Mathematics Hundred Years Before and Now

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Abstract: Conservation of manuscript and using history of mathematics is the Goal of this paper. It is based on fundamental unifying concept of classical and modern mathematics. The actual differences in classical and modern mathematics are paradigm. In early mathematics there is no variation in branches but modern mathematics is divided into different branches. Geometry and Calculus are modified in context of applications and process. In practice we recognized and accepted old as gold, so this paper tries to link up classical to modern approach to change the paradigm in mathematics with focusing the evidences of mathematical developments.

Keywords: Conservation, Modern Mathematics, Classical Mathematics, Paradigm Shift

1. Introduction

I'm trying to think about a paradigm shift in mathematics with comparing mathematics before 100 years and now on the basis of contents. A Paradigm shift is a major change from one way of thinking to another way of thinking in mathematics. Since awareness is prerequisite to all acceptable changes of theory and Science. The awareness helps to paradigm shift in mathematics. Change of paradigm occurs if the old design runs out of intellectual stream in context of existing mathematics.

People think that New mathematics never overturns old mathematics and they think that they don't get paradigm shift in mathematics. But mathematics based in nature and practice that are always changeable. Thus the mathematics is obviously shifts in accordance of science and technology with arts. A few things come to mind as paradigm shifts in mathematics basically views in mathematics. Late 19th century, the invention and use of symbolic logic helps to shift the mathematics in conceptual base. In 19th - 20th century is the time of rise of abstract algebra, which combined and generalized large areas of existing mathematics into a more cohesive whole. It also gave birth too many of the major fields of modern advanced mathematics like algebraic geometry, algebraic number theory, the rise of analysis. An important part of mathematics, which is calculus of derivative, was used by Bhaskaracharya in 12th century. In 17th century Newton and Leibnitz made calculus rigorous. The shifting in mathematics is the changing between classical and modern concept of using

and solving strategies of mathematics.

The Early 20th century is the fall of the Italian school of algebraic geometry. This is more of a local paradigm shift. Mid of the 19th century there is rise of hyperbolic/non - Euclidean geometry and the fall of the parallel postulate. Early 20th century is the foundational crisis in mathematics because there was not new development in mathematics but there was rigorous review in mathematics. This review helps to linkage to old mathematics to new mathematics.

In every aspect we accept that old is gold. So we should linkup between classical and modern mathematics. We lack but open eye and ear to find the orient's marvels here through the comparison between old and new mathematics. Paradigm shifting in mathematics is the basis of shifting of development. But our different levels mathematics is only rote learning. There is no modification, creativity and historical basis. This is due to the lack up historical linkage in mathematics. The following slide expressed the classical approach of mathematics which is very basis of the daily used mathematics.

In modern concept the above prosodic mathematical statement can be explain as, O marvelous girl! Out of a group of swans, $\frac{7}{2}$ times the square root of the number is playing on the shore of a tank. The two remaining ones are playing with amorous fight, in the water. What is the total number of swans? The solution of this problem can be obtain as, let total number of swans = x then by question, $(\frac{7}{2}) \sqrt{x} + 2 = x$. That is, $4x^2 - 64x + 16 = 0$, provide $x = 16$ and $\frac{1}{4}$ but $\frac{1}{4}$ is impossible then required number of swans = 16 [4]

Figure 1. A poetic (prosody) form of a practical example of an equation.

2. Evidences of Old Manuscripts

Let us share few evidences of mathematics before 100 years. The few old mathematical documents are expressed here for example:

व्यवकलनोदाहरण ।

(३) लगे बीजगणित लण्डन् चौथसै अठहत्तर ।
 इशवीमा आज कतिभो वर्षलौ भन उत्तर ॥ २ ॥

जस्तै = आज कल इशवी १६३३ भयेको हुनाले यस्मा १४७८ घटाउँदा
 $१६३३ - १४७८ = १५५$ भयो अर्थात् १५५ वर्ष भयो ॥

गुणने रीति ।

(४) जुन अङ्कले गुणकमा गइ भागदींदा ।
 निःशेष हुन्छ उहि अङ्कुर भागलींदा ॥
 लब्धी मिल्यो जउन ती दुइले गुणेर ।
 राखी दिनू गुणनफल हुँन जाछ हेर ॥ १ ॥

पहीला भागमा दियेका उदाहरण मा गुणक १२ र गुण्य १३५ मा यो
 रीतिले ल्याउँदा गुणक १२ मा ३ ले भाग दींदा $१२ \div ३ = ४$ निःशेष हुन्छ
 अतः लब्धि ४ले र भाजक ३ ले गुणक १३५ लाई गुणादा $१३५ \times ३ = ४०५$
 भयो अनि ४ ले गुणादा $४०५ \times ४ = १६२०$ भयो ।

Figure 2. A poetic expression for addition & multiplication.

कोही एक सुखमाथिये दुइ वगाल् शुगगा बमेका पनी ।
 ताहाँ एकथर भन्दथ्या तिमिःरू यौटा पठागौ भनी ॥
 हुज्झौ दावर, भन्दथ्या अरुथरी एक आउँदामा यहाँ ।
 हुज्झौ हामि बराबरी भनि, कनी? सुगगा रहेछन् तहाँ ॥२॥
 ८० को १० लाइ १६० मा जोड्दा २०० भयो परन्तु तैले १० दिये
 तं भन्दा ।
 जस्तै=यहाँ इष्ट ११ र १३ कल्पना गरियो । अत्र $१३ + १ = १४$ ।
 अनी $११ - १ = १०$ $१० \times २ = २०$ भयो यो १४ मा घटाउँदा ऋण शेष ६ रह्यो ॥
 फेरी इष्ट १५ र १७ कल्पना गर्दा $१७ + १ = १८$ । अत्र $१५ - १ =$
 $१४ \times २ = २८$ भयो यो अर्का १८ मा घटाउँदा १० ऋणशेष भयो ॥

Figure 3. A prosody expression of the simultaneous linear equations.

* गणिततरङ्गः *

३९

❀ अथ श्रेढीव्यवहारः (१) ❀

—*—*—*—

सङ्कलितको सीती ।

(१२८) एकजोडेर पदले पदार्थकन गूणन् ॥
 भो अङ्कको योग यही सङ्कलित भनि जानन् ॥१॥
 उदाहरण ।

(१२९) एकवज्जा एकहान्यो दुइ दुइ वज्जा पहिरिल्ले गराइ ।
 हान्यो भन् बाह्रवज्जा कतिपटक यहाँ योगइन्को बनाई ॥
 जस्तै=यहाँ पद १२ छ यसमा १ जोड्दा $१२ + १ = १३$ भयो । यसले
 पद १२ को आधा $१२ \div २ = ६$ लाइ गुणादा $६ \times १३ = ७८$ भयो ॥
 अथवा $१ + २ + ३ + ४ + ५ + ६ + ७ + ८ + ९ + १० + ११ + १२ =$
 ७८ हुन्छ ॥

सङ्कलितकायोगको सीती

(१३०) सङ्कलितलाई पदमा दुइजोडि गुणी अनी ।
 सङ्कलितको योग हुन्छ तिनले भाग दियो भनी ॥२॥
 उदाहरण

एक दिनमा एकदीयो तिन दुइ दिनमा तीन दिनमा छ फेरी ।
 यैरिल्ले बाइ दिनमा भन कति दियहो सङ्कलित योग हेरी ॥१॥
 जस्तै=१२ को सङ्कलित ७८ लाइ पद १२ मा दुइ जोडेर $१२ \div २ =$
 १४ ले गुणादा $७८ \times १४ = १०९२$ भयो यस्ताइ ३ ले भागदा $१०९२ \div$
 $३ = ३६४$ भयो अर्थात् १२ दिनमा ३६४ रुपैया दिये छ ।
 यथोत्तर चयमा अन्य मध्यादि धनल्याउनुयो सीती-

(१३१) एक घटाइ पदमा चयलाई ।
 गूणि आदिधन जोड रमाई ॥
 अन्त्यधन हुनगयो यसलाई ।
 आदि जोडिकन आधि गराइ ॥३॥

(१) मोटा चेटे कै १, २, ३, ४ इत्यादि क्रमसँग अङ्कका कामलाई श्रेढीव्यवहार भन्दछन् ।
 (२) जति सम्म अङ्क भन्दछ तसलाई पद भन्दछन् । जस्तै= ६ सम्मका अङ्कको योगगर
 यस प्रश्नमा ६ पद भयो । यहाँ ६ मा १ जोडेर $६ + १ = ७$ ले ६ को आधा ३ लाइ गुणादा
 $३ \times ७ = २१$ भयो यसमा पनी एक देखि क्रमसँग नौवम्मा अङ्क जोडदा पनि
 तैसी हुन्छ जस्तै= $१ + २ + ३ + ४ + ५ + ६ + ७ + ८ + ९ = ४५$ ॥
 २-अथवा $१-१-३-१-६-१-१०-१-१५-१-२१-१-२८-१-३६-१-४५-१-५५-१-६६-१-७८ = ३६४$
 रनलाई जोडदा पनि उत्ती ३६४ भयो ॥

Figure 4. The poetic expression for finding arithmetic sum of the numbers.

This problems can be solved by converting these prosody

style of mathematical equation into symbolic form, like as:
 Let x and y be the number of parrots in first and second group then as described in prosody we have, $x+1=2(y-1)$ or $x-2y=-3$ (i) and $x-1=y+1$ or $x-y=2$... (ii) solving (i) and (ii) we get $x=7$ and $y=5$. Hence 7 & 5 are the number of parrots. Such types of problems are found now so in comparison with before and now there is not seen more changes the verbal problems.

In above figure the first statement explains the formula for the sum of the numbers which are in arithmetic series. According to this statement the sum of the first 'n' natural numbers can be calculated by the formula $\frac{n(n+1)}{2}$, where 'n' is the number of terms. This formula can be used as, $S = \frac{n}{2} \{2a + (n-1)d\}$ equivalently write as,

$$\sum_{k=0}^{n-1} (a + kd) = \frac{n}{2} \{2a + (n-1)d\}$$

For truth of the above expression can be illustrated; if $k=0$ then first term is 'a'; if $k=1$ then the second term is $a+d$; if $k=2$ then third term is $a+2d$ similarly if $k=3$ then fourth term is $a+3d$ and their arithmetic sum is arrange like as
 $a+a+d+a+2d+a+(4-1)d = 4a+6d = 2\{2a+(4-1)d\} = \frac{4}{2} \{2a+(4-1)d\}$.

But these formulae are in verbal form in that expression. By using this formula an example is described here. The next statement explained the formula for the sum of the numbers as in the series $1+3+6+10+15+21+\dots+78$; the sum of this series can be find by using the formula, $\frac{n(n+1)}{3}$ Where 'n' is the last term; n, the number of term, that is the division of 3 to the multiple of last term to the sum of the number of term with 2. The third statement states the rule as the formula, $\frac{n\{2a+(n-1)d\}}{2}$. Thus these types of popular formulae are used in verbal way.

Similarly other examples of mathematical developments that consider as link to classical to modern by differed the method. Here that is the rule of three for calculating cube

root which differed to Bhaskaracharya view was given by Gopal Pande in his book Vyaktachandrika. It is described as below with an example of calculation of cube root of '123456789'. The following steps to be adopted by Gopal Pande for the calculation of cube roots of the given number.

- 1) Mark the points above the digits at last on right then leave two digits toward left and put points each time. Such as the first point is marked above 9, second above 6 and third above 3 as shown, 123456789
- 2) Count the number of points marked above the digits which gives the number of digits of the cube roots with leaving the digits for decimal place of the cube roots. Here the number of digits in the cube root is 3, since there are 3 points.
- 3) Assume that the number bears the first point from left (or last point from the right) as a single number and subtract the possible cube of a number from it. Here the first point from left bears up to 123, so we should subtract the cube of 4 that is 64 and take down the digits up to 2nd point and note that 4 is the first digit from the left of the cube root.

$$\begin{array}{r} 123456789(4 \\ -64 \\ \hline 59456 \end{array}$$

- 4) From (3) we get 59456 then we should multiply 59456 by ten and divide it by the number as the multiple of 1000 in the relation of the difference of cube of (first digit of cube root +1) and the cube of the first digit of the cube root. Here this two numbers are 5^3 and 4^3 and their differences as explained above is $125 - 64 = 61$.
- 5) Then from (4) we have,

$$61000)594560(9$$

Then we get 2nd digit of the cube root i.e. 9. Thus the number is 49. Again processed as (4) we get, $50^3 - 49^3 = 7351$ and $45^3 = 117649$

- 6) Again subtract cube of 49 from 123456789 and take down the digit up to 3rd point as follow;

$$\begin{array}{r} 123456789 \\ -117649 \\ \hline 5807789 \end{array}$$

- 7) Divided by 7351000 to (5807789×10) as mentioned above, i.e.,

$$\begin{array}{r} 7351000)58077890(7 \\ -51457000 \\ \hline 6620890 \end{array}$$

- 8) $(497)^3 = 122763473$ then

$$\begin{array}{r} 123456789 \\ -122763473 \\ \hline 693316 \end{array}$$

Hence digits of cube root before decimal place are 497. This completes the digits of cube roots before decimal place. Next for after decimal place we should introduce three zeros to the remainder obtained in above and proceed as (4) and (5). It is note that the difference of cube mentioned as 4 can be obtained as.

$$(498)^3 - (497)^3 = 3 \{ (497)^2 + 497 \} + 1 = 742519$$

- 9) 742519000)6933160000(9

Thus the cube root of 123456789 = 497.9 [1]

The following figures explain the rule of calculation of cube root. For evidences they are shown as picture.

Figure 5. A page of the book Vyaktachandrika of Gopal Pande & application of rule of three in cube root.

Here is the question; how much our learning paradigm shift in content base, algorithm base and problem solving style? Even we are saying our mathematics is changed. In comparison with content, algorithm and problem solving style cannot see changes with assimilating the learners understanding. So linking with historical base & follow up

$$\text{Or } f(x) = \frac{1}{2x}$$

$$= \frac{1}{2x} \Rightarrow x = \pm \frac{1}{2}.$$

Therefore $f(x) = \frac{1}{2}$ at $x=1$ and $f(x) = -\frac{1}{2}$ at $x=-1$. Hence turning points are $(1, \frac{1}{2})$ and $(-1, -\frac{1}{2})$

2.3. Derivatives and Factorization

The following relation gives the formula quadratic equations.

First order derivative = sum of the factors Example: $x^2 + 3x + 2$

The factors are $(x+1)$ and $(x+2)$; derivatives of the expression is, $f'(x) = 2x+3$.

Therefore $f'(x) = (x+1) + (x+2) = \Sigma A$, where ΣA is the sum of the factors.

2.4. Formula for Finding the Factors Cubic Expression with the Help of Derivative and Vice Versa

Formula $f'(x) = \Sigma A.B$, where $\Sigma A.B$ is the sum of the all pair of the factors.

Example: factorize the expression $x^3 + 6x^2 + 11x + 6$

Therefore $f'(x) = (x+1)(x+2) + (x+1)(x+3) + (x+2)(x+3) = \Sigma A.B$

That is $f'(x) = 3x^2 + 12x + 11$.

$f''(x) = 6x + 12 = 2!(3x+6) = 2!(x+1+x+2+x+3) = 2! \Sigma A$, where ΣA is the sum of the factors

$f'''(x) = 3!$. Similarly for bi - quadratic expression

$F'(x) = \Sigma ABC$, $f''(x) = 2! \Sigma AB$, $f'''(x) = 3! \Sigma A$, $f^{iv}(x) = 4!$, etc

2.5. Successive Derivatives of Product of Algebraic and Exponential Functions

Formula: vertically crosswise

Example: Differentiate, $f(x) = x^5 e^{2x}$

$$\Rightarrow f'(x) = 2 e^{2x} x^5 + 5 e^{2x} x^4$$

2.6. Formula for 2nd Order: Repeat the Vertically and Crosswise in the 2nd Column Keeping First Column Fixed

$$\Rightarrow f''(x) = 4e^{2x}x^5 + 2 \cdot 10 e^{2x}x^4 + 20e^{2x}x^3$$

2.7. Formula for 3rd Order: Repeating the Above Process

$$f'''(x) = (8e^{2x})x^5 + 3(20e^{2x})x^4 + 3(40e^{2x})x^3 + (60e^{2x})x^2$$

Note: 1. Binomial expansion (Pascal's Triangle) is used here.

Note: 2. The above process is equivalent to Leibnitz theorem for successive derivatives.

2.8. Special Cases for Integration By Parts

Formula 1: One more than the one before,

$$\int x^7 dx = \frac{1}{8} x^8 + C, \text{ where } C \text{ is the constant of integration.}$$

Formula 2: Vertically successive derivative of algebraic function and successive integration of trigonometric function then used vertically and crosswise multiplication, the sign is adopted as +, -, + ... alternately.

Example 1: $\int x^2 \sin x$

Now,

or

In the 2nd way last term remains to be integrating.
 $\int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$, where C is the constant of integration.

Note: This can be done by the process of integration by parts. But the process expanding here is very short and easy. This process can be done until the derivative becomes zero.

Example 2: $\int x^4 \sin x dx$

Similarly proceed for the above expression as,
 $\int x^4 \sin x dx = x^4 \cos x + 4x^3 \sin x + 12x^2 \cos x - 24x \sin x - 24 \cos x + C$, where C is the constant of integration.

Example 3: $\int x^3 \log x dx$ (Algebraic and logarithmic functions)

Now,

$$\begin{array}{ccc} \log x & \searrow & x^4 \\ 1/x & \longrightarrow & x^5/5 \end{array}$$

$$\int x^3 \log x \, dx = \left(\frac{1}{5} x^5\right) \log x - \int \left(\frac{1}{5x}\right) x^5 \, dx = \left(\frac{1}{5} x^5\right) \log x - \frac{1}{25} x^5$$

$dx + C$, where C is the constant of integration.

Example 4: $\int e^x \sin x \, dx$ (Exponential and trigonometric functions)

$$I = \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x + \int e^x (-\sin x) \, dx$$

$$= -e^x \cos x + e^x \sin x - I$$

$$\therefore (1/2)e^x (-\cos x + \sin x) + C = \frac{1}{2} e^x (\sin x - \cos x) + C$$

$$\begin{array}{ccc} e^x & \searrow & \sin x \\ e^x & \searrow & -\cos x \\ e^x & \longrightarrow & -\sin x \end{array}$$

Example 5: $\int e^{ax} \sin bx \, dx$

$$\begin{array}{ccc} e^{ax} & \searrow & \sin bx \\ e^{ax} & \searrow & -\frac{1}{b} \cos bx \\ e^{ax} & \longrightarrow & \sin bx/b^2 \end{array}$$

$$I = (e^{ax}/b) \cos bx + (a/b^2)e^{ax} \sin bx - (a^2/b^2) \int e^{ax} \sin bx \, dx$$

$$\therefore I = e^{ax} (a \sin bx - b \cos bx) / 2(a^2 + b^2) + C \quad [3]$$

3. Conclusions

Evidences show that before 100 years and now the contents are not significantly changed but algorithm and problem solving style is change. Also there was a very close way of thinking and value in mathematics. But later the major differences in their thinking of mathematics are only elaboration rather than the assimilation. But in algorithm and problem solving style the learner cannot assimilate for real understanding. Thus reformation of content, algorithm and process of problem solving need to be change with incorporating our classical mathematical developments.

I hope that there is beginning to see the paradigm shift that needs to happen. We need a major shift in societal expectation to include both language and mathematics in those preschool years. We need make mathematical culture for mathematics what they have always done for language and practical uses. That foundation in mathematics is every bit as important as it is for language. Our children need to be

constantly reinforced and encouraged and praised for their persistence for mathematics as much as for language with evident the context in historical development. And maybe most important is that our children need a self concept that says "I am smart enough and I can learn this." For this we relate mathematics from early stage to now. They must never doubt their own ability to learn with relating to life and developmental facts. Hence in conclusion everyone in society needs to understand that they are an essential part of creating the foundation for a successful future in mathematics for our pre - school children by the linkage of classical to modern mathematics.

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