Optimal Design and Modelling of an Innovated Structure of DC Current Motor with Concentrated Winding

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Abstract: In this paper we presented the structure and methodology of designing of an innovated DC motor with permanent magnets and axial flux. Progress in the field of sliding contacts manufacturing, the simplicity of the structure of the engine as the control simplicity of DC motors make this structure an attractive solution to the problem of electric cars drive. In this context, a dimensioning model of this engine structure is developed. This model is based on the analytical design method of electric actuators. The overall design approach is based on justified simplifying assumptions, leading to a simplification of the resolution of the sizing problem. Finally, this paper provides a comprehensive tool for sizing and modeling of this type of actuator.

Keywords: DC Motor, Permanent Magnets, Design, Analytical Method, Modelling, Optimization

1. Introduction

DC motors are the first engines used in industrial applications. These engines have many advantages including:

- Simplicity of the structure.
- Variable excitation for engines with wound inductor.
- Simple and easy control.
- By consequence, these engines have been somewhat neglected in the near past for they present drawbacks, namely:
  - Significant induced magnetic reaction making it impossible to overcome current.
  - Cost of maintenance of sliding contact.
  - Significant copper losses in the inductor.

Nowadays permanent magnet motors have taken the relief to motors with wound inductor. For this reason we are led to seek solutions combining the advantages of DC motors with wound inductor and those with permanent magnets particularly in light of interesting advances in the field of sliding contacts manufacturing. In this context, a spindle motor structure with permanent magnets and simple to perform combining the advantages of structures with wound inductor and those of permanent magnets is sought. A design and modeling program based on analytical method of this structure is developed and presented in this paper [1-7].

2. Motor Structure

An engine innovated structure with DC permanent magnet axial flux to one pole pair is illustrated in figure 1 and another with two pairs of poles is shown in figure 2. These two structures are simple to manufacture, compact and with concentrated winding. They have the following advantages:

- High power density.
- Low manufacturing cost.
- High efficiency (Absence of copper losses to the inductor).
- Modular design: possibility of overcoming power by adding additional modules perpendicular to the axis of the motor.
- Ease of control.
- Possibility of automation of the manufacturing process of these motor structures, especially the coils are concentrated type which facilitates the procedure for their insertions in a single block.
- The slots are straight and semi open.
3. Modelling and Sizing of the Motor

Electric motors sizing problem is usually solved by the finite element method [1-5]. A series of simulations is necessary in this case to solve the design problem. This method is accurate, but it is heavy and therefore it is not compatible with optimizations approaches. However, the analytical method provides solutions quickly and without iterations and provides a comprehensive design tool for electrical machines since it is based on simplifying assumptions justified. This method leads to design programs highly parameterized of electrical devices. So our choice is focused on the analytical method to solve the design problem of the studied engine structures [1-6].

3.1. Modeling of the Back Electromotive Force

Elementary back electromotive forces in the terminals of each two diametrically opposite coils are illustrated in figure 3. All rotor coils should be connected in series with a reversal of the direction of coil so as to have a continuous resulting electromotive force (Figure 3) [5].

Flux received by a coil is expressed by the following relationship [5]:

$$\Phi = \int_{\theta} B_e (\theta) \times ds(\theta)$$

Relation (1) can be converted to the following equation [5]:

$$\Phi_b = B_e \times s_d - 2 \times B_e \times \left( \frac{D_e^2 - D_i^2}{4} \right) \times \frac{\theta}{2}$$

Where Be is the flux density in the air-gap, $\theta$ is the mechanical angle, $s_d$ is the heads teeth section, $D_e$ and $D_i$ are respectively the internal and external diameters of the motor and $ds$ is the surface element through which the magnetic flux.

The back electromotive force can be derived from the following relationship:

$$E = -2 \times P \times N_{sb} \times \frac{d\Phi_b}{dt}$$

Where $P$ is pole pair number and $N_{sb}$ is the number of turns per coil.

The expression of the induced back electromotive force takes the following form:

$$E = P \times N_{sb} \times \left( \frac{D_e^2 - D_i^2}{2} \right) \times B_e \times \Omega$$

where $\Omega$ the angular speed of the motor.

This leads to the general expression of back electromotive force:

$$E = K_e \times \Omega$$
3.2. Sizing of the Motor

The rotor slot width is given by the following relationship:

$$L_{encr} = \frac{D - \Delta}{2} \times \sin \left( \frac{1}{2} \times \left( \frac{2 \times \pi}{N_b} - A_{dentrim} - A_{dentrim} \right) \right)$$  \hspace{1cm} (6)

where $A_{dentrim}$ is the average angular width of the rotor tooth, $A_{dentrim}$ is the average angular width of the rotor inserted tooth and $N_b$ number of coils.

The lower angular width of a slot is expressed by the following relationship:

$$A_{encr1} = 2 \times A \times \sin \left( \frac{L_{encr}}{D} \right)$$ \hspace{1cm} (7)

The upper angular width of a slot is expressed by the following relationship:

$$A_{encr2} = 2 \times A \times \sin \left( \frac{L_{encr}}{D} \right)$$ \hspace{1cm} (8)

The average angular width of a rotor tooth is given by the following relationship:

$$A_{dentrim} = \frac{2 \times \pi \times \alpha}{N_b}$$ \hspace{1cm} (9)

Where $\alpha$ is the opening ratio of a rotor tooth ($\alpha < 1$).

The average angular width of an interposed rotor tooth is given by the following relationship:

$$A_{dentrim} = r_{dit} \times A_{dentrim}$$ \hspace{1cm} (10)

Where $r_{dit}$ is the ratio between the average angular width of an inserted tooth and that of a tooth.

The average angular width of a slot is given by the following relationship:

$$A_{encr} = \frac{1}{2} \times \left( \frac{2 \times \pi}{N_b} - A_{dentrim} - A_{dentrim} \right)$$ \hspace{1cm} (11)

The lower angle of a tooth is given by the following relationship:

$$A_{dentrim1} = A_{dentrim} - A_{encr} - A_{encr1}$$ \hspace{1cm} (12)

The upper angle of a tooth is given by the following relationship:

$$A_{dentrim2} = A_{dentrim} - A_{encr} - A_{encr2}$$ \hspace{1cm} (13)

The lower angle of an inserted tooth is given by the following relationship:
The upper angle of an inserted tooth is given by the following relationship:

$$A_{dentrim} = \frac{2\pi}{N_b} - A_{dentr} - 2\times A_{aencr}$$  \hspace{1cm} (14)$$

The angle of dental development ($A_d$) is given by the following equation:

$$A_d = \frac{2\pi}{N_b} \times \beta$$  \hspace{1cm} (16)$$

where $\beta$ is the fulfillment of a rotor tooth coefficient ($\alpha < \beta < 1$).

The height of the teeth is expressed as follows:

$$H_d = \frac{N_{as} \times I_{dim}}{K_r \times \delta \times L_{encr}}$$  \hspace{1cm} (17)$$

$I_{dim}$ is the dimensioning current, $K_r$ is the filling factor and $\delta$ copper admissible current density.

The dimensioning current is expressed by the flowing relation:

$$I_{dim} = e \times R_e \times M_s \times \left( \frac{V_d}{t_d} + g \times \sin(\lambda) \right)$$  \hspace{1cm} (18)$$

Where $e<1$ it is usually close to 0.9, and $R_e$ the radius of the wheel of the car, $M_s$ is the mass of the car, $t_d$ is the reduction ratio, $t_d$ is the car's start time from speed equal 0 to the base speed ($V_b$) of the car, $g$ is the gravity acceleration and $\lambda$ is the angle with the horizontal road.

4. Electric Parameters of the Motor

The inductance of the rotor winding is given by the following relationship [11]:

$$L = \mu_0 \times N_k \times \left\{ \frac{s_d}{2} \times \frac{D_e - D_i}{2} \times \frac{H_d}{2} + \frac{2\pi}{N_b} \times A_d \times \frac{D_e + D_i}{4} \right\} \times N_{sh}^2$$  \hspace{1cm} (23)$$

The magnetic induction due to the power of the rotor winding by the demagnetization current of the magnet ($I_d$) is given by the following relationship:

$$B_{ci} = \frac{\mu_0 \times N_{sh} \times I_d}{H_s + e}$$  \hspace{1cm} (24)$$

The demagnetization of the magnets is provided that:

$$B_e - B_{ci} = B_e$$  \hspace{1cm} (25)$$

where $B_e$ is the demagnetization magnetic induction of the magnets.

$$L_{sp} = 2\times \left( \frac{D_e - D_i}{2} \times L_{encr} \right) + 2\times \left( \frac{D_e + D_i}{4} \right) \times A_{dentrim} + L_{encr}$$  \hspace{1cm} (27)$$
The resistivity of copper is expressed by the following relationship:

$$\rho = r_c \times (1 - \alpha_t \times (T_b - 20))$$  \hspace{1cm} (28)

where $T_b$ is the temperature of copper and $\alpha_t$ is the temperature coefficient at 20 °C.

Hence the expression for the resistance of the rotor winding is deduced by the following equation:

$$R = \rho \times L_{ip} \times N_{sh} \times N_b \times \frac{I_{em}}{\delta}$$  \hspace{1cm} (29)

The motor electrical constant is deduced from the equations (4) and (5).

$$K_e = P \times N_{sh} \times \left(\frac{D_s^2 - D_r^2}{2}\right) \times B_s$$  \hspace{1cm} (30)

The DC bus voltage is calculated in such a way that the vehicle can reach a maximum speed with a low torque undulation and without weakening. This voltage is calculated assuming that the engine runs at a stabilized maximum speed. At this operating point (Figure 4) the electromagnetic torque to be developed by the motor is expressed by the following equation [10-31]:

$$T_{udc} = \frac{P_f}{\Omega_{max}} + T_d + T_{sh} + T_p + T_f$$ (31)

The different torques are expressed by the following equations:

$$T_b = s \times \frac{v}{\sqrt{v}}$$  \hspace{1cm} (32)

$$T_{sh} = k \times \sqrt{v}$$  \hspace{1cm} (33)

$$T_p = R_s \times f_s \times M_s \times g$$  \hspace{1cm} (34)

$$T_e = R_s \times \left(\frac{M_s \times C_s \times A_s}{2}\right) \times v^2$$  \hspace{1cm} (35)

$$T_f = M_i \times g \times \sin(\lambda)$$  \hspace{1cm} (36)

Figure 4 shows the evolution of useful torque ($T_u$) and load torque ($T_R$) for operation at maximum speed ($\Omega_{max}$):

From figure 4, we deduce the expression of the DC bus voltage:

$$U_{dc} = K_e \times \Omega_{max} + R \times I_{dc}$$  \hspace{1cm} (38)

Where $I_{dc}$ is the current drawn by the motor at maximum stabilized speed.

$$I_{dc} = \frac{T_{udc}}{K_e}$$  \hspace{1cm} (39)

![Figure 4. Evolution of useful torque and load torque for operation at maximum speed.](image)

5. Motor Model

The transient voltage equation of the engine is given by the following equation:

$$u(t) = R \times i(t) + L \times \frac{di(t)}{dt} + e(t)$$  \hspace{1cm} (40)

where $i$ is the current drawn by the motor.

The transient back electromotive force is expressed as follows:

$$e(t) = K_e \times \Omega(t)$$  \hspace{1cm} (41)

The electromagnetic torque is given by the following relationship:

$$T_{em} = K_e \times I$$  \hspace{1cm} (42)

The iron losses is approximated by the following relation:

$$P_h = q \times \left(\frac{f}{50}\right)^{1.5} \times \left(2 \times P \times M_d \times B_s^2 + 2 \times P \times M_{cs} \times B_{cs}^2\right)$$  \hspace{1cm} (43)

$$f = \frac{P \times \Omega}{2 \times \pi}$$  \hspace{1cm} (44)

where $f$ is frequency of the elementary back electromotive forces, $M_d$ is the teeth mass, $M_{cs}$ is the stator yoke mass, $B_s$ is the flux density in the inductor yoke and $q$ is the quality factor of metal sheet.

Mechanical losses are expressed by the flowing relation:

$$P_m = \frac{\Omega \times v \times \Omega \times q \times \left|\Omega\right|}{\Omega}$$  \hspace{1cm} (45)
where $s$ is the dry friction coefficient, $\nu$ is the viscous friction coefficient and $k$ is the fluid friction coefficient.

6. Motor Efficiency Optimization Problem

The motor efficiency is expressed by the following relation:

$$\eta = \frac{u \times R \times i^2 - P_a - P_m}{u \times i}$$  \hspace{1cm} (46)

The efficiency can be optimized by Genetic Algorithms method. The formalization of the optimization problem is summarized as follow [6-12]:

Maximiser $\eta$

100 mm $\leq$ $D_1$ $\leq$ 250 mm
300 mm $\leq$ $D_s$ $\leq$ 700 mm
$U_{dc}$ $\leq$ 150V
5A / mm$^2$ $\leq$ $\delta$ $\leq$ 7A / mm$^2$
10 $\leq$ $N_{sh}$ $\leq$ 200
0.4 Tesla $\leq$ $B_s$ $\leq$ 1.6 Tesla
0.4 Tesla $\leq$ $B_o$ $\leq$ 1.6 Tesla

$$\text{(47)}$$

7. Torque Ripple Minimization

The torque ripple is directly related to the ripple of the resultant back electromotive force. Two parameters strongly influence the torque ripple are namely:

- The $\alpha$ parameter close to 1. This parameter should be the maximum possible, but for values of $\alpha$ very close to 1 a triggering of short circuits is activated between magnetic heads teeth, for that we are going to offer to optimize this parameter by finite element method.
- The $\beta$ parameter ($\alpha < \beta < 1$). This parameter affects the ripple torque, the length in the axial direction of the engine and also leads to local saturation at levels of teeth. This parameter setting is also optimized by the finite element method.

A series of simulations of the evolution of the electromagnetic torque and saturations at the teeth were allowed to set $\alpha = 0.7$ and $\beta = 0.9$ as optimal values minimizing torque ripple and local saturations.

8. Conclusion

In this paper we presented and studied an innovated DC engine structure with permanent magnet and axial flux with reduced production cost and high power density. A sizing and modeling program highly parameterized is developed. This program has led to joint optimization problems of performance, torque ripple and local saturations.

As prospects, this study can be validated by the finite element method and experimentally on a realized prototype.