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Change in internal energy & enthalpy of the black holes

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Abstract: The present paper derives an expression for the change in internal energy and enthalpy of the black holes using first law of thermodynamics and showing that the change in internal energy and enthalpy are the manifestations of same thing at constant pressure and volume. The paper also shows that the spinning black holes with spin $a_* \rightarrow 1$ do not radiate energy and mass of the black hole becomes infinity for zero surface gravity.

Keywords: Internal Energy, Enthalpy, Entropy and Black Holes

1. Introduction

Classically, the black holes are perfect absorbers and do not emit anything; their temperature is absolute zero. However, in quantum theory black holes emit Hawking radiation with a perfect thermal spectrum. This allows a consistent interpretation of the laws of black hole mechanics as physically corresponding to the ordinary laws of thermodynamics (Wald 2001).

In 2010, Kanak et al. have derived the formula for the energy of non-spinning black holes (E_{BH}) in terms of the radius of the event horizon (Kanak et al.2010). In 2011, Dipo Mahto et al. have derived the formula for the energy of spinning black holes (E'_{BH}) in terms of the radius of the event horizon (Mahto et al.2011). In the same year, Dipo Mahto et al. derived also an expression for the change in energy and entropy of Non-spinning black holes taking account the first law of the black hole mechanics relating the change in mass M, angular momentum J, horizon area A and charge Q, of a stationary black hole with Einstein's mass-energy equivalence relation (Mahto et al. 2011).

The present paper derives an expression for the change in the internal energy and enthalpy of the black holes using first law of black holes which shows that the change in internal energy and enthalpy are the manifestations of same thing at constant pressure and volume. The paper also shows that the spinning black holes with spin $a_* \rightarrow 1$ do not radiate energy and mass of the black hole becomes infinity for zero surface gravity.

2. Theoretical Discussion

There are systems which may show no apparent mechanical energy, but may still be capable of doing work. They are said to possess internal energy. If an amount of heat δQ be added to a thermodynamical system, say a perfect gas, which expands by a volume dV against the pressure P, then from first law of thermodynamics (Saha & Shrivastava,1969).

$$\delta Q = dU + PdV \tag{1}$$

Using equation (1), we have

$$dU = \delta Q - PdV \tag{2}$$

Define a quantity called entropy S as follows: if we add that heat δQ to a body at the temperature T , then change in entropy δS is given by (Dabholkar 2005).

$$\delta S = \frac{\delta Q}{T} \tag{3}$$

For a reversible process the total change in entropy is zero ($\delta S = 0$), but for an irreversible process entropy change always increases ($\delta S > 0$) which leads to the second law of thermodynamics (Mahto et al.2012). The equation (3) can be written as:

$$\delta Q = T\delta S \quad (4)$$

Putting the above equation into the equation (2), we have

$$dU = T\delta S - PdV \quad (5)$$

It may be supposed that the volume of the black hole remains the same when it receives some amount of energy δQ and hence

$$dV = 0 \quad (6)$$

From equation (5), we have

$$dU = TdS \quad (7)$$

The enthalpy or Heat content (H) of a homogenous system is defined by the following equation

$$H = U + PV \quad (8)$$

Now the above equation is differentiated, we have

$$dH = dU + VdP + PdV \quad (9)$$

Putting equation (5) in to above equation, we have

$$dH = TdS - PdV + VdP + PdV$$

$$dH = TdS + PdV \quad (10)$$

Hence

$$dH = TdS \quad (11)$$

From equation (7) and (11), it is clear that the internal energy and enthalpy are the manifestations of same thing at constant pressure and volume. The term $\kappa / 2\pi$ truly is the physical temperature of a black hole, not merely a quantity playing a role mathematically analogous to temperature in the laws of black hole mechanics (Wald 2001, Mahto et al. 2011).

i.e.

$$T = \frac{\kappa}{2\pi} \quad (12)$$

Using (12) into (7) and (11), we have

$$dH = dU = \frac{\kappa}{2\pi} dS \quad (13)$$

The entropy of the black hole is related to the area of event horizon given by the following equation (Wald,2001 & Mahto et al. 2011).

$$dS = dA/4 \quad (14)$$

The equation (13) becomes by the use of above equation as:

$$dH = dU = \frac{\kappa}{8\pi} dA \quad (15)$$

The above equation shows the change in internal energy with corresponding change in the area of the event horizon of black holes. For the spherically, symmetric and stationary, or Schwarzschild black hole's mass M, the horizon's radius is $R_{bh} = 2GM/c^2$ and its area is naturally given by the following relation (Bekenstein 2008, Mahto et al. 2011 &2012).

$$A = 4\pi R_{bh}^2 \quad (16)$$

Putting equation (16) into the equation(15) and solving, we have

$$dH = dU = \kappa R_{bh} dR_{bh} \quad (17)$$

The above equation shows the change in internal energy with corresponding change in the radius of the event horizon of black holes.

In the case of spinning black holes, the change in internal energy will have different values than that of non-spinning black holes, because the surface gravity(κ) in this case is given by the Kerr solution(Bardeen et al 1973).

$$\kappa = \frac{\left(M^4 - J_H^2\right)^{1/2}}{2M\left\{M^2 + \left(M^4 - J_H^2\right)^{1/2}\right\}} \quad (18)$$

where

$$J_H = a_* M^2 \quad (19)$$

The radius is smaller in the case of spinning BH, tending to GM/c^2 as $a_* \rightarrow 1$ (Narayan 2005), then eqn(19) yields as

$$J_H = M^2 \quad (20)$$

and hence ,we have from equation (18)

$$\kappa = \frac{\left(M^4 - M^4\right)^{1/2}}{2M\left\{M^2 + \left(M^4 - M^4\right)^{1/2}\right\}} \quad (21)$$

$$\kappa = 0 \quad (22)$$

Hence from (17), we have

$$dH = dU = 0 \quad (23)$$

This means that there is no change in internal energy and enthalpy, when the black hole has maximum spin ($a_* \rightarrow 1$) and with the help of equation (13), we have and

$$dS = 0 \quad (24)$$

With

$$T=0 \quad (25)$$

Hence it may be concluded that there is no change in internal energy and entropy at zero temperature in the case of spinning black holes having spin $a_* \to 1$. This also suggests that the spinning black holes having spin $a_* \to 1$ should not radiate energy which corresponds classical treatment of black holes.

For the Schwarzschild case of black holes,the surface gravity (κ) is given by(Transchen 2000 &Mahto et al. 2011).

$$\kappa = \frac{1}{4M} \quad (26)$$

With the help of equation(22), the above equation becomes as follows:

$$M = \infty \quad (27)$$

This means that the mass of the black hole becomes infinity for zero surface gravity. The existence of infinite mass of the black hole has vital role for the evolution of universe with Big-Bang theory.

3. Results and Discussion

In the present work, we have derived the formula for the change in internal energy and enthalpy of non-spinning black holes with corresponding change in the

(i) entropy by the equation(13)
(ii) horizon area by the equation(15)
(iii) radius of event horizon by the equation(17)

The surface gravity of the black hole has vital role to characterize the black holes, because it is directly related to the mass and spin of the black holes by equation (18). The radius is smaller in the case of spinning BH, tending to GM/c^2 as $a_* \to 1$. For this case the surface gravity reduces to zero leading the infinite mass of the black holes. This makes no change in internal energy, entropy and mass of spinning black holes at zero temperature and it may be concluded that the spinning black holes with spin $a_* \to 1$ do not radiate energy and justify the classical black holes.

4. Conclusion

In course of the present research work,, we can draw the following conclusions such as:

(i) The spinning black holes with spin $a_* \to 1$ do not radiate energy.
(ii) There is no change in internal energy and entropy in the case of spinning black holes having spin equals to $a_* \to 1$.
(iii) The mass of the spinning black hole becomes infinity for zero surface gravity.

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