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Quantum Interactions of Small-Sized Neurotransmitters and of Entangled Ionotropic Receptors Accentuate the Impact of Entanglement to Consciousness

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Abstract: This contribution concentrates on the evaluation of quantum processes in the brain that essentially contribute to the protection and activation of entanglement and their impact to consciousness. The corresponding calculations occur in the Fock space that represents discrete quantum fields, where the corresponding computations occur in the following succession. First, three possible weak interactions of emitted, small-sized neurotransmitters are described. These interdependencies are the attraction by electric dipole-dipole interaction, the attraction by the Morse potential and the repulsion characterized by s-wave scattering. Second, this article focus on ionotropic receptors that are embedded in a dense non-rigid grid. Anharmonic oscillators approximate these molecules, where their interactions cause grid vibrations. The determination of the expectation values of the total energy of the oscillating receptors, situated in two entangled ground states, demonstrate the existence of gap functions that shield the entanglement. This protected entanglement represents a bridge to the materialistic consciousness, and as well it refutes the dominant criticism against the quantum processes in the brain that decoherence destroys in picoseconds the entanglement (quantum coherence). The entangled entropy of the protected entangled states is not zero; what is a clear sign of entanglement. Third, consciousness activates the protected entanglement that reveals distinct positive effects, concerning the acquisition of information. Thus, the working space (associative cortices) that operates in a conscious state instantly gets compressed information on the current particular states of the cortical and subcortical components. Thereby, the emergence of consciousness is a synergetic process, which is created by the mutual interdependencies (causal circularity) of the components of the working space (synergetic agents) and the subcortical areas (synergetic "slaves").

Keywords: Interactions of Neurotransmitters, Vibrations of Ionotropic Receptors, Protected Entanglement, Consciousness, Synergetics

1. Introduction

Since many decades, particularly biologists and physicists fiercely debate the role of quantum processes in the brain. Elementary quantum processes are the release of neurotransmitters (exocytosis), their transmission and finally their reception. These basic actions take place in the brain at each level. The most outstanding quantum effect of the higher level is the entanglement that the working space (associative cortices) activates during its conscious state. Thereby, the working space immediately achieves relevant, compressed information from all locations, where the entanglement is enabled.

Generally, the impact of quantum states substantially depends on the robustness of their coherence, which is one essential doorway to quantum effects in the brain. However, the opinions distinctly differ on this statement. The community of supporters very engaged advocate for the existence of quantum effects in the brain, where the group of repudiators vividly refuse any presence of such effects.

Sequentially, some of the dominant proponents of the "quantum brain" are cited. One of the first advocates was Fröhlich [1]. He described the states of cell membranes in the "hot brain" by a kind of Bose-Einstein condensate, but his approach was experimentally not confirmed [2]. The authors Beck and Eccles [3] characterized the vesicular emission by a tunneling effect. However, the experimentally verified probability of the exocytose (without tunneling) is not defined by the corresponding quantum probability, although by the Poisson probability (resp. binominal distribution), [4-5]. The proponents Penrose [6] and Hameroff [7] suggest the creation of a shielded quantum coherence by the concept of the orchestrated-objected reduction (OOR). The respective effects of the interactions between the tubulin dimers cause coherent quantum vibration of these molecules. Experiments falsified this suggestion.

The group of opposers especially emphasizes the effect of decoherence on coherent quantum states. It destroys the coherence in a very short time, typically in picoseconds [8]. Furthermore, the adversaries vividly attack the suggestion that quantum effects could build a bridge to consciousness.

The first main objective of this contribution is the demonstration that quantum coherence in the brain can be established without its destruction by decoherence. For that, anharmonic oscillators approximate ionotropic receptors, where the interactions (couplings) of corresponding entangled oscillators shelter their entanglement (quantum coherence).

The second essential aim of this work is the stating of the thesis that the protected entanglement represents the preferred method of the consciousness to collect immediately all relevant information from the entangled brain locations. This statement even reveals the convincement that consciousness is materialistic.

The synergetic approach, which represents the theory of self-organization, describes the transition of the cortex between unconscious and conscious phases. The principle of the causal circularity of synergetics connects these two states. The four essential cortexes (prefrontal, parietal, temporal and occipital lobes) constitute autonomous synergetic agents, which establish the working space. These agents are autonomous and negotiate with themselves to make final, aligned decisions that, for instance regulate the subcortical areas. This corresponds to an adaptive distributed control. Thereby, the subcortical areas represent synergetic “slaves”, which, for instance deliver unrequested, relevant sensor data to the synergetic agents, when they call up these data. The synergetic agents interpret this information, and request further specific inputs, if they need additional disclosures. In opposition to the standard mathematical model of the cortex, this approach also includes the self-reflections of the working space without subcortical inputs.

2. Particles, Processes and Methods

2.1. Particles and Processes

The considered biological particles are small sized neurotransmitters (e.g. Glutamate, Dopamine) and small ionotropic receptors, for instance, ACh receptors, with overall diameter, including the channels, of about 8 nm, [5]. Four or five subunits comprise a direct gating receptor, which represent a macromolecule of a size of about up to 10 nm. Examples are the two subtypes of cholinergic receptors or the two types of glutamate receptors (AMPA, NMDA), [9]. The ionotropic receptors compose a non-rigid grid with resemblance to the molecular grids that occur in solid states [10-11].

The physical particles that represent small sized neurotransmitters and ionotropic receptors are spinless Bosons, which are members of different non-relativistic quantized fields. For two main reasons, these particles are characterized as Bosons and not as Fermions. First, these molecules are in general nonpolar, diamagnetic and the inner saturated electron shell shields the nuclear spin-spin interactions. Therefore, the molecular spin is neglected. Second, the Pauli exclusion principle of Fermions forbids their clustering. Fermions aggregate only in the cases of superconductivity (singlet state) and superfluidity of ^{3}He (triplet state), [12] at very low temperatures, because interacting pairs of electrons behave as Bosons. The relevant effect of the superconductivity is the shielding of the electron pairs by an energy gap [13]. This gap prevent the Cooper-pairs to disperse [14].

Classical approaches characterize the transmission of neurotransmitters through the synaptic cleft by an ordinary diffusion [15-16]. Throughout this paper, the neurotransmitters and receptors are disparate field quanta. This description opens the gateway to various quantum processes. Examples are the quantum diffusion [17] and the three aforementioned possible weak interactions between neurotransmitters. Further processes are the interactions of entangled receptors, the protection of these receptors against decoherence, and the essential correlation between entanglement and consciousness.

The assumption that the regarded molecules are indistinguishable Bosons constitutes the substantial precondition of the Bose-Einstein statistics. Thus, the receptors are, for instance elements of a grand canonical ensemble that is in a thermal equilibrium phase [18]. One crucial consequence of this viewpoint is the integration of such ensembles in the modern approach of the finite temperature quantum thermodynamics [19].

The cortex is an open system that is in a non-equilibrium phase, provided it is in a conscious state. This assumption immediately directs the investigations to synergetic specifications of the processes occurring in the brain [20]. This assumption forward leads to the conclusion that synergetic processes self-dependently generate the meaning of the available information on the base of mutual, expensive message exchanges between them. The subcortical areas acquire raw sensor information and perform a preprocessing of them (thalamus), before the associative cortices evaluate this information [21]. The synergetics supports the formation of order parameters (e.g. data structure like a priority map, salience map, and grid cell or activity patterns) by the synergetic agents (working space). The customary synergetic approach describes man made processes with abrupt phase transitions, for example the laser and the Belousov-Zhabotinsky reaction [20], where the principal functionality of the participating particles does not changes. Only their macroscopic behavior alternates. In the case of a laser, the uncorrelated light of a lamp transfers to a coherent light wave.

In contrast, to technical processes the number of the main actors (synergetic agents, synergetic “slaves”) in the brain processes strongly fluctuates. Further, the amount and the functionality of cells, neurons and synapses steadily changes, where mainly the synapses learn and to store information. The aggregation of such changing activities represent biological synergetic processes, where the amount and the types of message exchanges (interactions) between all agents diversify. Related to these information exchanges, the local interpretations of the incoming messages differ. Nevertheless, the synergetic agents of the working space determine the information interpretation, with the highest probability.

2.2. Methods

The physical framework of this contribution is the non-relativistic quantum field theory [22], which is accomplished in the Fock space [23] of the symmetrized product states. The operators acting in this space are defined in the time-independent Schrödinger representation or in the time-dependent Heisenberg picture [24]. However, the standard Fock space only comprises incoherent states, where the extended version of the Fock space contains coherent states. These states are introduced to describe weak interactions between neurotransmitters. Inherently, the Lennard-Jones potential (shorthand L-J potential), [25] delineates these interactions; though, this potential is analytically unsolvable by the corresponding radial Schrödinger equation. Therefore, the three dominant effects of the L-J potential are evaluated with suitable approximations.

The vibrations of the grid-embedded receptors, approximated by coupled anharmonic oscillators, are calculated by a modified method that is applied in solid states physics (many-particle interactions). Herewith, the various vibrations of the Fourier components of the spatial density of the receptors are computed. The self-interactions of receptors provide the basis to combine them with entangled ground states to construct bosonic gap functions, which shield the entangled states against decoherence. This approach defines the protected entanglement. In this case, the techniques that are applied to deduce the superconductivity of electrons is distinctly modified to the protection of entangled Bosons.

The evaluation of partition functions of canonical and grand canonical ensembles constructs the scaffolding to determine the corresponding density operators and the entangled entropy that measures the grade of the entanglement. We employ this entropy on the density operators of the ground states of the protected entanglement to evaluate the grade of the protected entanglement.

The transition from the unconscious phase to the conscious phase represents a non-equilibrium phase transition of second order occurring between two open systems. This transfer induces a spontaneous symmetry braking that decreases the Shannon entropy and consequently increases the order (decreases the Shannon entropy) of the cortex.

The essential methods of the synergetics that evaluate biological processes are characterized by the permanent acquisition and consolidation of information in systems that continuously experience structural diversifications. Therefore, learning will become an essential ingredient of this kind of application of synergetics. Furthermore, the process of entanglement represents a new efficient method of information handling.

3. Dominant Features of the Fock Space of Bosons

The Fock space is grid-based, where each of its spatial discrete points can contain an unlimited number of indistinguishable particles, which represent a quantum field. When the momentum and the energy of the field are relevant, then the switch to the $\boldsymbol{k}$-based version of the Fock space is appropriate, where k denotes the wave number vector.

The spatial Fock space is constructed by replacing the continuous version of operators by a grid-based version. For example, a continuous creation operator is replaced by $\hat{a}^{\dagger}(\boldsymbol{x}) \rightarrow \frac{\hat{a}_i^{\dagger}}{\sqrt{v}}$, where v specifies the elementary volume at which the lattice point is located. In this article, the “hat” ^ marks all operators, hence a clear differentiation exist between operators and probability amplitudes (numbers).

The Hamiltonian $\hat{H}$ describes the energy of particles moving in an external potential field V_i. The number operator $\hat{N}$ counts the total number of particles located in this field. These two dominant operators act in the Fock space:

$$\hat{H} = \sum_i \hat{a}_i^{\dagger} \left[-\frac{\hbar^2}{2m} \Delta_i + V_i \right] \hat{a}_i \equiv \sum_i E_i \hat{a}_i^{\dagger} \hat{a}_i, \quad (1)$$

$$\hat{N} = \sum_i \hat{a}_i^{\dagger} \hat{a}_i = \sum_i \hat{N}_i. \quad (2)$$

In equation (1), the expression Δ_i denotes the discrete Laplace operator, and E_i is the energy of a particle.

The active release of neurotransmitters at the presynaptic membrane provides each molecule with the momentum (kinetic energy) to traverse the synaptic cleft. This traversal leads to a molecular flow with molecular losses (modified continuity equation) and to molecular scattering [17]. Since these effects are already outlined, one focus of this paper lies on the computation of the supplementary molecular effects that are caused by the Lennard-Jones potential.

3.1. Many-Particle Representation of Non-interacting Bosons in the Symmetrized Fock Space

The Fock space $\mathcal{H}^{(+)}$ of Bosons is constructed by the direct orthogonal sum of all $N = 0, 1, 2, \ldots$. Hilbert spaces of the physical relevant, symmetrized product states denoted by the superscript $(+)$

$$\mathcal{H}^{(+)} = \bigoplus_{N=0,1,2,\ldots} \mathcal{H}_N^{(+)}. \quad (3)$$

The number of different permutations of the basis states of $\mathcal{H}_N^{(+)}$ constitutes the dimension d of this space. The complete Fock space of the physical states is $\mathcal{H} = \mathcal{H}^{(+)} \oplus \mathcal{H}^{(-)} \oplus \mathcal{H}^{(0)}$. The Fock space $\mathcal{H}^{(-)}$ of unsymmetrized product states represents Fermions and the space $\mathcal{H}^{(0)}$ constitutes the vacuum.

The symmetrized tensor product of creation operators establish the orthonormal basis of $\mathcal{H}^{(+)}$, where for simplicity, the symbol $\otimes$ of tensor multiplication is suppressed

$$|n_1, n_2, \ldots\rangle = \frac{(\hat{a}_1^\dagger)^{n_1}(\hat{a}_2^\dagger)^{n_2}\ldots}{\sqrt{n_1}!\sqrt{n_2}!\ldots}|0,0,\ldots\rangle. \quad (4)$$

Here $n_i = 0,1,2,\ldots (i \in \mathbb{N})$ denotes the number of particles that are localized at the lattice position i. The ket $\lceil 0,0,\ldots\rangle$ indicates the vacuum state. The equation (4) defines the many-particle state of non-interacting Bosons. The state vector is symmetric under the permutation of any two different creation operators, since they commute. The terms $\sqrt{n_i}!$.in the dominator of formula (4) eliminate the factors $\sqrt{n_{i+1}}$ that appear, when the creation operators are applied on the many-particle state (7). The adjoint many-particle state reads

$$\langle \ldots n_2, n_1| = \langle \ldots, 0, 0| \frac{\ldots(\hat{a}_2)^{n_1}(\hat{a}_1)^{n_2}}{\ldots\sqrt{n_2}!\sqrt{n_1}!} \quad (5)$$

The many-particle states (4) are orthonormal, where the sum of the corresponding projection operators subjects the completeness relation

$$\sum_{n_1,n_2\ldots=0}^{\infty} |n_1, n_2, \ldots\rangle\langle \ldots n_2, n_1| = \mathbf{1}. \quad (6)$$

When the dimension of a finite space $\mathcal{H}_N^{(+)}$is calculated, then it is obvious that identical particles are indistinguishable and therefore cannot separately counted. For example, if four states are available ($d = 4$) and 2 particles are to distributed to these four states (locations), then $\dim \mathcal{H}_4^{(+)} = \binom{N+d-1}{N} = \binom{5}{2} = 10$. The dimension is not $d^N = 4^2 = 16$, because six states are identical due to their indistinguishability, e.g. $|n_1 = 1, n_2 = 1, 0, 0\rangle = |n_2 = 1, n_1 = 1, 0, 0\rangle$.

The extension of the standard (incoherent) many-particle states of the Bosons (4) to coherent states facilitate the possibility to attach different weights to the grid locations. Thus, not only non-interacting particles can describe, but also interacting particles. More formally, these states accomplish an eigenvalue equation, whereas the state vector (4) does not fulfill an eigenvalue equation.

3.2. Many-Particle Representation of Interacting Bosons

The application of a creation or annihilation operator on the bosonic Fock space $\hat{a}_i^\dagger: \mathcal{H}_N^{(+)} \to \mathcal{H}_{N+1}^{(+)}$, $\hat{a}_i: \mathcal{H}_N^{(+)} \to \mathcal{H}_{N-1}^{(+)}$ leads to state transitions in this space. However, the many-particle state $|n_1, n_2, \ldots\rangle$ introduced by equation (4) is not an eigenstate of a creation operator or an annihilation operator, since it obeys the two following relations

$$\hat{a}_i^\dagger\, |n_1, n_2, \ldots\rangle = \sqrt{n_i + 1}\, |n_1, n_2, \ldots, n_i + 1, \ldots\rangle. \quad (7)$$

$$\hat{a}_i\, |n_1, n_2, \ldots\rangle = \sqrt{n_i}\, |n_1, n_2, \ldots, n_{i-1}, \ldots\rangle. \quad (8)$$

A coherent state is established in the Fock space by extending the original many-particle state (4) to

$$\lfloor\phi\rangle = \exp\{-\textstyle\sum_i \lfloor\psi_i\rfloor^2/2\} \sum_{n_1,n_2\ldots=0}^{\infty} \frac{\psi_1{}^{n_1}\psi_2{}^{n_2}\ldots}{\sqrt{n_1}!\sqrt{n_2}!\ldots}\, |n_1, n_2, \ldots\rangle, \quad (9)$$

where the ψ_i´s are complex numbers [27] and are called coherent amplitudes. Each of these amplitudes is an eigenstate (coherent state) of the annihilation operator $\hat{a}_i$; the adjoint state $\langle\phi|$ represents an eigenstate of the creation operator $\hat{a}_i^\dagger$

$$\hat{a}_i\lfloor\phi\rangle = \psi_i\lfloor\phi\rangle \text{ and } \langle\phi|\hat{a}_i^\dagger = \psi_i^*\langle\phi|. \quad (10)$$

The adjoint coherent state reads

$$\langle\phi| = \exp\{-\textstyle\sum_i \lfloor\psi_i\rfloor^2/2\} \sum_{n_1,n_2\ldots=0}^{\infty} \langle \ldots n_2, n_1| \frac{\ldots\psi_1{}^{*n_1}\psi_2{}^{*n_2}}{\ldots\sqrt{n_2}!\sqrt{n_1}!}. \quad (11)$$

However, different coherent states overlap and are not orthogonal, if $i \neq j$

$$\langle\phi_i|\phi_j\rangle = \exp\left\{\textstyle\sum_k \left(\psi_{k,i}^*\psi_{k,j} - \lfloor\psi_{k,i}\rfloor^2/2 - \lfloor\psi_{k,j}\rfloor^2/2\right)\right\} \quad (12)$$

Only in the case of $i = j$, they are normalized.

The completion relation for coherent states is

$$\prod_i \left[\int \frac{d\psi_i^* d\psi_i}{2\pi}\right] |\phi\rangle\langle\phi| = \mathbf{1}_{coh}. \quad (13)$$

The adaption of the continuous ψ_i coefficients into the state $\lfloor\phi\rangle$, (10) offers the possibility to extend ψ_i by the transformation $i \to x$ to a continuous coherent amplitude $\psi(x)$.

4. Different Weak Interactions of Neurotransmitters in Approximate Potential Fields

In this chapter, the coherent amplitudes are identified with two different wave functions $\psi(x)$ that solve the radial Schrödinger equations for the Morse potential (subchapter 4.4.) and for the pseudo potential, which describes a scattering process (subchapter 4.5.). Subchapter 4.3. characterizes the potential of the electrical dipole-dipole interactions.

4.1. Features of Weak Interactions of Neurotransmitters Revealed by the Lennard-Jones Potential

The empirical potential that describes the three selected molecular interactions is the L-J potential

$$V_{L-J}^{(int)}(r) = E_m\left[\left(\frac{r_m}{r}\right)^{12} - 2\left(\frac{r_m}{r}\right)^{6}\right], \quad (14)$$

where $E_m > 0$ [eV] denotes the well depth of the potential at the distance r_m at which the potential reaches its minimum $V_{L-J}(r_m) = -E_m$, where r [Å] = [.1 nm] is the distance between two particles, which is measured by the distance of their nuclear centers. The force between two molecules is attractive $-2\left(\frac{r_m}{r}\right)^6$ or repulsive $\left(\frac{r_m}{r}\right)^{12}$. The distance r_m defines the equilibrium at which the attractive and repulsive forces between two neutral molecules are equal. Therefore, the corresponding negative interaction energies near the well depth describe weakly bounded states of both molecules. At the smaller distance, $r_\sigma = r_m/2^{1/6}$ the potential is zero. At this distance, two molecules just touch themselves.

When the distance is further decrease $r < r_\sigma$, then they overlap, because each molecule strikes the other with a kinetic energy of $\frac{3}{2}k_B T$, where k_B is the Boltzmann constant and T denotes the temperature (a body temperature of 37 °C corresponds 310 K, [28]). Thereby, they become slightly deformed and the repulsive forces push both particles apart. At this distance, the repulsive force is greater than the attractive force.

The synaptic cleft is full of water, salt and ions; therefore, it is improbably that neurotransmitters traverse the cleft without any interaction. However, the classical approach describes this traversal by a standard diffusion, whose probability distribution is defined by the Fokker-Plank equation [15, 29]. Hence, the three different interactions attraction, bounding and repulsion that are outlined in this chapter are disregarded by the diffusion approach.

4.2. The Two-Body Approach for the Solution of the Radial Schrödinger Equation

The two-body approach solves the radial Schrödinger equation for the wave function $\psi(\boldsymbol{x})$, hereby $\boldsymbol{x}$ denotes the relative coordinate. Further, $\boldsymbol{X}$ represents the center of mass coordinate and m_r is the reduced mass

$$\boldsymbol{x} = \boldsymbol{x}_i - \boldsymbol{x}_j, \boldsymbol{X} = \frac{m_i x_i + m_j x_j}{m_i + m_j}, m_r = \frac{m_i m_j}{m_i + m_j}. \quad (15)$$

The wave function $\psi(\boldsymbol{x})$ satisfies the one-particle Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}\Delta_x + V(\boldsymbol{x})\right]\psi(\boldsymbol{x}) = E_{int}\,\psi(\boldsymbol{x}). \quad (16)$$

The radial version of this equation by inserting the L-J potential (14) is

$$-\frac{\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2} + \left[E_m\left(\left(\frac{r_m}{r}\right)^{12} - 2\left(\frac{r_m}{r}\right)^6\right) + \frac{l(l+1)}{r^2}\right]u(r) = E_{int}\,u(r), \quad (17)$$

where $u(r) = r\,R(r)$, with $r = |\boldsymbol{x}_i - \boldsymbol{x}_j|$.

However, this equation is analytically unsolvable. For example, when a power series solution is tried as it is customary for the hydrogen atom [24, 30], and then this approach fails. In this case, the standard procedure is to search for appropriate approximations that solve the Schrödinger equation (17) at different distance ranges r and calculate the solutions. Hereby, the distance r decreases from right to left, where the first corresponding approximation describes the attractive transient dipole-dipole interactions that occur for distances $r > r_m$. The subsequent calculation refers to the quantized band spectrum in the vicinity of r_m ($r \approx r_m$), where the L-J potential is substituted by the Morse potential. Finally, the repelling forces are calculated, when two molecules are scattered at distances $r < r_m$, where they are slightly deformed.

4.3. Transient Electrical Dipole−Dipole Interactions

The interaction of transient electrical dipoles depicts the attractive forces between two molecules that occur at greater distances from the bottom of the L-J potential. The appropriate interaction potential is

$$V_{d-d}^{(int)}(\boldsymbol{x}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\boldsymbol{p}\cdot\boldsymbol{x})(\boldsymbol{p}\cdot\boldsymbol{x})}{r^5} - \frac{(\boldsymbol{p}\cdot\boldsymbol{p})}{r^3}\right] + \frac{(\boldsymbol{p}\cdot\boldsymbol{p})}{3\varepsilon_0}\delta(\boldsymbol{x}), \quad (18)$$

where ε_0 denotes the electrical vacuum susceptibility and p defines the electrical dipole moment. The additional δ-term regulates the divergence of the first expression of (21) at the origin. The literature usually disregard this term, e.g. [31]. However, the textbook [32] quotes this δ-function, which originates from the identity

$$\frac{\partial^2}{\partial x_i \partial x_i}\frac{1}{|x|} = \frac{3x_i x_j - \delta_{ij}(x\cdot x)}{r^5} - \frac{4\pi}{3}\delta_{ij}\delta(\boldsymbol{x}). \quad (19)$$

The insertion of equation (22) in the formula (21) delivers

$$V_{d-d}^{(int)}(\boldsymbol{x}) = -\frac{1}{4\pi\varepsilon_0}\sum_{i,j=1}^{3} p_i\,p_i\,\frac{\partial^2}{\partial x_i \partial x_i}\frac{1}{|x|}, \quad (20)$$

where this formula represents the inverse Fourier transform of the multipole expansion of the Coulomb potential in the momentum space [33]

$$V_{d-d}^{(int)}(\boldsymbol{q}) = -\frac{1}{\varepsilon_0}\sum_{i,j=1}^{3} p_i\,p_j\,\frac{q_i q_i}{q^2}. \quad (21)$$

4.4. Morse Potential: Bounded Vibrating States

In the neighborhood of $r_e \approx r_m$, which is the area of the well depth of $V_{L-J}(r)$; this potential is replaced by the Morse potential [34] that defines the anharmonic oscillator

$$V_{Morse}^{(int)}(r) = D_e\left(1 - e^{-a(r-r_e)}\right)^2. \quad (22)$$

Where D_e characterizes the dissociation energy, whereas $a = \sqrt{\mu/2D_e}\ \omega$ [cm^{-1}] is a molecular specific parameter and ω denotes the frequency of the anharmonic oscillator. For very small distances $r \to 0$, this potential is not applicable, since it fails. At the equilibrium distance $r_e = r_m$ the potential is zero, whereas for $r \to \infty$ the potential becomes $V_{Morse}^{(int)}(r) = D_e$. Therefore, this potential also does not describe the attractive forces, which occur at large distances as the L-J potential proposes it.

The Morse potential is in good accordance with experimental data for diatomic molecules, which have only one degree of freedom that solely allows oscillations of the molecules in the binding direction (valence oscillations).

Thus, rotational oscillations stay disregarded in this contribution.

The corresponding radial one-particle Schrödinger equation for vibrations ($l = 0$) reads

$$-\frac{\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2} + [V(r) - E_{int}]\, u(r) = 0, \quad (23)$$

where

$$V(r) = D_e\big(1 - e^{-a(r-r_e)}\big)^2 - D_e = D_e\,\big(e^{-2a(r-r_e)} - 2e^{-a(r-r_e)}\big). \quad (24)$$

This potential has a negative minimum $-D_e$ at $r = r_e$; and it correctly converges to zero at $r = \infty$. The internal energy corresponds to the quantized vibrational energy $E_{int} = E_n$, which is the energetic solution of (23). The eigenvalues of the negative, vibrational bounded energies are

$$E_n = -D_e + \left(n + \tfrac{1}{2}\right)\hbar\omega - \left(n + \tfrac{1}{2}\right)^2 \frac{(\hbar\omega)^2}{4D_e},\; n = 0, 1, 2, \dots, n_{max}. \quad (25)$$

The dissociation energy D_e is equal to $E_{n_{max}}$. The second term of equation (25) formally corresponds to the energy levels of the harmonic oscillator, where the frequency of the anharmonic oscillator is

$$\omega = \sqrt{\frac{2D_e}{\mu}}\; a. \quad (26)$$

The energy of transition between the two levels $n + m$ and n ($n,\ m = 0, 1, 2, \dots$) is

$$\Delta E = E_{n+m} - E_n = m\left(1 - \frac{\hbar\omega}{4\,D_e}(2\,n + m + 1)\right)\hbar\omega. \quad (27)$$

Another approach to calculate the vibrational energy of the bounded states of the Morse potential is the algebraic one, which use the spectrum of the su (1, 1) Lie algebra [35]. However, this method cannot calculate the radial solutions $u(r)$ of equation (23), which are proportional to the fractional associated Laguerre polynomials L_{b+n}^{b} [34]

$$u(r) \approx r L_{b+n}^{b}(2\xi), \quad (28)$$

where $k = 2\pi d > 1$, $d = \frac{\sqrt{2\mu D_e}}{ah}$, $b = k - 2n - 1$, $\xi = de^{-a(r-r_e)}$ and $0 \leq 2n \leq (k-1)$.

Using the formula of fractional differentiation, the three following polynoms for $n = 0$ - 3 are

$$L_b^b(2\xi) = e^{i\pi(k-1)}\,\Gamma(k),\; L_{b+1}^{b}(2\xi) = e^{i\pi(k-2)}\,\Gamma(k-1)\,[2\xi - (k-2)], \quad (29)$$

$$L_{b+2}^{b}(2\xi) = e^{i\pi(k-3)}\,\frac{\Gamma(k-2)}{2!}\,[(2\xi)^2 - 2(k-3)\,2\xi + (k-3)(k-4)].$$

These solutions remind to the solutions of the hydrogen atom that are the non-fractional, associated Laguerre polynomials $L_{n-l-1}^{2l-1}(2\kappa_n r)$, where $\kappa_n = \frac{1}{n\,a_{Bohr}}$, [24, 30].

Notoriously, the Morse potential is only correct for diatoms, but not for the description of vibrational and rotational spectra of two polyatomic molecules (multiple degrees of freedom), where the corresponding calculations of different types of molecular vibrations and rotations are exhaustive. An elementary example of a polyatomic neurotransmitter is Glutamate $C_5\,H_8\,NO_4$. Between such molecules, or even bigger molecules, different bonds can confer binding specificities [9]. Thus, different bonds can occur in parallel, e.g. hydrogen bonds, hydrophobic interactions, ionic bonds, peptide bonds and last not least the dipole-dipole interactions.

In case of an ionotropic receptor, the anharmonic oscillator again approximates this molecule. Obviously, this is once more a simplified model for such kind of receptors. For example, the general structure of receptors that are transmitter-gated ion channels (e.g., ACh, GABA, Glycine, and Glutamate) already reveals relevant molecular features. These receptors possess membrane-spanning proteins consisting of four or five subunits, which form a central pore. The channel subunits are polypeptides that build helices and intrude entirely or partially in the membrane [4]. Thus, this approximation again disregards a great amount of details concerning the chemical and biological processes, e.g. [5]. Nevertheless, there is the assumption that this approach appropriately represents the relevant quantum features of the interactions between neurotransmitters and between receptors. Without the approximation of the neurotransmitters and the receptors by anharmonic oscillators, it is not possible to perform analytical quantum field computations.

4.5. S-Wave Scattering

The main objective of this subchapter is the presentation of the s-wave solution of the radial Schrödinger equation. This aim includes the calculation of the corresponding interaction potential of the s-wave scattering that defines a pseudo-potential. This potential replaces the usual hard sphere potential and represents the singularity of $\psi(x) \approx (u(r)/r)$ at the origin $r = 0$ by a delta function $\delta(r)$. This was the original intention of Fermi [36].

The two specifications $l = 0$ and $m = 0$ concerning the radial equation characterize the s-wave scattering. The singularity at the origin of the standard solution $u(r) = e^{ik\cdot r}/r$ of the simplified radial equation is described by a delta function

$$\left(k^2 + \frac{d^2}{dr^2}\right)\frac{e^{ik\cdot r}}{r} = -4\pi\delta(r)\,e^{ik\cdot r} = -4\pi\delta(r), \quad (30)$$

where $\delta(r)$ was already introduced by the identity referred in equation (19). A characteristic feature of the s-wave scattering is the scattering length a_s, which is calculated by the formula

$$a_s = \lim_{k\to 0}\,(1/k\,\cot\delta_0), \quad (31)$$

where δ_0 marks the phase shift for $l = 0$ and the corresponding cross-section σ_{scatt} converges to a constant $\sigma_{scatt} \to 4\pi a_s^2$. The particular solution of equation (30) that includes the scattering length a_s is

$$u_s(r) = c_s \frac{\sin k(a_s - r)}{r}, \qquad (32)$$

where c_s is the normalization constant $c_s = -\frac{1}{k\cos(ka_s)}$, [26].

Inserting the solution (32) in the equation (30), it becomes

$$\left(\frac{k^2\hbar^2}{2m_r} + \frac{\hbar^2}{2m_r}\frac{d^2}{dr^2}\right)\frac{c_s \sin k(a_s - r)}{r} = -\frac{4\pi\hbar^2 c_s \sin(k a_s)}{2\mu}\,\delta(r) = \frac{4\pi\hbar^2}{2m_r}\frac{\tan(ka_s)}{k}\,\delta(r). \qquad (33)$$

Hereby, this formula was multiplied with the factor $\hbar^2/2m_r$ to define the last term of equation (33) as the pseudo-potential for s-wave scattering

$$V_{scatt}^{(int)}(r) = \frac{4\pi\hbar^2}{2m_r}\frac{\tan(ka_s)}{k}\,\delta(r). \qquad (34)$$

The approximation $\frac{\tan(ka_s)}{k} \approx a_s$ is valid in the case of low energy scattering $ka_s \ll 1$, so the final formula for the pseudo-potential becomes

$$V_{scatt}^{(int)}(r) = \frac{4\pi\hbar^2 a_s}{2m_r}\,\delta(r). \qquad (35)$$

The s-wave scattering amplitude $f_{k,s}$ in the Born approximation [37] reads

$$f_{k_s}(e_r) = -\frac{2\mu}{\hbar^2}\int d^3y\, e^{-ie_r\cdot y}\, V_{scatt}^{(int)}(y)\sin k(a_s - y) \qquad (36)$$

$$= -4\pi\, a_s \sin(ka_s),$$

where $\boldsymbol{e_r} = \frac{r}{r}$ is the central unit vector of the differential solid angle $d\Omega$ into the particle is scattered.

The approach of shallow bound states (Low equation) operates with the Lippmann-Schwinger equation [38] and provides the still missing approximation of the scattering length a_s [24]

$$a_s = \hbar/\sqrt{2m_r B}. \qquad (37)$$

When, the shallow bound state B is near beneath the continuum level, then B equates with the maximal Energy that is given by the Morse potential (25)

$$B = E_{n_{max}} = \left(n_{max} + \tfrac{1}{2}\right)\hbar\omega - \left(\left(n_{max} + \tfrac{1}{2}\right)\right)^2 \frac{(\hbar\omega)^2}{4D_e} \approx D_o, \qquad (38)$$

where $D_o = E_{n_{max}} - E_0$ denotes the bond energy.

5. Oscillations of Ionotropic Receptors Generated by Particle-Particle Interactions

The grid of ionotropic receptors is embedded in the postsynaptic membrane, where again anharmonic oscillators approximate them. This means, that the interaction energy between these oscillators (receptors) is calculated by the insertion of the Morse potential. Hereby, the objective is the deduction of an equation for the expectation values of the Fourier components of the spatial receptor density. The density, for instance of ACh receptors is about 10^4 per square micrometer [5]. This corresponds a mean distance of .1 nm between two adjacent receptors. In consequence, the mean values of the Fourier components of the oscillator density perform oscillations, with different frequencies. To evaluate these frequencies, the method of Ehrenreich-Cohen is applied on the many-Boson representation, whereas these authors originally developed their method for many-electron problems [39].

In this chapter, the Heisenberg representation is well suited, where $\hat{b}_w^\dagger(t)$ is the creation operator of an anharmonic oscillator. Thus, for instance the creation field operator of a receptor, normalized in a box of volume V, becomes

$$\hat{\psi}^\dagger(x,t) = \frac{1}{\sqrt{V}}\sum_w \hat{b}_w^\dagger(t)e^{-iw.xx}. \qquad (39)$$

The Hamiltonian is bipartite: $\hat{H} = \hat{H}_0 + \hat{H}^{(int)}$

$$\hat{H} = \int \hat{\psi}^\dagger(x,t)\left(-\frac{\hbar^2}{2m}\Delta + V(x)\right)\hat{\psi}(x,t)\,d^3x + \qquad (40)$$

$$\frac{1}{2}\iint \hat{\psi}^\dagger(x,t)\,\hat{\psi}^\dagger(y,t)\,V_{Morse}^{(int)}(|x - y|)\,\hat{\psi}(y,t)\hat{\psi}(x,t)\,d^3x\,d^3y$$

where $V_{Morse}^{(int)}(|x - y|) = E_{pot}(r)$ and m is the mass.

Hence, expressed in creation and annihilation operators the Fourier transform of the Hamiltonian is

$$\hat{H} = \sum_{\boldsymbol{w}} \hbar\,\omega_{\boldsymbol{w}}(t)\,\hat{b}_{\boldsymbol{w}}^\dagger(t)\,\hat{b}_{\boldsymbol{w}}(t) + \qquad (41)$$

$$\frac{1}{2}\sum_{\boldsymbol{w}_1,\boldsymbol{w}_2,\boldsymbol{w}_3,\boldsymbol{w}_4} Int(\boldsymbol{w}_1,\boldsymbol{w}_2,\boldsymbol{w}_3,\boldsymbol{w}_4)\,\hat{b}_{\boldsymbol{w}_1}^\dagger(t)\,\hat{b}_{\boldsymbol{w}_2}^\dagger(t)\,\hat{b}_{\boldsymbol{w}_3}(t)\,\hat{b}_{\boldsymbol{w}_4}(t)$$

The integral expression *Int* reads

$$Int(w_1, w_{2,}, w_3, w_4) = \qquad (42)$$

$$\frac{D_e}{V^2}\iint e^{-iw_1\cdot x - iw_2\cdot y}\left(e^{-2a(r-r_e)} - 2e^{-a(r-r_e)}\right)e^{iw_3\cdot y + iw_4\cdot x}\,d^3x\,d^3y.$$

The outcome of the double Fourier integral (42) is

$$Int(w_1, w_{2,}, w_3, w) = \delta(w_1 + w_2 - w_3 - w_4)\,v_q\,, \qquad (43)$$

with $v_q = \frac{D_e}{2\pi^2}\left(e^{2ar_e}\frac{4a}{(q^2+4a^2)^2} - e^{ar_e}\frac{2a}{(q^2+a^2)^2}\right)$,

and $q = \frac{1}{2}(w_2 + w_4 - w_1 - w_3)$.

At first, the direct Fourier transform of e^{-2ar}, $a > 0$ is calculated to demonstrate the explicit evaluation of expression (43)

$$\mathcal{F}(e^{-2ar}) = \int_{-\infty}^{\infty} e^{-2ar}e^{-iq\cdot(x-y)}\,d^3x\,d^3y = \qquad (44)$$

$$4\pi\int_0^\infty e^{-2ar}\frac{\sin(qr)}{qr}r^2dr = 4\pi\left(\frac{4a}{(q^2 + 4a^2)^2}\right).$$

The evaluation of the inverse Fourier transform delivers the formula that is inserted into equation (42) to compute the expression $Int(w_1, w_{2,}, w_3, w_4)$

$$4\pi\,\mathcal{F}^{-1}\left(\frac{4a}{(q^2+4a^2)^2}\right) = \frac{4\pi}{V}\int_{-\infty}^{\infty}\left(\frac{4a}{(q^2+4a^2)^2}\right) e^{iqr} d^3q \quad (45)$$

$$= \frac{(4\pi)^2}{V}\int_0^{\infty}\frac{\sin(qr)}{qr}\left(\frac{4a}{(q^2+4a^2)^2}\right) q^2 dq = -\,i\,\frac{(4\pi)^2}{2r\,V}\int_{-\infty}^{\infty}\left(\frac{4a}{(q^2+4a^2)^2}\right) q\, e^{iqr} dq$$

$$= \frac{(4\pi)^2}{V}\,\frac{\pi}{2}\,e^{-2ar}.$$

The dominator $(q^2+4a^2)^2$ appearing in equation (45) has two imaginary poles of second order that lies at $q_+ = i2a$ and $q_- = -i2a$. The residue of the integrand at q_+ is *res* $(q_+) = \frac{r}{8\,a}\,e^{-2ar}$. The result (45) is obtained, with the contour of integration of a semicircle in the upper plane, which includes the pole q_+.

Subsequently, the equation of motion of the Fourier transform $q(t)$ of the spatial density $\hat{\rho}(x,t) = \hat{\psi}^\dagger(x,t)\,\hat{\psi}(x,t)$ is calculated, where the expectation value

$$\langle\phi|\hat{\rho}_q(t)|\phi\rangle = \frac{1}{V}\sum_w \hat{b}^\dagger_{w+q}(t)\,\hat{b}_w(t) \quad (46)$$

represents the mean density of the receptors in the *w*-space.

The details of the corresponding elaborate calculations are skipped, and in lieu, the equation of motion of the mean value is directly quoted [11]

$$i\hbar\frac{d}{dt}\langle\phi|\hat{b}^\dagger_{w+q}\,\hat{b}_w|\phi\rangle = \left(E_w - E_{w+q}\right)\langle\phi|b_w|\phi\rangle + \quad (47)$$

$$2v_q\left(\bar{n}_{w+q} - \bar{n}_w\right)\sum_{w'}\langle\phi|b^\dagger_{w'+q}\,\hat{b}_{w'}|\phi\rangle,$$

where $\bar{n}_{w+q} = \langle\phi|\hat{b}^\dagger_{w+q}\,\hat{b}_{w+q}|\phi\rangle = \langle\phi|\hat{N}_{w+q}|\phi\rangle = \frac{1}{e^{(E_{w+q}-\mu)/k_BT}-1}$, (48)

and

$$\bar{n}_w = \langle\phi|\hat{b}^\dagger_w\,\hat{b}_w|\phi\rangle = \langle\phi|\hat{N}_w|\phi\rangle = \frac{1}{e^{(E_w-\mu)/k_BT}-1}. \quad (49)$$

Hereby, the two formula (48) and (49) express the commitment that the receptors are members of a grand canonical ensemble, and therefore their number fluctuates. Thus, for example half of the AMPA receptors are replaced every 15 minutes [5]. The chemical potential μ regulates the varying number of particles.

To solve equation (47) this equation is reformulated

$$\langle\phi|\hat{b}^\dagger_{w+q}(t)\,\hat{b}_w(t)|\phi\rangle = \frac{2\,v_q\left(\bar{n}_{w+q}-\bar{n}_w\right)}{i\,\hbar\frac{d}{dt} + E_{w+q} - E_w}\sum_{w'}\langle\phi|\hat{b}^\dagger_{w'+q}(t)\,\hat{b}_{w'}(t)|\phi\rangle. \quad (50)$$

The summation over *w*, on both sides delivers the equation

$$\langle\phi|\hat{\rho}_q(t)|\phi\rangle = 2\,v_q\left(\sum_w\frac{\bar{n}_{w+q}-\bar{n}_w}{i\,\hbar\frac{d}{dt} + E_{w+q} - E_w}\right)\langle\phi|\hat{\rho}_q(t)|\phi\rangle. \quad (51)$$

The exponential oscillatory ansatz for the solution of equation (50) is

$$\left\langle\phi\left|\hat{b}^\dagger_{w+q}(t)\,\hat{b}_w(t)\right|\phi\right\rangle = \left\langle\phi\left|\hat{b}^\dagger_{w+q}(0)\,\hat{b}_w(0)\right|\phi\right\rangle e^{i(\omega_q t-\gamma t)}, \quad (52)$$

where γ denotes a damping factor. Hence, equation (51) becomes the relation

$$1 = 2\,v_q\sum_w\frac{\bar{n}_{w+q}-\bar{n}_w}{-h\omega_q - ih\gamma + E_{w+q} - E_w} = f(\omega_q, \omega_{qw_n}), \quad (53)$$

where two different frequencies occur

$$\omega_q \text{ and } \omega_{qw_n} = \frac{1}{\hbar}\left(E_{w_n+q} - E_{w_n}\right). \quad (54)$$

To get the graphical (numerical) solution of (53), the function $f\left(\omega_q, \omega_{qw_n}\right)$ should be plotted as an ordinate and the frequencies ω_q respectively ω_{qw_n} as an abscissa. The projections of the intersections of $f\left(\omega_q, \omega_{qw_n}\right)$, with the 1-line on the abscissa deliver especially the sequence of frequencies $\omega_{qw_1}, \omega_{qw_2}, \ldots$. They remind to the sequence of energetic transitions that is observable for anharmonic oscillators (27).

The oscillations of such a “paracrystalline” receptor grid (similar to the vesicular grid) probably generate phonons, which vice versa interact with their generating receptors. The deflection operator of phonons is $\hat{q}_w(t) \propto \hat{p}_w(t) + \hat{p}^\dagger_{-w}(t)$, where $\hat{p}_w(t)$ and $\hat{p}^\dagger_{-w}(t)$ represent phonons. Only, when the number of phonons is constant, then the expectation value of the phonon deflection vanishes $\langle\hat{q}_w(t)\rangle = 0$. In a state of a grand canonical ensemble, where the number of phonons fluctuates, the mean value does not disappears $\langle\hat{q}_w(t)\rangle \neq 0$. Hence, it cannot be excluded that phonons interact with receptors as in rigid grids of solids [10], whereby they might destroy the unprotected entanglement.

6. Quantum Information, Entanglement and Decoherence

Qubits describe the quantum information of receptors. Originally, they get popular as the working memory of the quantum computer, e.g. [40]. Nowadays, this concept also entered into the quantum biology.

The tensor product of creation operators of one particle constructs the spanning vectors of the finite Hilbert space $\mathcal{H}_N^{(+)}$. Each operator creates one particle on the *n* different energy levels

$$\left|n_{k_1}=1, n_{k_2}=1, \ldots, n_{k_n}=1\right\rangle = \hat{a}^\dagger_{k_1}\,\hat{a}^\dagger_{k_2}\ldots\hat{a}^\dagger_{k_n}\,|0,0,\ldots,0\rangle, \quad (55)$$

where this basis state represents a pure state.

The two dimensional $\mathcal{H}_1^{(+)}$ space represents a 2-states system, with the basis vectors $|\,n_{k_1}\rangle$ and $|\,n_{k_2}\rangle$. The superposition of two basis vectors constructs a pure state in this one-particle space ($\hat{\rho}^2 = \hat{\rho}$, Tr $\hat{\rho}^2 = $ Tr $\hat{\rho}$: see subchapter 8.1).

$$|\,\psi\rangle = c_1|\,n_{k_1}\rangle + c_2|\,n_{k_2}\rangle = c_1\,|\,0\rangle + c_2|\,1\rangle, \quad (56)$$

where $c_1 = r_1 e^{i\varphi_1}$ and $c_2 = r_2 e^{i\varphi_2}$ are normalized complex numbers

$$\lfloor c_1 \rfloor^2 + \lfloor c_2 \rfloor^2 = 1. \quad (57)$$

A system with only two states is often denoted by $|\,0\rangle$ and $|\,1\rangle$. The set of all states of the form (56) subjected to the normalization condition (57) constitutes a *1 − qubit* that defines the concept of quantum information. For example, an ionotropic oscillator is formally concretized in the context of a *1−qubit*. The two particular states *open* (occupied) or *closed* (unoccupied) of a receptor are assigned to two different energy levels of the anharmonic oscillator as they are calculated by the Morse potential (25). The higher energy level E_{n+1} corresponds to the state *closed* and the lower level E_n is assigned to the state *open*. The superposition of the two corresponding energetic basis vectors (two states system) defines a state that corresponds to a *1−qubit*, which is equivalent to that one defined by equation (56). When E_n and E_{n+1} are determined, then $\dim \mathcal{H}_1^{(+)} = 2$, where the anharmonic oscillator can occupy two different energy levels. When all different energy levels of an anharmonic oscillator are allowed, and then $\dim \mathcal{H}_1^{(+)} = \infty$.

The direct product of *N 1 − qubits* composes an untangled *N − qubits*. For example, the state of an untangled *2−qubit* is

$$|\,\psi_{unt}\rangle = |\,\psi_A\,\rangle \otimes |\,\psi_B\,\rangle = (c_0|\,0\rangle_A + c_1|\,1\rangle_A) \otimes (d_0|\,0\rangle_B + d_1|\,1\rangle_B), \quad (58)$$

where the coefficients c_{ij} factorize $c_{ij} = c_i\, d_j$. The two following equations separately comply with the normalization request

$$\sum_{i,=0}^{1} \lceil c_i \rceil^2 = \sum_{j=0}^{1} \lceil d_j \rceil^2 = 1. \quad (59)$$

The expectation value of an operator $\hat{A}$, for instance in the state $|\,\psi_A\,\rangle$, (58) reads

$$\langle \psi_A|\hat{A}|\psi_A\rangle = \lfloor c_0 \rfloor^2 {}_A\langle 0|\hat{A}|0\rangle_A + \lfloor c_1 \rfloor^2 {}_B\langle 1|\hat{A}|1\rangle_B + c_0^* c_1\; {}_A\langle 0|\hat{A}|1\rangle_B + c_1^* c_0\; {}_B\langle 1|\hat{A}|0\rangle_A. \quad (60)$$

This formula contains two "non-diagonal" factors $c_0^* c_1$ and $c_1^*\, c_0$. These two terms are essential for the coherent superposition. However, due to interactions with the local environment the expectation values of these two terms vanish, since their relative phases can take all possible values (noise). The coherence gets lost and decoherence arises, since the interference terms disappear.

The Hilbert space of the entangled state is $\widetilde{\mathcal{H}}_{2AB}^{(+)} = \mathcal{H}_{1A}^{(+)} \otimes \mathcal{H}_{1B}^{(+)}$. The tensor product of two non-interacting systems constructs the space of the composite system. The basis vectors of $\mathcal{H}_{1A}^{(+)}$ are $|\,0\rangle_A$ and $|\,1\rangle_A$; the basis states of $\mathcal{H}_{1B}^{(+)}$ are $|\,0\rangle_B$ and $|\,1\rangle_B$. The state vector $|\,\psi_{ent}\rangle \in \widetilde{\mathcal{H}}_{2AB}^{(+)}$ represents an entangled *2−qubit,* which is composed by two *1 − qubits* that describe distinguishable particles (different creation operators in systems *A* and *B*).

$$|\,\psi_{ent}\,\rangle = c_{00}|\,0\rangle_A\,|\,0\rangle_B + c_{01}|\,0\rangle_A\,|\,1\rangle_B + c_{10}|\,1\rangle_A\,|\,0\rangle_B + c_{11}|\,1\rangle_A\,|\,1\rangle_B, \quad (61)$$

$$= (c_{00}\,\hat{a}_0^\dagger\,\hat{b}_0^\dagger + c_{01}\,\hat{a}_0^\dagger\,b_1^\dagger + c_{10}\,\hat{a}_1^\dagger\,\hat{b}_0^\dagger + c_{11}\,\hat{a}_1^\dagger\,\hat{b}_1^\dagger)|0,0\rangle.$$

The coefficients c_{ij} are complex, do not factorize and are subjected to the normalization condition

$$\sum_{i,j=0}^{1} \lceil c_{ij} \rceil^2 = 1. \quad (62)$$

To explain the effect of entanglement more detailed, the special case of the state (61) is well suited: $c_{00} = c_{01} = 1/\sqrt{2}$; $c_{10} = c_{11} = 0$. It is one of the four possible Bell states for Bosons that represents an entangled pure state for two particles, however not for one particle [41-42].

The four *k*-based entangled *2 − qubits* Bell states for indistinguishable Bosons are

$$|\,\Phi_{ent}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle_A\,|\,0\rangle_B \pm |\,1\rangle_A\,|\,1\rangle_B) = \frac{1}{\sqrt{2}}(\hat{a}_{k_0}^\dagger\,\hat{a}_{-k_0}^\dagger \pm \hat{a}_{k_1}^\dagger\,\hat{a}_{-k_1}^\dagger)|0,0\rangle, \quad (63)$$

$$|\,\psi_{ent}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle_A\,|\,1\rangle_B \pm |\,1\rangle_A\,|\,0\rangle_B) = \frac{1}{\sqrt{2}}(\hat{a}_{k_0}^\dagger\,\hat{a}_{-k_1}^\dagger \pm \hat{a}_{k_1}^\dagger\,\hat{a}_{-k_0}^\dagger)|0,0\rangle. \quad (64)$$

These four states construct the basis of the entangled *2−qubits* Hilbert space of Bosons $\widetilde{\mathcal{H}}_{2AB}^{(+)}$.

For instance, the state $|\,\Phi_{ent}^{(+)}\rangle$ is well qualified to describe the effect of entanglement in some details [43-44]. There exist two contingences to "measure" this state. When the first access takes place at system *A,* then this influence impacts that the *1−qubit* of this system gets, for instance to the state $|\,0\rangle_A = \hat{a}_{k_0}^\dagger|0\rangle$. Every subsequent access to the system *B* causes a *1−qubit,* which is equivalent to the state $|\,0\rangle_B = \hat{a}_{-k_0}^\dagger|0\rangle$. The whole system *AB* "collapses" instantaneously, and independently of the distance between both systems, to the product state $|\,0\rangle_A\,|\,0\rangle_B = \hat{a}_{k_0}^\dagger\hat{a}_{-k_0}^\dagger|0,0\rangle$. Any subsequent access to system *B* steadily transfers it to the state $|\,0\rangle_B$.

7. Density Operators, Entangled Entropy and Decoherence

Entangled and mixed states do not correspond to pure states, whereas density operators describe such composite systems. Reduced (partial) density operators extract from the total density operator the particular parts, e.g. of system *A*. These particular operators are very useful to quantify the entanglement entropy [24].

7.1. Density Operators and Entangled Entropy of the Bell States

Here, the corresponding calculation concentrates on the particular density operator of the Bell state $|\,\Phi_{ent}^{(+)}\rangle$, defined in (63), since each respective calculation of the remaining three Bell-states represents an ordinary repetition of the particular computation, and therefore, it is redundant.

The density operator of the Bell state $|\,\Phi_{ent}^{(+)}\rangle$, with normalized basis states reads

$$\hat{\rho}_{AB} = |\,\Phi_{ent}^{(+)}\rangle\langle\Phi_{ent}^{(+)}| = \frac{1}{2}\big(|\,0\rangle_A\,|\,0\rangle_B\langle 0\,|_B\langle 0\,|_A + |\,0\rangle_A\,|\,0\rangle_B\langle 1\,|_B\langle 1\,|_A + |\,1\rangle_A\,|\,1\rangle_B\langle 0\,|_B\langle 0\,|_A + |\,1\rangle_A\,|\,1\rangle_B\langle 1\,|_B\langle 1|_A\big). \tag{65}$$

The trace Tr of this density operator is

$$\mathrm{Tr}\,\hat{\rho}_{AB} = \mathrm{Tr}\left(|\,\Phi_{ent}^{(+)}\rangle\langle\Phi_{ent}^{(+)}|\right) = \langle\Phi_{ent}^{(+)}|\Phi_{ent}^{(+)}\rangle = \tag{66}$$

$$\frac{1}{2}\left({}_A\langle 0|0\rangle_A\;{}_B\langle 0|0\rangle_B + {}_A\langle 1|0\rangle_A\;{}_B\langle 1|0\rangle_B + {}_A\langle 0|1\rangle_A\;{}_B\langle 0|1\rangle_B + {}_A\langle 1|1\rangle_A\;{}_B\langle 1|1\rangle_B\right) = 1.$$

The reduced density operator $\hat{\rho}_A$ reads

$$\hat{\rho}_A = \mathrm{Tr}_B\,\hat{\rho}_{AB} = \frac{1}{2}\big(|\,0\rangle_A\langle 0\,|_A\;{}_B\langle 0|0\rangle_B + {}_B\langle 1|1\rangle_B|\,1\rangle_A\langle 1\,|_A\big) = \frac{1}{2}\big(|\,0\rangle_A\langle 0\,|_A + |\,1\rangle_A\langle 1\,|_A\big), \tag{67}$$

where Tr_B denotes the trace with respect to the basis states of the system B. Formula (67) demonstrates that the reduced density operator of an entangled state represents a mixed system, since $\hat{\rho}_A^2 = 1/2\,\hat{\rho}_A \neq \hat{\rho}_A$.

The entanglement entropy of $\hat{\rho}_A$ becomes

$$S_A = -\mathrm{Tr}\,(\hat{\rho}_A \ln \hat{\rho}_A) = -\frac{1}{2}\left({}_A\langle 0|0\rangle_A \ln\frac{1}{2}\,{}_A\langle 0|0\rangle_A + {}_A\langle 1|1\rangle_A \ln\frac{1}{2}\,{}_A\langle 1|1\rangle_A\right) \tag{68}$$

$$= -\frac{1}{2}\left(\ln\frac{1}{2} + \ln\frac{1}{2}\right) = \ln 2.$$

This outcome demonstrates that the state $|\,\Phi_{ent}^{(+)}\rangle$ is maximally entangled due to the uniform probability distribution. More generally, bipartite states of a composed system are maximally entangled, when their entanglement entropy is maximal.

7.2. Density Operators of Canonical and Grand Canonical Ensembles

The Hamiltonian $\hat{H} = \sum_k \hbar\,\omega_k\,\hat{a}_k^\dagger\,\hat{a}_k = \sum_k E_k\,\hat{N}_k$ substantially describes the partition function Z_{can} of a canonical ensemble of N Bosons

$$Z_{can} = \mathrm{Tr}\,\langle e^{-\beta\hat{H}}\rangle = \sum_{n_1,n_2\ldots,n_d=0}^{comb} \langle n_d,\ldots n_2,n_1|\;e^{-\beta\hat{H}}\,|n_1,n_2,\ldots,n_d\rangle = \sum_{n_1,n_2\ldots=0}^{comb} \langle n_d,\ldots n_2,n_1|\;e^{-\beta\sum_k E_k n_k}\,|n_1,n_2,\ldots,n_d\rangle = \sum_{n_1,n_2\ldots=0}^{comb} \prod_k e^{-\beta E_k n_k} = \prod_k \sum_{n_k=0}^{comb} e^{-\beta E_k n_k}. \tag{69}$$

The parameter β represents the well-known formula $\beta = 1/k_B\,\mathrm{T}$, where the superscript *comb* indicates that the summation is performed over all combinations of the particle numbers n_k, which accomplish the condition $\sum_k n_k = N$, where N is finite.

There exist one exception for canonical ensembles, where $N = \infty$. These are canonical ensembles of harmonic oscillators and the anharmonic oscillators (both are Bosons). For the harmonic oscillator, with the frequency ω_0, the partition function becomes

$$Z_{can}^{(har)} = Tr\big(e^{-\beta\hat{H}}\big) = \sum_{n=0}^{\infty} e^{-\beta E_n} = e^{-\frac{\beta\hbar\omega_0}{2}} \sum_{n=0}^{\infty} e^{-\beta\hbar\omega_0 n} = \frac{e^{-\frac{\beta\hbar\omega_0}{2}}}{1-e^{-\beta\hbar\omega_0}}. \tag{70}$$

The partition function of the anharmonic oscillator, with the frequency ω (26) reads

$$Z_{can}^{(anh)} = \sum_{n=0}^{\infty} e^{-\beta E_n} = e^{-\beta\left(-D_e+\hbar\omega\,\frac{(8D_e-\hbar\omega)}{16D_e}\right)} \sum_{n=0}^{\infty} e^{-\beta\frac{\hbar\omega}{4D_e}(4D_e-\hbar\omega)\,n}\; e^{\beta\frac{(\hbar\omega)^2}{4D_e}n^2}, \tag{71}$$

where the formula (25) for the energy E_n is applied, and the following condition is expected $n_{max} = \infty$.

The partition function of a grand canonical ensemble of Bosons in a non-coherent representation reads

$$Z_{gc}^{(nc)} = \mathrm{Tr}\,\langle e^{-\beta(\hat{H}-\mu\,\hat{N})}\rangle = \sum_{n_1,n_2\ldots=0}^{\infty} \langle\ldots n_2,n_1|\;e^{-\beta(\hat{H}-\mu\,\hat{N})}\,|n_1,n_2,\ldots\rangle = \tag{72}$$

$$\sum_{n_1,n_2\ldots=0}^{\infty} \langle\ldots n_2,n_1|\,e^{-\beta\sum_k (E_k-\mu)n_k}\,|n_1,n_2,\ldots\rangle = \prod_k \sum_{n_k=0}^{\infty} e^{-\beta(E_k-\mu)n_k} = \prod_k \frac{1}{\left(1-e^{-\beta(E_k-\mu)}\right)},$$

where μ is the chemical potential. Further, the constraint $(E_k - \mu) > 0$ must be granted, so that $e^{-\beta(E_k-\mu)} < 1$.

The density operator of a canonical ensemble is

$$\hat{\rho}_{can}^{(nc)} = \frac{1}{Z_{can}}\,e^{-\beta\sum_k^{comb} E_k\,n_k}\,|n_1,n_2,\ldots,n_d\rangle\langle n_d,\ldots n_2,n_1| = \frac{1}{Z_{can}}\,e^{-\beta\hat{H}}, \tag{73}$$

where the completeness relation refers to the *N*-particles representation of the Fock space.

The density operator of non-coherent many-particle states of a grand canonical ensemble is due to the identity relation (6), given by

$$\hat{\rho}_{gc}^{(nc)} = \frac{1}{Z_{gc}^{(nc)}} e^{-\beta(\hat{H}-\mu\hat{N})} |n_1, n_2, \ldots\rangle\langle \ldots n_2, n_1| = \frac{1}{Z_{gc}^{(nc)}} e^{-\beta(\hat{H}-\mu\hat{N})}. \quad (74)$$

The imaginary time $t = -i\tau$, $\tau \ni [-\hbar\beta, \hbar\beta]$ was introduced to define the thermodynamic Green´s functions (propagators), for instance for a grand canonical ensemble [19]. The method of path integration [45] extensively uses such propagators to describe, for instance particle scatterings by Feynman diagrams [22]. However, such details divert this contribution from its main goals; thus, this topic is not of further interest.

7.3. Decoherence and Entanglement

To characterize the effect of decoherence [46] more concisely, the respective considerations start with an uncorrelated, composed system, for example in the space $\mathcal{H}_{(N+M)SE}^{(+)} = \mathcal{H}_{NS}^{(+)} \otimes \mathcal{H}_{ME}^{(+)}$. Here, *S* represents the system; *E* characterizes the environment, whereas *N* and *M* respectively define the number of the participating particles. Further, all molecules of the system S and of the environment *E* are Bosons, and are elements of the composed Fock space. Consequently, the impinging of the environmental particles of *E* on *S* (typically scattering, [7]) cause the perturbations of the system S. This influences afterwards effect the rapidly change of the phases of the states of the system *S*.

When the system *S* does not interact with the environment *E*, then the composed system *SE* evolves unitarily. The superposed state vector of the bipartite system at time *t* is

$$|\,\psi_{coh}(t)\,\rangle_{SE} = \hat{U}(t)\,|\,\psi_{coh}(0)\,\rangle_{SE} = \sum_j c_j\ |\,\psi_j(t)\,\rangle_S\,|\,\phi_j(t)\rangle_E\,, \quad (75)$$

where the set of the states $|\,\psi_j(t)\,\rangle_S$ denotes a complete orthonormal basis. These states can be eigenfunctions of a Hermitian operator $\hat{O}_S$, but not necessarily. The environmental states are $|\,\phi_j(t)\rangle_E$.

When the environmental interactions are turned on, then the state vector (75) changes to a decoherent state vector

$$|\,\Psi_{dec}\,(t)\rangle_{SE} = \sum_j e^{i\alpha_j}\,c_j\ |\,\psi_j(t)\,\rangle_S\,|\,\phi_j(t)\rangle_E, \quad (76)$$

where the α_j denote additional ($c_j = r_j e^{i\varphi_j}$) randomly fluctuating phases. In consequence, the expectation value of the operator $\hat{O}_S$ becomes

$$\langle \hat{O}_S\,\rangle = \sum_j \lfloor c_j\rfloor^2 \langle\, \langle \phi_j(t)|_E \langle \psi_j(t)|_S\, \hat{O}_S\ |\,\psi_j(t)\,\rangle_S\,|\,\phi_j(t)\rangle_E\rangle, \quad (77)$$

since the expectation values of the different interference terms of the superposition (76) average to zero, due to the vanishing averages of the phases $\alpha_j + \varphi_j$.

The probability to observe the particular state $|\,\psi_j(t)\,\rangle_S\,|\,\phi_j(t)\rangle_E$ is $\lfloor c_j\rfloor^2$ as the two dominant initiators Bohr [47] and Born [37] of the Copenhagen interpretation proposed. The set of all states $\{\,|\,\psi_i(t)\,\rangle_S\,|\,\phi_j(t)\rangle_E\}$ " collapses" (reduces) to the one observed state. Usually, this result is described in the context of a measurement, where a corresponding apparatus, which is often called pointer [48-49], replaces the environment. Thus, any influence of a measuring device acts as an environmental perturbation.

A refined insight into the process of quantum decoherence is obtained, when this phenomenon is qualified by a decoherent density operator, which becomes

$$\hat{\rho}_{dec}(t) = |\,\Psi_{dec}\,(t)\rangle_{SE}\langle\Psi_{dec}\,(t)|_{SE} \quad (78)$$

$$= \sum_{i,j}\, c_i\, c_j^*\,|\,\psi_i\,(t)\rangle_S\,|\,\phi_i(t)\rangle_E \otimes \langle\phi_j(t)|_E\,\langle\psi_j(t)|_S\,,$$

with redefined $c_i = r_i e^{i(\alpha_i+\varphi_i)}$ and $c_j = r_j e^{i(\alpha_j+\varphi_j)}$.

To get the reduced density operator the states of the environment are traced out

$$\hat{\rho}_S(t) = \mathrm{Tr}_E\big(\,\hat{\rho}_{dec}(t)\big) = \sum_{i,j}\, c_i\, c_j^*\,|\,\psi_i\,(t)\rangle_S\,\langle\psi_j(t)|_S\,\langle\phi_i(t)|\phi_j(t)\rangle_E \quad (79)$$

$$\approx \sum_{i,j} c_i\, c_j^*\,|\,\psi_i\,(t)\rangle_S\,\langle\psi_j(t)|_S\,\delta_{ij} = \sum_i \lfloor c_i\rfloor^2\,|\,\psi_i\,(t)\rangle_S\,\langle\psi_i(t)|_S\,.$$

To perform this calculation, the orthogonality condition of decoherence $\langle\,\phi_i(t)|\phi_j(t)\rangle_E \to \delta_{ij}$, for $t \to \infty$ is applied [50]. This condition is crucial for the elimination of the off-diagonal elements.

Even, if the density operator is diagonal, then only one particular state $|\,\psi_i\,(t)\rangle_S\,\langle\psi_i(t)|_S$ is observed. What happens with the remaining diagonal terms? One answer provides the many-worlds interpretation [51], where all other possible states continue to exist in the world and split into different paths (branches). However, this interpretation will not be further deepened, because the connection of entanglement and decoherence lies in the focus of this subchapter.

It is obvious that the same decoherence effects are observable for entangled states, where the Bell state $|\,\Phi_{ent}^{(+)}\rangle$, (63) may serve as a specimen. When the environmental influence decoheres this state

$$|\,\Phi_{dec}^{(+)}\rangle = \frac{1}{\sqrt{2}}\left(e^{i\alpha_1}|\,0\rangle_A\,|\,0\rangle_B + e^{i\alpha_2}|\,1\rangle_A\,|\,1\rangle_B\right), \quad (80)$$

then the decohered density operator becomes

$$\hat{\rho}_{dec}(t) = |\,\Phi_{dec}^{(+)}\rangle\langle\Phi_{dec}^{(+)}| = \quad (81)$$

$$\frac{1}{2}|0\rangle_A|0\rangle_B \otimes \langle 0|_B\langle 0|_A + \frac{1}{2}e^{i(\alpha_1-\alpha_2)}|0\rangle_A|0\rangle_B \otimes \langle 1|_B\langle 0|_A +$$

$$\frac{1}{2}e^{i(\alpha_2-\alpha_1)}|1\rangle_A|1\rangle_B \otimes \langle 0|_B\langle 0|_A + \frac{1}{2}|1\rangle_A|1\rangle_B \otimes \langle 1|_B\langle 1|_A =$$

$$\frac{1}{2}|0\rangle_A|0\rangle_B \otimes \langle 0|_B\langle 0|_A + \frac{1}{2}|1\rangle_A|1\rangle_B \otimes \langle 1|_B\langle 1|_A,$$

where the interfering terms converge to zero.

The robustness (persistence) of entanglement under decoherence is increased, when another, maximally entangled state is introduced. This is, for example the GHZ state (Greenberger-Horne-Zeilinger) that represents an $M-qubit$, with $M > 2$), [52]

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes M} + |1\rangle^{\otimes M}). \quad (82)$$

This state reduces to the Bell state $|\Phi_{ent}^{(+)}\rangle$ for $M = 2$.

The direct consequence of the above cited example of disentanglement is clear; entanglement is very fragile under decoherence (destroying effects). The only possibility to stabilize the entanglement of a system is to shield it. The question that instantly arises is; how an entangled system can be shielded.

This means that coupled ionotropic receptors (anharmonic oscillators) should be in the state of stable quantum coherence that is caused by protected entangled receptors. Moreover, all entangled molecules should be in the same low energy state (no phase decoherence, but phase coherence) as e.g. in the case of superfluidity of ^{4}He atoms (Bosons). The ideal case of the phase coherence of entangled receptors is achieved, when all receptors are in the same basis state, and gap functions protect this entangled state.

8. Gap Functions and Protected Entanglement

The interactions of entangled anharmonic oscillators are investigated in correspondence to the superconductivity [11, 14], where the Morse potential is again utilized. Hereby, the set of interacting, entangled oscillators is regarded as a hot (body temperature) grand canonical ensemble of Bosons, where the energy of the interacting entangled oscillators is lower than the energy of free, entangled oscillators. This energy gap shelters the interacting entangled oscillators against the influences that come from the free, entangled oscillators.

8.1. Gap Functions

Energy gaps between the free and interacting states ensure that the interacting entangled oscillators are a in unique ground states that are protected as the Cooper-pairs. Thus, the coupled, entangled anharmonic oscillators create a quantum coherence as in the case of superconductivity.

The total Hamiltonian $\hat{H}$ is the initial point to describe the coherence of the basic states of the entangled anharmonic oscillators

$$\hat{H} = \int d^3x\ \hat{\psi}^{\dagger}(\boldsymbol{x})\left(-\frac{\hbar^2}{2m}\nabla^2 - \mu\right)\hat{\psi}(\mathbf{x}) + \quad (83)$$

$$\frac{1}{2}\iint d^3x\ d^3x'\ \hat{\psi}^{\dagger}(\boldsymbol{x})\ \hat{\psi}^{\dagger}(\boldsymbol{x}')\ V_{Morse}(|\boldsymbol{x}-\boldsymbol{x}'|)\ \hat{\psi}(\boldsymbol{x}')\ \hat{\psi}(\boldsymbol{x}).$$

The one-particle potential $V(x)$ appearing in equation (40) is replaced by the chemical potential μ that regulates the equilibrium of a system (e.g. grid of receptors), when the particle number *N* changes for instance in case of the depletion of some receptors or due to the variation of the temperature *T*. In consequence, the set of anharmonic oscillators (ionotropic receptors) is modelled as a grand canonical system, where the particle number fluctuates.

Formally, μ specifies a Lagrange multiplicator. One has $\mu < E_{w_0}$, where E_{w_0} denotes the lowest, negative bound state energy. Since, the mean number of Bosons with energy E_w

$$\langle \hat{N}_w \rangle = n_w = \frac{1}{e^{\beta(E_w-\mu)}-1} \quad (84)$$

is positive and not divergent, therefore the completion of the condition $(E_w - \mu) > 0$ is again required. The chemical potential controls a "hot" bosonic, grand canonical ensemble at the brain temperature of about 310.15 K (37℃). The parameter $\beta = 1/_{k_B T}$ gets the value $\beta = 5.15\ [\frac{1}{eV}]$ at the brain temperature, where $k_B = 6.25\ 10^{-5}\ [eV]$.

When, the Fourier transform is applied on equation (83), then the Hamiltonian becomes

$$\hat{H} = \sum_w E_w'\ \hat{b}_w^{\dagger}\ \hat{b}_w + \frac{1}{2}\sum_{w,w',q} v_q\ \hat{b}_{w+q}^{\dagger}\ \hat{b}_{w'-q}^{\dagger}\ \hat{b}_{w'}\hat{b}_w, \quad (85)$$

where the energy of the non interacting particles is represented by

$$E_w' = \frac{\hbar^2 w^2}{2m} - \mu \quad (86)$$

and v_q denotes the Fourier transform of the Morse potential (24)

$$v_q = \int V_{Morse}(|\boldsymbol{x}-\boldsymbol{x}'|)e^{-\boldsymbol{q}\cdot(x-x')}\ d^3x\ d^3x' = \quad (87)$$

$$\frac{D_e}{2\pi^2}\left(e^{2ar_e}\frac{4a}{(q^2+4a^2)^2} - e^{ar_e}\frac{2a}{(q^2+a^2)^2}\right) < 0.$$

Subsequently, it will demonstrated that there exist a shielding effect of the entangled ground states by so-called gap functions, which depend from the sign of the interaction energy. This potential is attractive, thus the interaction term is reformulated

$$-\frac{1}{2}\sum_{w,w',q}\ |v_q|\ \hat{b}_{w+q}^{\dagger}\ \hat{b}_{w'-q}^{\dagger}\ \hat{b}_{w'}\hat{b}_w \quad (88)$$

to mark the negative sign for it.

The generalization of the Bell states $|\Phi_{ent}^{(\pm)}\rangle$, (63) defines the ground states $|\phi^{(\pm)}\rangle$ of $\hat{H}$ (85) by factorizing the exponential expansion of $|\phi^{(\pm)}\rangle$ and truncating the power expansion after the second term

$$|\phi^{(\pm)}\rangle = \prod_{w_0,w_1}\left(\tfrac{1}{\sqrt{2}}c_{w_0} + \left(\tfrac{1}{\sqrt{2}}c_{w_0}\,\hat{b}^{\dagger}_{v_0}\,\hat{b}^{\dagger}_{-w_0} \pm c_{w_1}\hat{b}^{\dagger}_{w_1}\,\hat{b}^{\dagger}_{-w_1}\right)\right)|\phi_0\rangle \tag{89}$$

$$= \prod_{w_0,w_1}|\phi^{(\pm)}_{w_0,w_1}\rangle.$$

Since, the operators for different w_i-values in this formula commutate, the exponential function can be split in a product of exponential functions. Hereby, the two particular ground states are introduced

$$|\phi^{(\pm)}_{w_0,w_1}\rangle = \left(\tfrac{1}{\sqrt{2}}\,c_{w_0} + \left(\tfrac{1}{\sqrt{2}}c_{w_0}\,\hat{b}^{\dagger}_{w_0}\,\hat{b}^{\dagger}_{-w_0} \pm c_{w_1}\hat{b}^{\dagger}_{w_1}\,\hat{b}^{\dagger}_{-w_1}\right)\right)|\phi^{(\pm)}_{0,w_0,w_1}\rangle, \tag{90}$$

where both coefficients c_{q_0} and c_{q_1} are real and $|\phi^{(\pm)}_{0,q_0,q_1}\rangle$ specifies the corresponding vacuum states. The form of the individual ground states (90) is justified by two arguments. First, indistinguishable particles with entangled momenta are considered. Second, the pair states $(\hat{b}^{\dagger}_{w_0}\,\hat{b}^{\dagger}_{-w_0})$ and $(\hat{b}^{\dagger}_{w_1}\,\hat{b}^{\dagger}_{-w_1})$ show a formal (not physical) conformity with Cooper-pairs [13], which represent Bosons. Hence, similar methods as they are used for the evaluation of the superconductivity are applied to calculate the total energies and the gap functions for Bosons. Further, there exist the expectation that in a living system (brain) the effect of entanglement should generates features of a coherent "condensate" that shields the entanglement.

The specific states subject to the normalization condition are

$$\langle\phi^{(\pm)}_{w_0,w_1}|\phi^{(\pm)}_{w_0,w_1}\rangle = c^2_{w_0} + c^2_{w_1} = 1, \tag{91}$$

where, these ground states are not orthogonal

$$\langle\phi^{(-)}_{w_0,w_1}|\phi^{(+)}_{w_0,w_1}\rangle = c^2_{w_0} - c^2_{w_1} \neq 0. \tag{92}$$

In the next step, the expectation value of the total energy is calculated, whereas, at first the mean value of the kinetic energy is pointed out

$$\langle\phi^{(\pm)}|E_{kin}|\phi^{(\pm)}\rangle = \langle\phi^{(\pm)}|\textstyle\sum_w E'_w\,\hat{b}^{\dagger}_w\,\hat{b}_w\,|\phi^{(\pm)}\rangle = \sum_{w_0} E'_{w_0}\frac{c^2_{w_0}}{2} \pm \sum_{w_1} E'_{w_1}c^2_{w_1}. \tag{93}$$

Before the expectation value of the interaction energy is evaluated, the sequence of the operators is reordered at the second term $\hat{b}^{\dagger}_{w+q}\,\hat{b}^{\dagger}_{w'-q}\,\hat{b}_{w'}\hat{b}_w$ of equation (85). In consequence, this product of operators is replaced by

$$\hat{b}^{\dagger}_{w'}\,\hat{b}^{\dagger}_{-w'}\,\hat{b}_w\hat{b}_{-w}\,. \tag{94}$$

This order of operators is achieved, when the following interchanges in the original sequence of operators (85) is conducted: $w' \to -w$ and $q = w' - w$. These interchanges refer to the whole sum of equation (88); therefore, the value of the sum stays unchanged.

The Hartree approximation $q = 0$, [53]

$$\langle\phi^{(\pm)}|\hat{b}^{\dagger}_w\,\hat{b}_w\,|\phi^{(\pm)}\rangle\langle\phi^{(\pm)}|\hat{b}^{\dagger}_{w'}\,\hat{b}_{v'}\,|\phi^{(\pm)}\rangle =$$

$$\langle\phi^{(\pm)}|\hat{N}_w|\phi^{(\pm)}\rangle\langle\phi^{(\pm)}|\hat{N}_{w'}|\phi^{(\pm)}\rangle \neq 0 \tag{95}$$

is avoided, because these regular terms do not deliver contributions that comprise the pair states mentioned above.

Thus, the following expectation value of the interaction energy are evaluated

$$\langle\phi^{(\pm)}|E_{int}|\phi^{(\pm)}\rangle = -\tfrac{1}{2}\langle\phi^{(\pm)}|\textstyle\sum_{w,w'}\,|v_{w'-w}|\,\hat{b}^{\dagger}_{w'}\,\hat{b}^{\dagger}_{-w'}\,\hat{b}_w\,\hat{b}_{-w}\,|\phi^{(\pm)}\rangle. \tag{96}$$

The evaluation of the following operator's expression is partitioned

$$\langle\phi^{(\pm)}|\hat{b}^{\dagger}_{w'}\,\hat{b}^{\dagger}_{-w'}\,\hat{b}_w\,\hat{b}_{-w}|\phi^{(\pm)}\rangle = \langle\phi^{(\pm)}_{w_0,w_1}\phi^{(\pm)}_{w_0',w_1'}|\hat{b}^{\dagger}_{w'}\,\hat{b}^{\dagger}_{-w'}\,\hat{b}_w\,\hat{b}_{-w}|\phi^{(\pm)}_{w_0,w_1}\phi^{(\pm)}_{w_0',w_1'}\rangle \tag{97}$$

into two parts. The first part reads

$$\hat{b}_w\,\hat{b}_{-w}|\phi^{(\pm)}_{w_0,w_1}\phi^{(\pm)}_{w_0',w_1'}\rangle = \left(\frac{1}{\sqrt{2}}c_{w_0} \pm 2\,c_{w_1}\right)|\phi^{(\pm)}_{w_0',w_1'}\rangle$$

The adjoint counterpart becomes

$$\langle\phi^{(\pm)}_{w_0,w_1}\phi^{(\pm)}_{w_0',w_1'}|\,\hat{b}^{\dagger}_{w'}\,\hat{b}^{\dagger}_{-w'} = \langle\phi^{(\pm)}_{w_0,w_1}|\left(\tfrac{1}{\sqrt{2}}c_{w_0'} \pm 2\,c_{w_1'}\right). \tag{98}$$

Both parts are composed to achieve the two final forms of equation (97)

$$\left(\tfrac{1}{\sqrt{2}}c_{w_0} \pm 2\,c_{w_1}\right)\left(\tfrac{1}{\sqrt{2}}c_{w_0'} \pm 2\,c_{w_1'}\right)\langle\phi^{(\pm)}_{w_0,w_1}|\phi^{(\pm)}_{w_0',w_1'}\rangle = \tag{99}$$

$$\tfrac{1}{2}\,c_{w_0}\left(\tfrac{1}{\sqrt{2}}c_{w_0} \pm 2\,c_{w_1}\right)c_{w_0'}\left(\tfrac{1}{\sqrt{2}}c_{w_0'} \pm 2\,c_{w_1'}\right),$$

where

$$\langle\phi^{(\pm)}_{w_0,w_1}|\phi^{(\pm)}_{w_0',w_1'}\rangle = \tfrac{1}{2}\,c_{w_0}c_{w_0'} \tag{100}$$

Now, the minimum of the expectation value of the total energy will be sequentially computed. For this reason, the respective energetic expressions are refined to the form

$$\langle\phi^{(\pm)}|E_{tot}|\phi^{(\pm)}\rangle = \left(\textstyle\sum_{w_0} E'_{w_0}\frac{c^2_{w_0}}{2} \pm \sum_{w_1} E'_{w_1}c^2_{w_1}\right) - \tag{101}$$

$$\frac{1}{2}\sum_{w_0,w_1,}\Delta^{(\pm)}_{w_0,w_1}\,c_w\left(\frac{1}{\sqrt{2}}c_{w_0} \pm 2\,c_{w_1}\right)$$

Next, the two gap functions are interposed into equation (101)

$$\Delta^{(\pm)}_{w_0,w_1} = \sum_{w_{0'},w_{1'}} 2\,V_{w_0,w_{0'},w_1,w_{1'}}\ \tfrac{1}{2} c_{w_{0'}} \left(\tfrac{1}{\sqrt{2}} c_{w_{0'}}\ \pm\ 2\,c_{w_{1'}}\right). \quad (102)$$

These functions represent two order parameters [20], which characterize the different energy levels between the interacting receptors and the free receptors. In addition, the following shortcut is introduced

$$2\,V_{w_0,w_0{}',w_1,w_1{}'} =\ \left|v_{w_0-w_0'}\right| + \left|v_{w_1-w_1'}\right|. \quad (103)$$

The derivative of equation (101), with respect to c_{w_1} is in the case of $|\phi^{(+)}\rangle$

$$\left(2\,E'_{w_1} - E'_{w_0}\right) c_{w_1} + \Delta^{(+)}_{w_0,w_1}\left(-\,c_{w_0} + \frac{c_w^2}{c_{w_0}} + \frac{1}{\sqrt{2}}\,c_{w_1}\right) = 0, \quad (104)$$

whereas for $|\phi^{(-)}\rangle$ the derivative becomes

$$\left(2\,E'_{w_1} + E'_{w_0}\right) c_{w_1} +\ \Delta^{(-)}_{w_0,w_1}\left(-c_{w_0} + \frac{c_{w_1}^2}{c_{w_0}} - \frac{1}{\sqrt{2}}\,c_{w_1}\right) = 0. \quad (105)$$

Thereby, the normalization condition (91) is applied to perform the following differentiation

$$\frac{d}{dc_{w_1}}\left(c_{w_0}\,c_{w_1}\right) = \left(c_{w_0} - \frac{c_{w_1}^2}{c_{w_0}}\right). \quad (106)$$

The fractional solutions of equation (104) becomes

$$\left(\frac{c_{w_0}}{c_{w_1}}\right)^{(+)} = \frac{1}{\Delta^{(+)}_{w_0,w_1}}\left(\,\tilde{\varepsilon}^{(+)}_{w,w_1,}\ \pm\ \tilde{E}^{(+)}_{w_0,w_1}\,\right) \equiv \frac{1}{\Delta^{(+)}_{w_0,w_1}}\left(\,\tilde{\varepsilon}^{(+)}_{w_0,w_1,}\ +\ \tilde{E}^{(+)}_{w_0,w_1}\,\right), \quad (107)$$

together with the abbreviation

$$\tilde{\varepsilon}^{(+)}_{w_0,w_1} = \left(\left(E'_{w_1} - \frac{E'_{w_0}}{2}\right) + \frac{1}{2\sqrt{2}}\Delta^{(+)}_{w_0,w_1}\right), \quad (108)$$

and the definition of the excitation energy

$$\tilde{E}^{(+)}_{w_0,w_1} = \sqrt{\left(\tilde{\varepsilon}^{(+)}_{w_0,w_1,}\right)^2 + \left(\Delta^{(+)}_{w_0,w_1}\right)^2}. \quad (109)$$

Note that the minus sign at the expression $\pm\,\tilde{E}^{(\pm)}_{w_0,w_1}$ is omitted in equations (107), since this solution should be positive for energetic reasons.

Without the interactions of the receptors is $\Delta^{(+)}_{w_0,w_1} = 0$. The excitation energy $\tilde{E}^{(+)}_{w_0,w_1}$ becomes then $\left(E'_{w_1} - \frac{E'_{w_0}}{2}\right)$ and continuously grows up.

The multiplicative solution of equation (104) gets

$$\left(c_{w_0}\,c_{w_1}\right)^{(+)} = \frac{\Delta^{(+)}_{w_0,w_1}\left(\left(c^{(+)}_{w_1}\right)^2 - \left(c^{(+)}_{w_0}\right)^2\right)(-1 \pm 1)}{4\,\tilde{\varepsilon}^{(+)}_{w_0,w_1}} \equiv \frac{-\Delta^{(+)}_{w_0,w_1}\left(\left(c^{(+)}_{w_1}\right)^2 - \left(c^{(+)}_{w_0}\right)^2\right)}{2\,\tilde{\varepsilon}^{(+)}_{w_0,w_1}}, \quad (110)$$

Here, the solution, with $(-1-1=-2)$, is stipulated, since otherwise it vanishes.

To evaluate this solution, the difference $\left(c^{(+)}_{w_1}\right)^2 - \left(c^{(+)}_{w_0}\right)^2$ has to be calculated. For this reason, the solution (107) is squared and the normalization restriction (91) is applied, thus the two quadratic expressions result

$$\left(c^{(+)}_{w_0}\right)^2 = \frac{1}{2}\left(1 + \frac{\tilde{\varepsilon}^{(+)}_{w_0,w_1}}{\tilde{E}^{(+)}_{w_0ww_1}}\right), \left(c^{(+)}_{w_1}\right)^2 = \frac{1}{2}\left(1 - \frac{\tilde{\varepsilon}^{(+)}_{w_0,w_1}}{\tilde{E}^{(+)}_{w_0,w_1}}\right). \quad (111)$$

Hence, the formula (110) obtains the final form

$$\left(c_{w_0}\,c_{w_1}\right)^{(+)} = \frac{\Delta^{(+)}_{w_0,w_1}}{2\,\tilde{E}^{(+)}_{w_0,w_1}}. \quad (112)$$

The fractional solutions of equation (105) becomes

$$\left(\frac{c_{w_0}}{c_{w_1}}\right)^{(-)} = \frac{1}{\Delta^{(-)}_{w_0,w_1}}\left(\,\tilde{\varepsilon}^{(-)}_{w_0,w_1,}\ \pm\ \tilde{E}^{(-)}_{w_0,w_1}\,\right) \equiv \frac{1}{\Delta^{(-)}_{w_0,w_1}}\left(\,\tilde{\varepsilon}^{(-)}_{w_0,w_1,}\ +\ \tilde{E}^{(-)}_{w_0,w_1}\,\right), \quad (113)$$

with the two abbreviations

$$\tilde{\varepsilon}^{(-)}_{w_0,w_1} = \left[\left(E'_{w_1} + \frac{E'_{w_0}}{2}\right) - \frac{1}{2\sqrt{2}}\Delta^{(-)}_{w_0,w_1}\right], \tag{114}$$

$$\tilde{E}^{(-)}_{w_0,w_1} = \sqrt{\left(\tilde{\varepsilon}^{(-)}_{w_0,w_1,}\right)^2 + \left(\Delta^{(-)}_{w_0,w_1}\right)^2}. \tag{115}$$

The freely selectable sign of the expression $\pm\,\tilde{E}^{(-)}_{w_0,w_1}$ in equation (113) is set down to the plus sign.
The multiplicative solution of equation (105) becomes

$$\left(c_{w_0}\, c_{w_1}\right)^{(-)} = \frac{-\Delta^{(-)}_{w_0,w_1}\left(\left(c^{(-)}_{w_1}\right)^2 - \left(c^{(-)}_{w_0}\right)^2\right)}{2\,\tilde{\varepsilon}^{(-)}_{w_0 w_1,}}. \tag{116}$$

The formulas of these quadratic coefficients do not change the internal signs

$$\left(c^{(-)}_{w_0}\right)^2 = \frac{1}{2}\left(1 + \frac{\tilde{\varepsilon}^{(-)}_{w_0,w_1}}{\tilde{E}^{(-)}_{w_0,w_1}}\right), \left(c^{(-)}_{w_1}\right)^2 = \frac{1}{2}\left(1 - \frac{\tilde{\varepsilon}^{(-)}_{w_0,w_1}}{\tilde{E}^{(-)}_{w_0,w_1}}\right), \tag{117}$$

hence the multiplicative solution again delivers a positive result, provided $\Delta^{(-)}_{w_0,w_1} > 0$,

$$\left(c_{w_0}\, c_{w_1}\right)^{(-)} = \frac{\Delta^{(-)}_{w_0,w_1}\cdot}{2\,\tilde{E}^{(-)}_{w_0,w_1}}. \tag{118}$$

The calculation of the kinetic energy of equation (101), with respect to $|\phi^{(+)}\rangle$ delivers the result

$$\left(\frac{E'_{w_0}}{2}\left(c^{(+)}_{w_0}\right)^2 + E'_{w_1}\left(c^{(+)}_{w_1}\right)^2\right) = \frac{1}{2}\left(\frac{E'_{w_0}}{2} + E'_{w_1}\right) + \frac{1}{2}\left(\frac{E'_{w_0}}{2} - E'_{w_1}\right)\frac{\tilde{\varepsilon}^{(+)}_{w_0,w_1}}{\tilde{E}^{(\pm)}_{w_0,w_1}} \tag{119}$$

$$= \frac{1}{2}\left(\frac{E'_{w_0}}{2} + E'_{w_1}\right) - \frac{\left(\tilde{\varepsilon}^{(+)}_{w_0,w_1}\right)^2}{2\,\tilde{E}^{(+)}_{w_0,w_1}} + \frac{1}{4\sqrt{2}}\,\Delta^{(+)}_{w_0,w_1}\frac{\tilde{\varepsilon}^{(+)}_{w_0,w_1}}{\tilde{E}^{(+)}_{w_0,w_1}}.$$

The corresponding potential energy reads

$$-\frac{1}{2}\,\Delta^{(+)}_{w_0,w_1}\left(\frac{1}{\sqrt{2}}\left(c^{(+)}_{w_0}\right)^2 + 2\left(c_{w_0}\, c_{w_1}\right)^{(+)}\right) = -\frac{1}{2}\,\Delta^{(+)}_{w_0,w_1}\left(\frac{1}{2\sqrt{2}}\left(1 + \frac{\tilde{\varepsilon}^{(+)}_{w_0,w_1}}{\tilde{E}^{(+)}_{w,w_1}}\right)\right. \tag{120}$$

$$\left. + \frac{\Delta^{(+)}_{w_0,w_1}}{\tilde{E}^{(+)}_{w_0,w_1}}\right) = -\frac{1}{4\sqrt{2}}\,\Delta^{(+)}_{w_0,w_1} - \frac{1}{4\sqrt{2}}\,\Delta^{(+)}_{w_0,w_1}\frac{\tilde{\varepsilon}^{(+)}_{w_0,w_1}}{\tilde{E}^{(+)}_{w_0,w_1}} - \frac{1}{2}\frac{\left(\Delta^{(+)}_{w_0,w_1}\right)^2}{\tilde{E}^{(+)}_{w_0,w_1}}.$$

For comprehensibility, the summation over w_0 and w_1 is not indicated in the equations (119) resp. (120). When, the two equations (119) and (120) are put together, and then particular total energy is obtained

$$E^{(+)}_{tot,w_0,w_1} = \frac{1}{2}\left(\frac{E'_{w_0}}{2} + E'_{w_1}\right) - \frac{1}{4\sqrt{2}}\,\Delta^{(+)}_{w_0,w_1} - \frac{1}{2\,\tilde{E}^{(+)}_{w_0,w_1}}\left(\left(\tilde{\varepsilon}^{(+)}_{w_0,w_1}\right)^2 + \left(\Delta^{(+)}_{w_0,w_1}\right)^2\right) \tag{121}$$

$$= \frac{1}{2}\left(\frac{E'_{w_0}}{2} + E'_{w_1}\right) - \frac{1}{4\sqrt{2}}\,\Delta^{(+)}_{w_0,w_1} - \frac{1}{2}\,\tilde{E}^{(+)}_{w_0,w_1}.$$

The interactions between entangled receptors decrease the expectation value of the appropriate total energy under the free energy $\frac{1}{2}\left(\frac{E'_{w_0}}{2} + E'_{w_1}\right)$, where the total energy (121) of the ground state $|\phi^{(+)}_{w_0,w_1}\rangle$ has the lowest energy. The interacting entangled particles are sheltered against environmental attacks (decoherence), since further down; it will be demonstrate that $\Delta^{(+)}_{w_0,w_1}$ is positive, and $\tilde{E}^{(+)}_{w_0,w_1}$ (109) is anyway positive.

The calculation of the particular kinetic energy corresponding to the ground state $|\phi^{(-)}_{w_0,w_1}\rangle$ is

$$\left(\frac{E'_{w_0}}{2}\left(c^{(-)}_{w_0}\right)^2 - E'_{w_1}\left(c^{(-)}_{w_1}\right)^2\right) = \frac{1}{2}\left(\frac{E'_{w_0}}{2} - E'_{w_1}\right) + \frac{1}{2}\left(\frac{E'_{w_0}}{2} + E'_{w_1}\right)\frac{\tilde{\varepsilon}^{(-)}_{w_0,w_1}}{\tilde{E}^{(-)}_{w_0,w_1}} \tag{122}$$

$$= \frac{1}{2}\left(\frac{E'_{w_0}}{2} - E'_{w_1}\right) + \frac{1}{2}\frac{\left(\tilde{\varepsilon}^{(-)}_{w_0,w_1}\right)^2}{\tilde{E}^{(-)}_{w_0,w_1}} + \frac{1}{4\sqrt{2}}\,\Delta^{(-)}_{w_0,w_1}\frac{\tilde{\varepsilon}^{(-)}_{w_0,w_1}}{\tilde{E}^{(-)}_{w_0,w_1}}.$$

The evaluation of the corresponding particular potential energy provides the result

$$-\frac{1}{2}\ \Delta^{(-)}_{w_0,w_1}\left(\frac{1}{\sqrt{2}}\left(c^{(-)}_{w_0}\right)^2-2\left(c_{w_0}\ c_{w_1}\right)^{(-)}\right)= \quad (123)$$

$$-\frac{1}{4\sqrt{2}}\ \Delta^{(-)}_{w_0,w_1}-\frac{1}{4\sqrt{2}}\ \boldsymbol{w}\frac{\tilde{\varepsilon}^{(-)}_{w_0,w_1}}{\tilde{E}^{(-)}_{w_0,w_1}}+\frac{1}{2}\frac{\left(\Delta^{(-)}_{w_0,w_1}\right)^2}{\tilde{E}^{(-)}_{w_0,w}}.$$

To get the respective particular total energy the two equations (122) and (123) are again gathered

$$E^{(-)}_{tot,w_0,w_1}=$$

$$\frac{1}{2}\left(\frac{E'_{w_0}}{2}-E'_{w_1}\right)-\frac{1}{4\sqrt{2}}\ \Delta^{(-)}_{w_0,w_1}+\frac{1}{2\ \tilde{E}^{(-)}_{w_0,w_1}}\left(\left(\tilde{\varepsilon}^{(-)}_{w_0,w_1}\right)^2+\left(\Delta^{(-)}_{w_0,w_1}\right)^2\right) \quad (124)$$

$$=\frac{1}{2}\left(\frac{E'_{w_0}}{2}-E'_{w_1}\right)-\frac{1}{4\sqrt{2}}\ \Delta^{(-)}_{w_0,w_1}+\frac{1}{2}\ \tilde{E}^{(-)}_{w_0,w_1}.$$

In this case, even the total energy belonging to $|\phi^{(-)}_{w_0,w_1}\rangle$ increases, since $\tilde{E}^{(-)}_{w_0,w_1}>\frac{1}{2\sqrt{2}}\ \Delta^{(-)}_{w_0,w_1}$. Below, it will be indicated that the sign of this gap function $\Delta^{(-)}_{w_0,w_1}$ is also positive and its value is comparable to that one of $\Delta^{(+)}_{w_0,w_1}$. Therefore, the gap function $\Delta^{(-)}_{w_0'w_1'}$ provides a less protection of the entangled ground states against decoherence as the gap function $\Delta^{(+)}_{w_0'w_1'}$ delivers. This reduction of the sheltering by $\Delta^{(-)}_{w_0'w_1}$ will be approved in the next subchapter, where the entangled entropies that correspond to both gap functions are calculated. In addition, contemporary conditions of the environmental influences can affect the amount of the protection,

To determine $\Delta^{(+)}_{w_0,w_1}$, the equation (102) for this this gap function is rewritten

$$\Delta^{(+)}_{w_0,w_1}=\sum_{w_0',w_1'}V_{w_0,w_0',w_1,w_1'}\ c_{w_0'}\left(\frac{1}{\sqrt{2}}c_{w_0'}+2\ c_{w_1'}\right)= \quad (125)$$

$$\sum_{w_0',w_1'}V_0\left(\frac{1}{2\sqrt{2}}\left(1+\frac{\tilde{\varepsilon}^{(+)}_{w_0'w_1'}}{\tilde{E}^{(+)}_{w_0'w_1'}}\right)+\frac{\Delta^{(+)}_{w_0'w_1'}}{\tilde{E}^{(+)}_{w_0'w_1'}}\right).$$

Hereby, it is assumed that for a small region the potential is constant

$V_{w_0,w_0',w_1,w_1'}=V_0$, for $\left|E'_{w_1'}-\frac{E'_{w_0'}}{2}\right|<\hbar\omega$, where ω denotes the mean frequency of the oscillator (receptor) vibration. Further, the gap function is set to a constant

$$\Delta^{(+)}_{w,w_1}=\Delta^{(+)}. \quad (126)$$

Thus, the rewritten formula (125) becomes a self-consistent (iterative) equation for $\Delta^{(+)}$

$$\frac{2\sqrt{2}\,\Delta^{(+)}}{V_0}=\sum_{w_0',w_1'}\left(1+\frac{\tilde{\varepsilon}^{(+)}_{w_0'w_1'}+2\sqrt{2}\,\Delta^{(+)}}{\tilde{E}^{(+)}_{w_0'w_1'}}\right). \quad (127)$$

To solve this equation, the sum is replaced by an integral. Thus, the integration variable gets $\mathcal{E}'=E'_{w_1}-\frac{E'_{w_0}}{2}$ and $N(\mathcal{E}')=\frac{\mho}{(2\pi)^3}D(\mathcal{E}')$ represents the usual replacement of a sum by an integral, where $D(\mathcal{E}')$ indicates the density of states, and $\mho$ denotes the volume. Supplementary, the state density is approximately constant $D(\mathcal{E}')\approx D(0)$. The integration bounds runs from zero until a mean positive vibrational binding energy $\hbar\omega$. Thus, the integral form of equation (127) becomes

$$\frac{2\sqrt{2}\,\Delta^{(+)}}{V_0\,N(0)}=\int_0^{\hbar\omega}\left\{1+\frac{\mathcal{E}'+\Delta^{(+)}/2\sqrt{2}}{\tilde{E}^{(+)}(\mathcal{E}')}+\frac{2\sqrt{2}\,\Delta^{(+)}}{\tilde{E}^{(+)}(\mathcal{E}')}\right\}d\,\mathcal{E}'= \quad (128)$$

$$\hbar\omega+\sqrt{(\hbar\omega)^2+\frac{\Delta^{(+)}\hbar\omega}{\sqrt{2}}+\frac{9}{8}\Delta^{(+)2}}-\sqrt{\frac{9}{8}}+2\sqrt{2}\,\Delta^{(+)}\left(\operatorname{arsinh}\left(\frac{\hbar\omega}{\Delta^{(+)}}+\frac{1}{2\sqrt{2}}\right)-\operatorname{arsinh}\left(\frac{1}{2\sqrt{2}}\right)\right),$$

where the excitation energy reads

$$\tilde{E}^{(+)}(\mathcal{E}')=\sqrt{\mathcal{E}'^2+\mathcal{E}'\,\Delta^{(+)}/\sqrt{2}+\frac{9}{8}\,\Delta^{(+)2}}. \quad (129)$$

This equation is solved by a basic iteration that starts with $\Delta^{(+)}_0=0$, where the calculation terminates at the second order term. Hence, the two iterations are

$$\frac{2\sqrt{2}}{V_0\,N(0)}\Delta^{(+)}_1=\int_0^{\hbar\omega}2\,d\,\mathcal{E}'=2\,\hbar\omega \quad (130)$$

$$\frac{2\sqrt{2}}{V_0\,N(0)}\ \Delta^{(+)}_2=\int_0^{\hbar\omega}\left\{1+\frac{\mathcal{E}'+\hbar\omega/\sqrt{2}}{\tilde{E}^{(+)}(\mathcal{E}')}+\frac{4\sqrt{2}\,\hbar\omega}{\tilde{E}^{(+)}(\mathcal{E}')}\right\}d\,\mathcal{E}'=\frac{\hbar\omega}{\sqrt{2}}C, \quad (131)$$

together with

$$\tilde{E}^{(+)}(\mathcal{E}')=\sqrt{\mathcal{E}'^2+\sqrt{2}\,\hbar\omega\,\mathcal{E}'+\frac{9}{2}(\hbar\omega)^2}, \quad (132)$$

and

$$C=\frac{1}{\sqrt{2}}\left(\sqrt{2}+\sqrt{11+2\sqrt{2}}-3+8\left(\operatorname{arsinh}\left(\frac{2+\sqrt{2}}{4}\right)-\operatorname{arsinh}\left(\frac{1}{2\sqrt{2}}\right)\right)\right) \quad (133)$$

$$\approx 2.6>0.$$

The result of this simplified approximation indicates that the expression $\hbar\omega$ dominates the value of $\Delta^{(+)}_1$, where all constant factors are irrelevant. When the mean frequency is one milliseconds, then $\hbar\omega=(6.58\,10^{-16}\text{eV sec})\times\left(2\pi\,\frac{10^{-3}}{sec}\right)\approx 41.32\,10^{-19}$ eV. This very small, positive gap value dominantly downsizes, when a frequency of one picosecond $(\frac{10^{-12}}{sec})$ is selected, then $\hbar\omega\approx 41.32\,10^{-28}$ eV.

In case of $|\phi^{(-)}\rangle$, the corresponding formula reads

$$\frac{2\sqrt{2}}{V_0\,N(0)}=\frac{1}{\Delta^{(-)}}\int_0^{\hbar\omega}\left\{1+\frac{\text{e}'-\Delta^{(-)}/2\sqrt{2}}{\tilde{E}^{(-)}(\text{e}')}-\frac{2\sqrt{2}\,\Delta^{(-)}}{\tilde{E}^{(-)}(\text{e}')}\right\}d\,\text{e}'. \quad (134)$$

The modified integration variable is now $\text{e}'=\frac{E'_{w_0}}{2}+E'_{w_1}$;

further, the constancy of $\Delta^{(-)}_{w_0' w_1'} = \Delta^{(-)}$ is again expected, and the potential is estimated to be constant in a small region

$$V_{w_0,w_0',w_1,w_1'} = V_0, \text{ for } \left|\frac{E_{w_0}'}{2} + E_{w_1}'\right| < \hbar\omega . \tag{135}$$

The modified excitation energy gets

$$\tilde{E}^{(-)}(e') = \sqrt{e'^2 - e' \Delta^{(-)}/\sqrt{2} + \frac{9}{8}\Delta^{(-)2}}. \tag{136}$$

The two pursuant iteration steps are

$$\frac{2\sqrt{2}}{V_0 N(0)}\Delta^{(-)}_1 = \int_0^{\hbar\omega} 2\, de' = 2\hbar\omega \frac{2\sqrt{2}}{V_0 N(0)}\Delta^{(-)}_2 \tag{137}$$

$$= \int_0^{\hbar\omega}\left\{1 + \frac{e'-\hbar\omega/\sqrt{2}}{\tilde{E}^{(-)}(e')} - \frac{4\sqrt{2}\,\hbar\omega}{\tilde{E}^{(-)}(e')}\right\} d\,e' = \frac{\hbar\omega}{\sqrt{2}} C',$$

where $C' \approx .25 > 0$, hence the sign of $\Delta^{(-)}_2$ is again positive.

The entanglement of the two ground states that are guarded by the two gap functions $\Delta^{(\pm)}$ is called protected entanglement.

8.2. Non-vanishing Entanglement Entropies of the Ground States

This subchapter demonstrates that the sheltering interactions described in the previous subchapter do not destroy the entanglement between receptors, but only diminish the entanglement entropies of the two ground states. In consequence, the protected entanglement represents a source that generates quantum coherence. The ground states stay entangled, and therefore continue to be coherent. This property of quantum coherence guarantees, for instance the immediately synchronization of the oscillations between different neural networks.

The density operators of the ground states are

$$\hat{\rho}^{(\pm)}_{AB} = |\,\phi^{(\pm)}_{w_0,w_1}\rangle\langle\phi^{(\pm)}_{w_0,w_1}|\,, \tag{138}$$

where the traces of these two operators become

$$\mathrm{Tr}\,\hat{\rho}^{(\pm)}_{AB} = \mathrm{Tr}\left(|\,\phi^{(\pm)}_{w_0,w_1}\rangle\langle\phi^{(\pm)}_{w_0,w_1}|\,\right) = \langle\phi^{(\pm)}_{w_0,w_1}|\phi^{(\pm)}_{w_0,w_1}\rangle = \left(c^{(\pm)}_{w_0}\right)^2 + \left(c^{(\pm)}_{w_1}\right)^2 = 1. \tag{139}$$

The reduced density operators of $\hat{\rho}^{(\pm)}_{AB}$ of subsystem *A* reads

$$\hat{\rho}^{(\pm)}_{A} = \mathrm{Tr}_B\,\hat{\rho}^{(\pm)}_{AB} = \frac{\left(c^{(\pm)}_{w_0}\right)^2}{2} + \frac{\left(c^{(\pm)}_{w_0}\right)^2}{2}|\,\hat{b}^{\dagger}_{w_0}\rangle\langle\hat{b}^{\dagger}_{w_0}| \pm \left(c^{(\pm)}_{w_1}\right)^2|\,\hat{b}^{\dagger}_{w_1}\rangle\langle\hat{b}^{\dagger}_{w_1}|. \tag{140}$$

Both density operators have a unit trace

$$\mathrm{Tr}\,\hat{\rho}^{(\pm)}_{A} = \left(c^{(\pm)}_{w_0}\right)^2 + \left(c^{(\pm)}_{w_1}\right)^2 = 1. \tag{141}$$

The entanglement entropies of the two ground states $|\phi^{(\pm)}_{0,w_0,w_1}\rangle$ are

$$S^{(\pm)}_A = -\mathrm{Tr}\left(\hat{\rho}^{(\pm)}_A \ln \hat{\rho}^{(\pm)}_A\right) = -\left(\left(c^{(\pm)}_{w_0}\right)^2 \ln\frac{\left(c^{(\pm)}_{w_0}\right)^2}{2} \pm \left(c^{(\pm)}_{w_1}\right)^2 \ln\left(c^{(\pm)}_{w_1}\right)^2\right) \tag{142}$$

$$= \left(c^{(\pm)}_{w_0}\right)^2 \ln 2 - \left(\left(c^{(\pm)}_{w_0}\right)^2 \ln\left(c^{(\pm)}_{w_0}\right)^2 \pm \left(c^{(\pm)}_{w_1}\right)^2 \ln\left(c^{(\pm)}_{w_1}\right)^2\right).$$

In the impermissible case, that $\left(c^{(\pm)}_{w_0}\right)^2 = \left(c^{(\pm)}_{w_1}\right)^2 = 1$, these entropies apparently get the maximal value of $\ln 2$ as for the corresponding Bell states (68). However, the equality of the two coefficients must be excluded, since it contradicts the normalization condition that only the sum of both coefficients is 1; both numbers are unequal and less than 1. A case analysis demonstrates this inequality.

When $\Delta^{(\pm)}_{w_0' w_1'} = 0$, then $\frac{\tilde{\varepsilon}^{(\pm)}_{w_0,w_1}}{\tilde{E}^{(\pm)}_{w_0,w_1}} = 1$, therefore, the results are $\left(c^{(\pm)}_{w_0}\right)^2 = \frac{1}{2}\left(1 + \frac{\tilde{\varepsilon}^{(\pm)}_{w_0,w_1}}{\tilde{E}^{(\pm)}_{w_0,w_1}}\right) = 1$, and $\left(c^{(\pm)}_{w_1}\right)^2 = \frac{1}{2}\left(1 - \frac{\tilde{\varepsilon}^{(\pm)}_{w_0,w_1}}{\tilde{E}^{(\pm)}_{w_0,w_1}}\right) = 0$. Thus, in the case of no interactions the entangled entropy is maximal $S^{(\pm)}_A = \ln 2$, according to the rule : $\text{limes}_{x\to+0}\, x \ln x = 0$. However, in this case the entanglement of the receptors stays unprotected.

In case $\Delta^{(\pm)}_{w_0' w_1'} \neq 0$, then $\frac{\tilde{\varepsilon}^{(\pm)}_{w_0,w_1}}{\tilde{E}^{(\pm)}_{w,v_1}} < 1$ and both coefficients are less than 1. In consequence, the entanglement entropies are no more maximal, but only decremented and do not vanish, where $S^{(+)}_A$ is even greater as $S^{(-)}_A$.

In summary, the entanglement entropies $S^{(\pm)}_A$ have despite the interactions non-zero values, what represents a clear sign of entanglement.

9. Consciousness Activates Entanglement

The phase transitions between unconscious and conscious perception and vice versa represent a powerful experimental method to analyze dominant features (signatures) of both phases. Proper experiments that carefully observe the effects of these transitions with probands substantiate the modern assertion that consciousness is materialistic [5, 21]. Unconscious activities of the areas of the subcortex collect and prepare relevant preconscious (precognitive) information that any time can become conscious, when the working space pays attention to it [54-55].

The entanglement in the cortex provides a bridge to the understanding of consciousness. However, this hypothesis also premises that each accomplished entangled state is renewable at any time. This effect corresponds an experimental setup, where entangled states are constantly recreated, and the information of these states is at once transferred in adjacent or remote regions [56].

9.1. Effects of Entanglement in the Brain

Conscious assignments consistently activate the entanglement of the ionotropic receptors at different regions. Impacts of supplementary action potentials initiate the entanglement processes and stipulate, for instance the exocytosis. When ionotropic receptors bound two neurotransmitters, then each receptor immediately cares a well-defined information. Thus, in a conscious state the cortex areas distinctly get very quick all entangled information in a compressed form.

In quantum physics, the entanglement is considered as a process of teleportation. However, in the context of this article, the phenomenon of entanglement represents an effort of information processing. Further, there exist a second speculative aspect of entanglement. In living systems like the brain, decoherent processes do not destroy the protected entanglement. Thus, the biological (natural) kind of entanglement causes the immediate firing of neurons, whose receptors are in the activated entangled state.

Entanglement supports the completion of a tight timing between different cortical areas (synchronous operations). Two representative examples for the necessity of a strong synchronization are the synaptic plasticity and the exocytosis. The synaptic plasticity is characteristic for learning and memorizing activities, where the long-term potentiation (LTP) and the long-term depression (LTD) play a dominant role. Which one of these two effects occurs depends from the timing between these two processes. The increase or decrease of synaptic strengths change the neural selectivity, where enduring weight modifications as a kind of reinforcement correspond to a learning process that stipulates the enduring settlement of the synaptic weights.

The initiation of the vesicular emissions of molecules is not a result of a mental intension [3], but an impact of conscious activations of the synchronizing action potentials. The appropriate probability distribution of the transmitter release is the Poisson distribution that approximates the binomial distribution, with a sufficient accuracy.

9.2. Synergetic (Self-organized) Model of Consciousness

In chapter 8, the grand canonical ensemble of receptors was in the state of thermal equilibrium, since this ensemble was considered as a closed system. Thus, the phase transition to the protected entanglement was calculated with a similar method that is applied in superconductivity [11, 14]. Furthermore, this equilibrium phase transition is of second-order.

The objective of synergetics [20, 57] is the description of the self-organization of open systems that includes the selection of stable solutions of the occurring processes. The human brain is an outstanding example of an open system. Each cell and each aggregation of cells receives continuously energy or molecules (matter) from its environment (e.g. heat, blood, chemicals from the secretory hypothalamus). All cells process the incoming energy/matter flux (regulation of blood oxygen, blood volume, blood pressure, glucose concentration, acidity; homeostasis) and emit the rest of the energetic influx to the environment.

Open systems are in a dynamical equilibrium, when their internal variables stay constant, provided the exchange with the environment is steady. However, the stationary non-equilibrium states of open systems are more relevant, since these states represent a higher degree of order as in the dynamical equilibrium states. The Shannon entropy S decreases in stationary non-equilibrium states (higher order), what is the occurrence of self-organization. The transition between the dynamical equilibrium and the stationary non-equilibrium causes the transfer of the brain states from unconsciousness to consciousness. Such a transition leads to a broken (reduced) symmetry, where this aspect complies with a higher order.

In the mathematical view, the different neural clusters and their interconnections in the brain represent nonlinear dynamical systems. The solutions of such dynamical systems represent, for example stable and strange attractors, repellers, local and global bifurcations, central manifolds, hyperbolic sets and chaos, e.g. [58]. These effects are referred, since they can likewise occur in the brain.

The consideration of the impact of fluctuations (noise) is important, because they, for instance can shift a stable solution into an unstable solution and two stable solution. On the other hand, noise can even push away a steady solution to an unstable solution, and the change of a control parameter causes structural instability. Therefore, fluctuations transform, for example the mode of operation of bifurcations of dynamical systems that are in an equilibrium to corresponding effects that occur in non-equilibrium states.

The higher cortical levels, for instance the prefrontal lobe and the posterior parietal lobe can be as well in an unconscious state (dynamic equilibrium phase) or in a conscious state (non-equilibrium phase). The subcortical levels can also be entangled; however, they remain each time in unconscious states and cannot transit to conscious states.

Consciousness activates the entanglement, where it directly controls the output of excitatory receptors (exited, e.g. by acetylcholine, noradrenaline or serotonin neurotransmitters) or inhibitory receptors (inhibited, e.g. by GABA and glycine neurotransmitters). In consequence, the synapses are strengthen (LTT) or weakened (LTD). These effects evoke the increase of neural excitations (action potentials) or the decreased propagation of signals, without excitatory support (degradation of the spread of action potentials).

The overall information that the working space collects should be immediately disposable, situation dependent, compressed and exhaustive. This represents a necessary condition. The sufficient condition demands that this information must be complete to be able to perform consistent and appropriate decisions.

The thresholds to perform a transition from unconscious states to conscious states are preset by appropriate values of the action potentials. Beneath this thresholds, the subcortex and the cortex remain in the unconscious states, thus, both systems stay in a dynamical equilibrium. When the bottom-up excitation via different subcortical areas is too weak, then only a subliminal perception occur.

The synergetics characterizes the self-organization by the circular causality (creation of meaning). The order parameters (data structures, e.g. salience and priority maps in visual cortical areas and parietal lobe, [4]) control the subcortical areas and the lower levels (brain stem and cerebellum). The cortices of the working space are autonomous synergetic agents [57] that treat the subcortical areas as synergetic "slaves", which conversely activate the synergetic agents. The exclusive focus of these agents to a single, salient order parameter (e.g. activity pattern) represents an act of thinking. When, the synergetic principle are applied on the communicating, synergetic agents (associative cortices), then one of these agents determines the single order parameter that controls (inhibits) the remaining order parameters. This process provides the unique interpretation of the common information. Thus, a single thought inhibits all other thoughts.

The autonomy of the synergetic agents also emphasizes their self-sufficiency. Thus, synergetic agents that are in a conscious state can define their own goals, without considering any external inputs. Consequently, conscious synergetic agents are able to think on a meta-cognitive level and thereby they develop the self-consciousness. Such an autonomy establishes the free will.

In the mathematical view, the prevailing order parameter represents a stable solution (attractor), because even escalating fluctuating forces cannot push away this stationary solution. This is the highest order of the cortex. Two thoughts can switch alternatively from one to the other (corresponds binocular rivalry). Each individual thought that is only stable for a short time represents an intermittently stable solution (transient stability) that is pushed away by critical fluctuations to other solutions of this type. This outcome of intermittently stationary solutions describes a reduced order. Unstable solutions are stable solutions that are rapidly damped out. This corresponds to thoughts that shortly flare up and then disappear. Such a process represents a minor kind of disorder.

The chaos represents the highest degree of disorder. The mathematical theory of chaos defines three typical routes to the chaos. These are the intermittency (saddle note bifurcation), quasi-periodic oscillations (Hopf bifurcation) and period doubling (fork bifurcation), [62]. Let us, for example concentrate on the intermittency. It describes the transition from a regular periodic behavior to a chaotic behavior. This means that a stable, periodic solution shows an increment of irregular bursts, with growing amplitudes. For example, when a thought is temporal unstable, then this idea cannot be retained, because it steadily disappears, and a series of new ideas emerge. In case of the two other previously mentioned ways to chaos, similar effects take place.

10. Conclusions

Neurotransmitters and ionotropic receptors are Bosons that operate in the Fock space. Anharmonic oscillators approximate the bounded states of neurotransmitters and the interactions of these receptors. Thereby, the interdependencies of the receptors is particularly significant, since they generate oscillations, where gap functions protect the vibrating receptors that are in entangled ground states. These states are essential for the protected entanglement, since decoherence does not destruct their quantum coherence. The calculation of the guarding gap functions exhibits some resemblance to the evaluation of the process of super-conductivity, since Cooper-pairs represent Bosons.

The fundamental thesis of this contribution is the commitment that consciousness activates entanglement in the brain. This entails the assumption that in living systems, in opposition to technical systems, the entanglement is robust and frequently renewable. The working space encompasses the associative cortices, which dispose the entanglement activations. The essential benefit of the entanglement is the phenomenon that the relevant information, which is located at different areas, is immediately disposable, contemporary and compressed. Thus, entanglement represents an essential path to understand consciousness. Originally, the entanglement was an effect of quantum physics, but in the light of this contribution, it converts to a tool of information handling.

The transition from unconscious states to conscious states in an open system constitutes a non-equilibrium phase transition of second order (spontaneous symmetry braking). This occurs in the framework of biological-oriented synergetics that represents the basic theory of self-organization. Synergetic agents define the working space, where synergetic "slaves" constitute the subcortical areas. In a conscious state, these agents autonomously control the "slaves". To perform these regulations, the agents construct order parameters (e.g. macroscopic observable patterns). Conversely, the spontaneous bottom-up sensor information that "slaves" acquire, represent a sufficient condition for the agents of the working space. Both levels directly depend on each other (circular causality); what is an essential virtue of self-organization.

In case that the entanglement as the most spectacular quantum effect plays a dominant role in the brain, then this occurrence clearly demonstrates the significance of quantum physics in the brain (living nature).

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