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Solution of Lagrange's Linear Differential Equation Using Matlab

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Abstract: MATLAB, which stands for Matrix Laboratory, is a software package developed by Math Works, Inc. to facilitate numerical computations as well as some symbolic manipulation. It strikes us as being slightly more difficult to begin working with it than such packages as Maple, Mathematica, and Macsyma, though once you get comfortable with it, it offers greater flexibility. The main point of using it is that it is currently the package you will most likely found yourself working with if you get a job in engineering or industrial mathematics. So we found that the Matlab method in differential equations is very important and useful mathematical tools which help us to solve and plot differential equations. The aims of this paper is to solve Lagrange's Linear differential equations and compare between manual and Matlab solution such that the Matlab solution is one of the most famous mathematical programs in solving mathematical problems. We followed the applied mathematical method using Matlab and we compared between the two solutions whence accuracy and speed. Also we explained that the solution of Matlab is more accuracy and speed than the manual solution which proves the aptitude the usage of Matlab in different mathematical methods.

Keywords: Solution, Lagrange, Linear Differential Equation, Matlab

1. Introduction

The derivative dy/dx of a function $y = \emptyset(x)$ is itself another function $\emptyset'(x)$ found by an appropriative rule. The function $y = e^{0.1x^2}$ is differentiable on the interval $(-\infty, \infty)$, and the chain rule its derivative is $dy/dx = 0.2\,xe^{0.1x^2}$if we replace $e^{0.1x^2}$ on the right- hand side of the last eqution by the symble$y$, the derivative becomes.

$$\frac{dy}{dx} = 0.2\,xy \quad (1)$$

Now imagine that a friend of yours simply hands you equation (1) you have no idea how it was constructed - and asks, what is the function represented by the symbol y? You are now face with one of the basic problems in the research. How do you solve such an equation for the unknown function $y = \emptyset(x)$. [1]

MATLAB, which stands for Matrix Laboratory, is a software package developed by Math Works, Inc. to facilitate numerical computations as well as some symbolic manipulation. It strikes me as being slightly more difficult to begin working with than such packages as Maple, Mathematica, and Macsyma, though once you get comfortable with it, it offers greater flexibility. The main point of using it in M442 is that it is currently the package you will most likely find yourself working with if you get a job in foe example engineering. [13]

2. Differential Equations

Definition (2.1): A differential equation involving ordinary derivatives of one or more dependent variable with respect to a single independent variable is called an ordinary differential equation. [12]

Definition (2.2): A linear ordinary differential equation of order n, in dependent variable y and the independent variable x, is an equation which is in, or can be expressed in, the form

$$a(x)\frac{d^n}{dx^n} + a1(x) + \frac{d^{n-1}}{dx^{n-1}} + \cdots +$$

$$an - 1(x)\frac{dy}{dx} + an(x)y = b(x)$$

Where a_o is not identically zero. [4]

3. Lagrange's Linear Equation

The partial differential equation of the Pp + Qq=R where P, Q, R are functions of *x, y, z* is the standard from the linear partial differential equation of the order one and it called Lagrange's linear Equation. [2]

3.1. Solution of Lagrange's Linear Equation

In by eliminating an arbitrary function f from

$$f(u, v) = 0 \tag{2}$$

Connecting two functions u and v we get the partial differential equation

$$Pp + Qq = R \tag{3}$$

Where

$$P = \frac{\partial u}{\partial y}.\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}.\frac{\partial u}{\partial z}$$

$$Q = \frac{\partial v}{\partial x}.\frac{\partial u}{\partial z} - \frac{\partial u}{\partial x}.\frac{\partial v}{\partial z}$$

And

$$R = \frac{\partial u}{\partial x}.\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}.\frac{\partial v}{\partial x}$$

Thus, equation (2) is the general of (3) and so we have to find the values of *u* and *v*. Let $u = a$ and $v = b$ be two equationswhere a and b are arbitrary constants. Differentiating them, we have

$$\frac{\partial u}{\partial x}.dx + \frac{\partial u}{\partial y}.dy + \frac{\partial u}{\partial z}.dz = 0$$

And$\frac{\partial v}{\partial x}.dx + \frac{\partial v}{\partial y}.dy + \frac{\partial v}{\partial z}.dz = 0$

Solving these, we have

$$\frac{dx}{\frac{\partial u}{\partial y}.\frac{\partial v}{\partial z} - \frac{\partial v}{\partial y}.\frac{\partial u}{\partial z}} = \frac{dy}{\frac{\partial u}{\partial z}.\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}.\frac{\partial v}{\partial z}} = \frac{dz}{\frac{\partial u}{\partial x}.\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}.\frac{\partial v}{\partial x}}$$

i.e.

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \tag{4}$$

Solution of the (4) differential equation are $u = a$ and$v = b$

Thus the solution of the given equation (2) it found and its $f(u, v) = 0$

Note that equations given by (4) are called Lagrange's auxiliary equations or subsidiary equations. Working method: for the solution of the partial differential equation.

$$Pp + Qq = R$$

From the auxiliary equations$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

Find two independent integrals of auxiliary equation say $u = a$ And $v = b$. Then the general integral of the equation is given by $f(u, v) = 0$when f is an arbitrary function. [3]

3.2. Geometrical Interpret of Lagrange's Linear Equations

Lagrange's linear equation is

$$Pp + Qq = R \tag{5}$$

Which can be written as?

$$Pp + Qq = R(-1) = 0$$

We know that the *d.c ′s* of the normal at point on the surface $f(x, y, z) = 0$ are proportional to

$$\frac{\partial f}{\partial x}:\frac{\partial f}{\partial y}:\frac{\partial f}{\partial z}$$

$$\Longrightarrow -\frac{\partial f}{\partial x}\left[\frac{\partial f}{\partial z}: -\frac{\partial f}{\partial y}\right]\frac{\partial f}{\partial z} = -1$$

$$\Longrightarrow \frac{\partial z}{\partial x}:\frac{\partial z}{\partial y} = -1 \Longrightarrow p:q = -1$$

Hence, the geometrical interpretation of (5) is that they normal to ascertain surface is perpendicular to a line whose direction cosines are in the ratio $P:Q:R$.

We saw that the simultaneous equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \tag{6}$$

Represented a family of curves that the tangent at point had direction cosine in the ratio $P:Q:R$

And that $f(u, v) = 0$ represented a surface through such curves, where $U = cons$. And $v = cons$Are two particular integrals of (6) through every point of such a surface passes curve of the family, lying wholly on the surface. Hence, the normal to this surface at any point must be perpendicular to the tangent to this curve, *I. e,* perpendicular to a line whose *d c′s* are proportional to $P:Q:R$ this is just what is required by the partial differential equation.

Thus, equation (5) and (6) define the same of surface and are thus equivalent. [5]

Example (3.2.1): Solve$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

Solution:

The subsidiary equations are

$$\frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{xy + zx} = \frac{dz}{xy - zx}$$

Taking $x, y. z$ as multiples, we have
Each fraction=$\frac{xdx+ydy+zdz}{0}$

$$\therefore xdx + ydy + zdz = 0$$

Integrating, $x^2 + y^2 + z^2 = c_1$
Again, taking the last two members, we have

$$\frac{dy}{y+z} = \frac{dz}{y-z}$$

$$=\gg (y-z)dy = (y+z)dz$$

$$=\gg ydy - (zdy + ydz) - zdz = 0$$

Integrating, we have

$$y^2 - 2yz - z^2 = c2$$

∴The general solution is

$$f(x^2 + y^2 + z^2, y^2 - 2yz - z^2) = 0. \text{ [7]}$$

4. Multiple Method of Differential Equation

We start discussing one of the more common methods for solving basic partial differential equations. The method of separation of variables cannot always be used and even when it can be used it will not always be possible to get much past the first step in the method. However, it can be used to easily solve the *1-D* heat equation with no sources, the *1-D* wave equation, and the *2-D* version of Laplace's equation, $\nabla^2 u = 0$.

In order to use the method of Separation of variables we must be working with a linear homogenous partial differential equation with linear homogenous boundary conditions at this point we're not going to worry about the initial condition (s).

Because the solution that we initial get will rarely satisfy the initial condition (s). As we'll see however there are ways to generate a solution that will satisfy initial condition (s) provided they meet same fairly simple requirements. [8]

The method of Separation of variables relies upon the assumption that a function of the form,

$$u(x, y) = \emptyset(x)G(t) \tag{7}$$

Will be a solution to a linear homogenous partial differential equation in *x* and *t*. this is called a product solution and provided the boundary conditions are also linear and homogenous this will also satisfy the boundary conditions. However, as noted. Above this will only rarely satisfy the initial condition. [10]

After all there really isn't. As we'll see it works because it will reduce our partial differential equation down to two ordinary differential equation and provided we can solve those then we're in business and the method will allow us to get a solution to the partial differential equation. So, let's do a couple of examples to see how this method will reduce a partial differential equation down to two ordinary differential equations. [11]

Example (4.1.1): Use Separation of variables on the following partial differential equation.

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x)\frac{\partial u}{\partial x}(0,t) = 0 \ \frac{\partial u}{\partial x}(l,t) = 0$$

Solutions:

In this case we're looking at the heat equation with no sources and perfectly insulated boundaries. So, we'll start off by again assuming that our product solution will have the form,

$$u(x,t) = \emptyset(x)G(t)$$

And because the differential equation itself hasn't changed here we will get the same result from plugging this in as we did in the previous example so the two

Ordinary differential equations that we'll need to solve are,

$$\frac{dG}{dt} = -k\lambda G \frac{d^2\emptyset}{dx^2} = -\lambda\emptyset$$

Now, the point of example was really to deal with the boundary conditions so let's plug the product solution into them to get,

$$\frac{\partial\big(G(t).\emptyset(x)\big)}{\partial x}(0,t) = 0 \ \frac{\partial\big(G(t).\emptyset(x)\big)}{\partial x}(l,t) = 0$$

$$G(t)\frac{d\emptyset}{dx}(0) = 0 \ G(t)\frac{d\emptyset}{dx}(l) = 0$$

Now, just as with the first example if we want to avoid the trivial solution and so we can't have $G(t)=0$for every t and so we must have,

$$\frac{d\emptyset}{dx}(0) = 0 \ \frac{d\emptyset}{dx}(l) = 0$$

Here is a summary of what we get by applying separation of variables to this problem

$$\frac{dG}{dt} = -k\lambda G \frac{d^2\emptyset}{dx^2} = -\lambda\emptyset = 0$$

$$\frac{d\emptyset}{dx}(0) = 0 \ \frac{d\emptyset}{dx}(l) = 0$$

Next, let's see what we get if use periodic boundary conditions with the heat equation. [12]

Example (4.1.2): Use Separation of variables on the following partial differential equation.

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x)u(-l,t) = u(l,t)\frac{\partial u}{\partial x}(-l,t) = \frac{\partial u}{\partial x}(l,t)$$

Solutions:

First note that these boundary conditions really are homogenous boundary conditions. If we write them as,

$$u(-l,t)-u(l,t)\frac{\partial u}{\partial x}(-l,t)-\frac{\partial u}{\partial x}(l,t)$$

It's a little easier to see.

Now, again we've done this partial differential equation so we'll start off with,

$$u(x,t)=\emptyset(x)G(t)$$

And the two ordinary differential equations that we'll need to solve are,

$$\frac{dG}{dt}=-k\lambda G\frac{d^2\emptyset}{dx^2}=-\lambda\emptyset$$

Plugging the product solution into the re written boundary conditions gives,

$$G(t)\emptyset(-l)-G(t)\emptyset(l)=G(t)[\emptyset(-l)-\emptyset(l)]=0$$

$$G(t)\frac{d\emptyset}{dx}(-l)-G(t)\frac{d\emptyset}{dx}(l)=G(t)\left[\frac{d\emptyset}{dx}(-l)-\frac{d\emptyset}{dx}(l)\right]=0$$

And we can see that we'll only get non-trivial solution if,

$$\emptyset(-l)-\emptyset(l)=0\ \frac{d\emptyset}{dx}(-l)-\frac{d\emptyset}{dx}(l)=0$$

$$\emptyset(-l)=-\emptyset(l)\frac{d\emptyset}{dx}(-l)=\frac{d\emptyset}{dx}(l)$$

So, here is what we get by applying Separationof variables of variables to this problem.

$$\frac{dG}{dt}=-k\lambda G\frac{d^2\emptyset}{dx^2}+\lambda\,\emptyset=0$$

$$\emptyset(-l)=-\emptyset(l)\tfrac{d\emptyset}{dx}(-l)=\tfrac{d\emptyset}{dx}(l).\ [9]$$

5. Matlab Program

5.1. Partial Differential Equations in One Spatial Dimension

We will use MATLAB unction pdepe to solve initial-boundary value problems for parabolic and elliptic PDEs in one spatial dimension more precisely, pdepe solves PDEs of the form:

$$c(x,t,u,u_x,)\mathrm{ut}=x^{-m}\big(x^m f(x,t,u,u_x)\big)+s(x,t,u,u_x),\ (x_L,t)\in(x_i,x_\tau)\times(t_\sigma,\mathrm{tf}) \tag{8}$$

With initial- boundary conditions

$$u(x,t_o)=uo(x),\qquad x_1\le x\le x_\tau,$$

$$p(x,t,u)+q(x,t)f(x,t,u,u_x)=0, fort_o\le t\le tf$$

And

$$x=x_L,x_\tau \tag{9}$$

Specifying constant m and functions c,f,s,p,q, we can solve various types of PDEs as we will use see. [12]

5.2. Single Parabolic PDEs

Consider the heat equation

$$\mathrm{ut}=u_{xx},\ (x,t)\in(0,1)\times(0,1),$$

$$u(x,0)=x^2,\qquad 0\le x\le 1,$$

$$u(0,x)=0,0\le t\le 1,$$

$$u(1,t)=1,0\le t\le 1$$

To solve this initial-boundary value problem, we need following steps:set the constant m in (4.4). Usuall y*m=0*, but other values of m can be useful. For example, if the solution u is radially symmetric in R^d, the Laplace operator in spherical coordinates is

$$\Delta u=r^{-m}(r^m u_r)\mathrm{r},\ \mathrm{m{=}d{-}1}.$$

In our examples, *m=0.*

$$\gg m=0,$$

1) Define your *PDE* by specifying the functions c,f,s in (8).

In our examples, we have

$$c(x,t,u,u_x)=1, f(x,t,u,u_x)=u_x, s(x,t,u,u_x)=0.$$

Hence a M-File function describing the above is %heateqn. m: defines the heat equation for "pdepe"
Function $[c,f,s]$=heateqn $(x,t,u,dudx)$

$$c=1;$$

$$f=dudx;$$

$$s=0;$$

$$end$$

2) Define the initial condition u_x in (9).

In our example, $u_x(x)=x^2$ so that a M-File function describing it is% icfun1.m: defines an initial condition for "pdepe"

Function u_o=icfun1 (x)

$$u_o=x\wedge 2;$$

$$end$$

3) 4-Define the boundary conditions by specifying the function p,q in (9). Not that the Aux function f isalready defined.

In our example, we have.

$$p(o,t,u)=u,\qquad p(1,t,u)=u-1,$$

$$q(0,t) = q(1,t) = 0.$$

Hence a MATLAB function describing the boundary conditions is % bcfun 1: m defines a boundary conditions for "pdepe"

Function$[p_1, q_1, p_r, q_r]$=bcfun1(x_1, u_1, x_r, u_r, t)

$$p_1 = u_1;$$

$$q_1 = 0;$$

$$p_r = u_r - 1;$$

$$q_r = 0,$$

$$end$$

Not that $p1, q1$ and$u1$ stand for$p(x_L, t, u)$ and $u(x_L, t)$ Respectively. Other variable are defined similarly.

4) Define space mesh and time interval.

$\gg x$ =Linspace$(0,1,20)$;

$\gg t$ =Linspace $(0,1,10)$;

5) Finally use the function pdepe to solve the PDE.

u =pdepe $(m, @\ he$ateqn, @ icfun1, @ bcfun1, $x, t)$;

The u is a $10 - b - 20$matrix; each row of u represents a solution at a specific time.

6) to observe evolution of the solution, we can use the function surf.

≫surf (x, t, u)

To plot the solution profile at a specific time, we need access arrow of u. For example, we can

≫plot $(x, u(end, :))$[6]

5.3. Solution of Lagrange's Linear Equation by Matlab

Example (5.3.1) Solve$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

Solution:

function f=Lagran

symsxyzdxdydz

f1=x*dx+y*dy+z*dz

C1=int(x)+int(y)+int(z)

f2=(y*dy-(z*dz+y*dy)-z*dz)

C2=int(y)-int(z)-int(y)-int(z)

$c=c_1+c_2$

Result

>>lagran

f1=dx*x + dy*y + dz*z

C1=x^2/2 + y^2/2 + z^2/2

f2=-2*dz*z

C2=-z^2

C=x^2/2 + y^2/2 - z^2/2

6. Compare Between Manuals Solutions and Matlab Solutions

After implementing all the steps described in this paper, we evaluated the Matlab solutions and the final results were discussed. Results obtained from the Matlab solutions were compared with the results obtained by manuals solutions. We made it clearer that the Matlab solutions results which presents it noticed that there was the difference between the results of the Manuals solutions and Matlab solutions.

7. Results

We found the following results: The paper explained that there was slight difference between manual and Matlab solutions in solving Lagrange's Linear differential equations using Matlab due to accuracy in Matlab. Also possibility of solving Lagrange's linear differential equations using Matlab solution is more speed and accurate than the manual solution.

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