopies (Sellers 2007).

As mentioned above, RS multi-spectral data of vegetation canopies, which cannot be directly observed, are predicted with DAGS, and biophysics component data of vegetation canopies, which are omitted with observation, can be also forecasted in the experiment. When the unknown data are forecast and prognosticated, the system itself can be also optimized at the same time. Accordingly, an interrelated model between the observed data and the simulated data is likewise built, so that the data simulated with the model are more objective and more accurate. The proposed algorithm eliminates the imperfection of the traditional approach for \( LAI \) inversion of vegetation canopies. In addition, the proposed algorithm simplifies the optimal iterative process of \( LAI \) inversion for different vegetation type, and quickens the convergent velocity of the optimal solution of the system. Consequently, the optimal iterative velocity of the model inversion of \( LAI \) can be considerably expedited, and the retrieval precision of \( LAI \) will be significantly improved as well (Suits 2008).

As discussed previously, adjusting the change step-length and mobility scale of the input parameters not only makes the change step-length and mobility scale of input parameters become smaller as much as possible, but also simulates the actual spectral of vegetation canopies as accurately as possible. In this way, the simulated spectral and actual spectral can be consistently matched as much as possible. In sensibility analysis, a sensitive parameter is assigned with a major weighted coefficient, and an insensitive and negligible parameter is assigned to a minimum weighted coefficient or the constant value. Consequently, the quantities of the retrieved parameters can be decreased greatly, and a less resource of the computer will be occupied, and the computation time of the parameter inversion will be likewise considerably shortened (Verhoef 2006).

Moreover, the knowledge of the spectral distribution range, the mean spectrum of different vegetation types, and the identification and classification results of the spectral is set up as a priori knowledge of \( LAI \) inversion. On this basis, the model inversion of \( LAI \) is implemented with DAGS. Furthermore, research result demonstrates that \( LAD \) of vegetation canopies is closely related to \( LAI \). If the value range of \( LAD \) is greater, then it will result in value of the
retrieved LAI getting a slightly greater, and vice versa. When LAI value is less than 3, then LAI value of the model inversion is appropriately adjacent and coincident with actually measured one. When LAI value is greater than 3, then LAI value of the model inversion is generally less than actually measured one, and the reflectance of vegetation canopies is no longer responsive to LAI. Since the value of the retrieved parameter only represents the average value of the parameters for the vegetation type, and it cannot completely represent all value of the parameters of the vegetation type, so when LAI whose LAI value is greater than 3 is retrieved, it is thoroughly significant to accurately ascertain the value of LAD for vegetation canopies (Veroustraete, et al 2006).

By the comparison of the simulated spectral with the actual spectral, it illustrates that the key to accurately retrieve LAI relies on improving the simulated precision of the model. In other words, it is essential for the precise inversion of LAI to accurately simulate LAI of vegetation canopies with the model. The above conclusion is obtained from LAI inversion of the level leaves of vegetation. However, this experiment validates that the same conclusion applies to LAI inversion of the level canopies of vegetation. The research indicates that it is necessary for the widespread inversion of LAI to achieve a balance between the complexity and the accuracy of the retrieved model. The more precise the descriptive model for the parameter state of vegetation canopies, the more complicated its own configuration will be, such as the three-dimensional ray-tracing model, as a result, the more the model parameters involved. However, the more complicated the model, the more strenuous the model inversion. It is conceivable that, if a complicated retrieval model of vegetation leaves is coupled with an intricate retrieval model of vegetation canopies, then the inversion of LAI is synthetically implemented, the difficulty of the inversion can be enormously increased. The proposed DAGS for the model inversion of LAI can be not only utilized to solve relatively simple optimizing problems, but also to solve moderately complicated optimizing problems, such as the inversion of biophysical component parameters of vegetation canopies by the aid of RS data of the hyper-spectral and multi-angle. The experimental result indicates that the proposed algorithm is an effective way to solve the inversion of the complicated model. The retrieval model of LAI is the inversion based on a definite condition, and different retrieval model possesses its adaptive vegetation types. For instance, both the coniferous forest and the broad-leaved forest are provided respectively with the distinctive canopy structure and reflectance characteristics of vegetation canopies, so the established model of the leaf reflectance is correspondingly different. Consequently, when the inversion of LAI is performed in the extensive range, it is necessary for the inversion of LAI to synthetically utilize the model of the canopy reflectance, which is suitable for the application in different vegetation type. Meanwhile, when the model inversion is implemented, it is also an incentive for LAI inversion of distinctive vegetation type to further research on the determination method of the value range for the sensitive parameters. Moreover, it has vital significance for decreasing the uncertainty of the parameter inversion to accurately ascertain the value range of the sensitive parameters for different vegetation type.

7. Conclusions

By making use of DAGS, this paper simulates the spectral reflectance of vegetation canopies, and sets up the component parameter of vegetation to be retained round an initial value, and then to achieve the model inversion of LAI. Experimental result shows that the less the value of LAI, then the greater the leaf structure parameter N of vegetation, and also the less the LAD of vegetation as well. This conclusion explains that the influence which LAI and LAD act on the reflectance of vegetation canopy is contrary to that of LISP, respectively. Further research discovers that the combination of different parameters can correspond to almost similar spectral. Therefore, the morbid problem of the model inversion is still existent, and so far it is still the bottle-neck issue of the model inversion research. Since both the traditional retrieval approach of LAI as well as the inversion method based on a priori knowledge is correspondingly limited to the parameter inversion of a certain specific RS observed moment, so the retrieval result is provided with greater uncertainty. The proposed DAGS of LAI inversion not only inherits the advantages of the retrieval approach based on a priori knowledge, but also expands the retrieval parameters in time-space. Meanwhile, by introducing dynamic parameters of the model, the interrelations among the inversion parameters of different retrieval moments are effectively restricted. Thereby, the retrieval efficiency and the reliability are evidently improved. By taking advantage of DAGS, this article achieves the model inversion of LAI, and ensures the reliability for the value range of the uncertainty of the retrieved parameters, and guarantees the accuracy of a priori knowledge for LAI inversion. This means that the inversion of LAI is the retrieval based on a priori knowledge. By the accumulation of a priori knowledge, the uncertainty of the retrieved parameter is gradually decreased, and the connotation of the priori knowledge for LAI is accordingly enriched. For multi-spectral RS data of different vegetation type, the richer the priori knowledge of LAI is, the higher the accuracy of LAI inversion will be. By making use of a priori knowledge of LAI for different vegetation type, the solution space of the retrieved parameters is strictly restricted. As a result, the uncertainty of the retrieved parameters can be reduced so that the reliability of LAI inversion is significantly improved. Since different vegetation type possesses disparate canopy configuration and reflectance characteristics, so when LAI inversion of is implemented, it is especially necessary for the model inversion of LAI to synthetically utilize the reflectance model of different vegetation type. The important conclusion to be drawn from this discussion is that the uncertainty of LAI inversion can be greatly reduced by means of precisely determining the value range of sensitive
parameters for diverse vegetation type. All mentioned above tell us that it is of great importance for decreasing the uncertainty of the retrieved parameter to precisely determine the value range of sensitive parameters for different vegetation type. However, one thing we have to notice is that it is critical for the model inversion of LAI to accumulate the priori knowledge of LAI retrieval, such as the know-how of the identification and recognition of different vegetation type by making use of RS multi-spectral data. Considering the possible reasons, it is rational for us to conclude that the proposed DAGS is relatively optimal for the model inversion of LAI. In a word, the proposed DAGS of the model inversion of LAI inherits the benefits of the traditional retrieval way based on a priori knowledge, and introduces the sensitive parameters in time-space so as to improve the retrieved accuracy of LAI. There is no doubt that, as discussed above, the proposed algorithm opens up a novel algorithmic pathway for the model inversion of LAI with the aid of DAGS.

References


