Market Design for CO\textsubscript{2} Abatement in Coal-fired Power Industry Based on Combinatorial Auction

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Abstract: Market design is the core issue for reducing CO\textsubscript{2} emissions from coal-fired power industry, however the current carbon market has some deficiencies in this area. Utilizing the combinatorial auction way, this article proposed an enhanced carbon market special for power industry by complementing current cap-and-trade system. Concretely, the enhanced market design is improved by lower- and upper-bound price, combinatorial auction for carbon allowances initial allocation, carbon submarket trade, and electricity-environment coordinated regulation. In the enhanced market, generator competes for initial carbon allowances as a form of delivering a demand function to market organizer, which can be depicted as and settled by a stochastic linear programming model. Given the carbon allowances market supply curve (i.e., total initial allowances issued), environment regulator matches the market demand curve (i.e., through adding up those individual bid demand curves together) in a uniform market clearing price (MCP) way; by this means, initial carbon market equilibrium is reached. Under this enhanced mechanism, price of bidders is ordered according to their operational advantage, moreover, respective quantity of bid allowances is also sequenced in the same way. Comparing with current cap-and-trade system, the enhanced market design can efficiently motivate generator to reduce CO\textsubscript{2} emissions through controlling CO\textsubscript{2} intensity per sold MWh. Numerical simulation further verified the efficiency of this enhanced carbon market design.

Keywords: CO\textsubscript{2} Abatement, Carbon Market Design, Coal-fired Power Industry, Combinatorial Auction

1. Introduction

On December 2009, the United Nations Framework Convention on Climate Change (UNFCCC) passed the Copenhagen Accord in Denmark. Since then the low carbon electricity has developed quickly. Year 2015 saw UNFCCC passed the Paris Agreement and it further advanced low carbon electricity development.

Carbon market, known as cap-and-trade system, has been implemented by regions and countries such as European Union Emission Trading Scheme (EU ETS), New South Wales Emission Trading Scheme (NSW ETS), Regional Greenhouse Gas Initiative (RGGI), and China pilot carbon markets. In these systems, an authority sets a cap on the total amount of carbon allowances and allocates them to appropriate firms by auctioning or grandfathering (free of charge). To minimize the cost caused by carbon cap regulation, allowances may be traded among firms [1-3].

As main service area, carbon market needs to be harmonized with the development of coal-fired power industry. However, existing market design has some deficiencies in this aspect. Nielsen and Jeppesen [4] find that Tradable Green Certificates (TGCs) of European Union (EU) countries have different definitions and market conditions, thereby dampen the development of EU electricity market. Sorrell and Sijm [5] also find that different abatement policies may conflict each other, as a result of that, decrease mitigation efficiency. Peace and Juliani [6], Weng and Xu [7] argue that combating climate change will induce cost burden to economy system and it can be relieved by better market design. It is profound to motivate generator to reduce CO\textsubscript{2} emission on condition of electric power supply satisfying national economy development [7-8]. In this aspect, power industry not only needs regulated electricity market, but also needs regulated carbon market. More concretely, it requires the two markets to be regulated coordinately and work harmonized [9-10].

The current pricing mechanism has little incentive that
motivates generator to build cleaner production facilities. Rosendahl [11] proves that allowances pricing through unconstrained market may decrease generator’s mitigation investment because of market externality. In reality, from 2005 to 2012, EU ETS induced carbon price fluctuated vigorously, which was the main problem of the mechanism [3, 12]. Meyer [13] and Zhao et al. [14] hold that price vibrating too much cannot encourage generator to invest in carbon capture and sequestration (CCS) because of risk consideration. Meanwhile, Klepper and Peterson [1] also give empirical evidences that EU ETS cannot support price high enough to motivate generator to abate CO2 emission. It is necessary to supplement price regulatory policy to encourage generator investing in CCS technology.

In carbon market, initial allowances allocation is a crucial issue in ensuring emission cap to be realized and generator to be motivated investing in CCS technology. But the grandfathering and National Allocation Plans (NAPs) allocation approach has some difficulties to achieve these objectives. Klepper and Peterson [1] apply NAPs to predict EU ETS price and find that, CO2 allowances supply abundance is the main reason why price decreased too much. Because of initial allowances allocation free of charge, generator gets windfall profits by trading allowances and transferring cost to consumers [12, 16]. Since 2013, EU ETS market design has revised the grandfathering approach by auctioning approach for initial allowances allocation in power sector.

Many literatures have contributed to designing an efficient carbon market. Montgomery [17] models a perfect market with emission certificate and proposes that there exists a minimum cost equilibrium for companies regulated by allowances cap. To understand abatement risk induced by carbon market, Hepburn [18] and Mandell [19] make a comparison to policy efficiency among price-based instruments and quantity-based instruments. Betz and Sato [20], Dormady and Healy [21] further suggest that different allowances allocation mechanism derives different cost, thereby eventually influences generator’s decision in electricity market. Focusing on minimizing cost at firm-level, Fankhauser and Hepburn [22-23] explore carbon market design in dimensions of both space and time. Pettersson [24] forms a linear programming to predict carbon price and policy efficiency for eastern European countries. This model contains major practical restrictions, such as load balance, installation capacity, fuel consumption, allowances cap, time value of capital, and technology advance. Incentivized by load-based cap-and-trade system, Gillenwater and Breidenbach [25] propose an unbundled Generation Emission Attribute Certificates (GEACs) to motivate generator to take abatement action. This mechanism concerns emission rate of certificate, default emission rate, and grid line loss rate, which can be compatible with electricity market. Concerning restrictions on load balance, fuel consumption, and allowances cap, Kockar et al. [26] build a cost minimization model and support that EU ETS can improve competitive ability of those generators featured as low emission intensity. Related topic is also debated in article [27-30], etc.

To effectively motivate generator to reduce CO2 emission on condition of electric power supply satisfying national economy development, based on the literature of [8, 24-26, 31], this paper developed a novel carbon market for CO2 abatement in coal-fired power industry. It refers to main features of existing market, such as allowances cap, default emission penalty, and allowances trade. Meanwhile, it is enhanced by combinatorial auction for initial allowances allocation, lower- and upper-price boundary in auction, and electricity-environment coordinated regulation on market design. As seen in the enhanced market, both price and allowances offered by bidders are ordered according to generator’s operational advantage, respectively. This is useful to motivate generator to abate CO2 emission and improve market efficiency. Comparing with current cap-and-trade system, the enhanced market can effectively reduce CO2 emission from power industry.

The following part is organized as, Section 2 depicts model assumption and framework of market design. Section 3 analyzes generator’s decision making in the enhanced carbon market. Useful features on generator bidding behavior are also explored. Section 4 is market equilibrium analysis. Section 5 does a comparison between the enhanced market and current cap-and-trade system to test validity of this market design. Section 6 further carries out a numerical simulation to the enhanced market. Finally, a brief conclusion is given by section 7.

2. Market Descriptions

2.1. Assumptions and Variables

As seen in section 1, current cap-and-trade system has deficiencies in encouraging coal-fired generator to reduce CO2 emission. This paper complements the system and proposes an enhanced carbon market special for CO2 mitigation in coal-fired power industry. To make market design more practical as well as standing on solid theoretical foundation, it sets the following assumptions:

(i) All load demands are satisfied. The primary goal of power industry is to supply electric energy to meet the requirement of national economy development [4, 9]. Elmaghraby [32] and Song et al. [33] also suggest that a critical component to a successful market design in power industry is the way in which load demand is satisfied.

(ii) The market has n coal-fired generators and each owns only one generation unit. Although the enhanced market can deal with the issue of generator owning multiple generation units, this assumption may simplify its fuel consumption function and avoid complex math formula.

(iii) Each generator’s emission intensity is above zero. As described in section 2.2, the enhanced carbon market is characterized as emission intensity-oriented. For solving generator’s demand function, it is necessary to
set this assumption. In reality, CO\textsubscript{2} intensity of coal-fired power plant is around 0.3 to 1.7 t/MWh [9, 26, 34].

(iv) Information is asymmetric and collusions is forbidden among generators and regulators. This means the cost information of each generator is privacy, therein no generator can exactly foresee market price.

To describe the enhanced market and analyze generator’s decision making, variables are defined in section Nomenclature.

2.2. Market Design

This section developed an enhanced carbon market for CO\textsubscript{2} abatement in power industry by overcoming deficiencies of existing cap-and-trade system. Its framework is described in Figure 1. As seen in sections 5 and 6, this market design can effectively motivate generator to reduce CO\textsubscript{2} emission on condition of power supply satisfying national economy development.

As a main improvement, the enhanced carbon market applies the first sealed-price combinatorial auction [35-37] to organize market transaction. Allowances allocation through auctioning way has many advantages contrast to grandfathering way, such as overcoming windfall profits problem, forbidding carbon leakage, and price discovery [25, 38].

The new carbon market is regulated by two coordinated regulators: electricity regulator and environment regulator, and is special for CO\textsubscript{2} reduction in power industry. As seen in sections 5 and 6, this mechanism can promote electricity-environment coordinated development.

Under the enhanced carbon market, electricity regulator need previously release electricity regulatory information, which include yearly predicted load demand, electricity price boundary, market share restriction, and grid line loss rate. In fact, they are basic requirements for stable operation of wholesale electricity market, electric power system technique constraint, and national economy development. Obviously, carbon market meeting these requirements may benefit electricity-environment coordinated development.

As a follower, environment regulator need thereafter release environmental regulatory information, which contain allowances cap, yardstick emission intensity calculated as total allowances divided by system operator procured electricity (i.e., \( e = E^{(S)}/[W^{(D)}(1 + s)] \)), carbon price boundary, default emission penalty rate, and second transaction charge rate.

Emission intensity slack factor \( m_j \) is defined as \( e_j (1 + m_j) = e \). It acts as two roles: to encourage (punish) generator to decrease (increase) CO\textsubscript{2} intensity and, to ensure safety of electrical power system at a cost of emissions increase appropriately. According to Figure 1, yardstick emission intensity does not include CO\textsubscript{2} emission induced by generator self-consumed electricity. However, CO\textsubscript{2} intensity of generator is calculated by both sold and self-consumed electricity. If \( e_j > e \) (i.e., \( m_j < 0 \)), then all its emissions will be charged with extra default emission penalty rate \( \alpha \). This regulatory policy increases default emission penalty. Concerning respective Euro 40/tCO\textsubscript{2} and Euro 100/tCO\textsubscript{2} fine degree in EU ETS first and second period, it holds the increasing penalty tendency. Obviously, this mechanism can motivate generator to improve operation management and invest in CCS technology.

Setting floor and ceiling carbon price can prohibit price fluctuating too much so as to increase market stability. This regulatory policy may overcome deficiency of existing cap-and-trade system in this aspect.

Generator may trade emission allowances with other generators or directly purchase allowances from environment regulator. However, for the former way, buyers will be charged with extra second transaction charge rate \( \beta \); for the latter way, they will pay the highest price composed of MCP,
This is because primary goal of power industry is to meet load demand for national economy development. If a collision cannot be avoided, the mechanism needs to ensure load demand at a cost of \( \text{CO}_2 \) increased appropriately. But in this case, environment regulator must punish those generators whose \( \text{CO}_2 \) intensity is higher than yardstick intensity.

Finally, the enhanced carbon market is operated by environment regulator. As described in Figure 1, generator needs to deliver a demand function to environment regulator. This means generator not only needs to bid allowances, but also needs to bid corresponding price. In auction terminology, it is a homogenous commodity quantity-price combinatorial auction [34, 39], which is also main enhancement for the new market design, of course, more challenge to generator.

3. Decision Making and Features

3.1. Decision Model

In enhanced carbon market, generator has a challenge to determine optimal demand function. Logistically, its decision making is based on all available information, both publicly and privately. According to section 2.2, generator makes decision for quantity-price combinatorial bid can be modeled as a stochastic math programming. The objective function is maximization of expected profit, which is comprised of (i) potential electricity revenue and fuel consumption cost induced by allowances, (ii) allowances cost and, (iii) default emission penalty induced by \( \text{CO}_2 \) intensity excess yardstick intensity.

Because generator cannot get accurate information on electricity price and operation active power before electricity market is cleared, it may assume them as uniform distribution stochastic variable in their reasonable range. Furthermore, generator cannot know other generators’ cost information (i.e., fuel price, \( \text{CO}_2 \) intensity, fuel consumption function, see assumption (iv)), its quantity-price bidding decision can only depend on public regulatory information and its own private information. Note that in the carbon second transaction (see Figure 1), generator selling surplus allowances will not only be charged with extra transaction cost, but also have a probability of transaction failure. On the other hand, it purchases shortage allowances will pay higher price than MCP, despite from other generators or from environment regulator.

Based on the above qualitative analysis, considering assumptions (i), (ii) and (iii), generator’s decision making is modeled as.

\[
\max \pi_j = \sum_{\rho_u} \int \rho_u \phi\left(\rho_u\right) d\rho_u - E_j \rho_{\alpha_j} - \int_{P_j} \left( a_j + b_j P_j + c_j P_j^2 \right) \frac{E_j \rho_{\alpha_j}}{P_j} \phi\left(P_j\right) dP_j + \min\left(0, m_j\right) E_{j} \alpha \rho_{\alpha_j}
\]

Such that,

\[
\frac{E_j}{e_j(1 + s_j)} \leq \frac{\tau E^{(S)}}{e} \quad (2)
\]

\[
e_j\left(1 + m_j\right) = e \quad (3)
\]

\[
P_j \leq P_j \leq P_j^{(N)} \quad (4)
\]

\[
\rho_{\alpha} \leq \rho_u \leq \rho_{\alpha} \quad (5)
\]

\[
\phi\left(P_j\right) = \begin{cases} \frac{1}{P_j^{(N)} - P_j} & \rho_u \in \left[P_j, P_j^{(N)}\right] \\ 0, \text{other} \end{cases} \quad (6)
\]

\[
\phi\left(\rho_u\right) = \begin{cases} \frac{1}{\rho_u - \rho_{\alpha}} & \rho_u \in \left[\rho_{\alpha}, \rho_{\alpha}\right] \\ 0, \text{other} \end{cases} \quad (7)
\]

\[
\rho_{\alpha} \leq \rho_{\alpha_j} \leq \rho_{\alpha} \quad (8)
\]

\[
E_j \geq 0 \quad (9)
\]

Eq. 1 shows that, generator’s expected revenue is calculated as expected electricity price multiple electric energy determined by bidding allowances. Its expected cost is composed by three components, (i) allowances cost, (ii) expected fuel cost calculated by generated electric energy, fuel price, and fuel consumption function, and (iii) possible emission penalty. The economic meaning of Eq. 2 is that potential sold electric energy calculated by bidding allowances is no more than the amount determined by market share. Eqs. 6-7 are probability density function of operation active power and electricity price, respectively.

3.2. Generator's Demand Function

By solving the above stochastic math programming, it can get generator’s demand function. For convenience, let’s denote marginal profit of emission as \( \xi_j \), expected electricity price as \( A \), and expected fuel cost per MWh as \( B_j \).

\[
A = \int \rho_u \phi\left(\rho_u\right) d\rho_u;
\]

\[
B_j = \phi\left(\rho_{\alpha_j}, F_j\left(P_j\right)\right) = \rho_{\alpha_j} \int_{P_j} \phi\left(P_j\right) dP_j
\]

In mathematics, suppose \( \text{prob}(\bullet) = 0 \) is equivalent to \( \bullet \) being impossible event. So restrictions of Eqs. 4-5 are included in restrictions of Eqs.6-7, respectively. Let’s put Eq. 3 and Eqs. 6-7 into Eq. 1, it gets

\[
A = \frac{\rho_{\alpha} + \rho_{\alpha}}{2}
\]
\[ B_j = \frac{\rho_j}{p_j^{(N)}} \left[ a_j \ln \left( p_j^{(N)} \right) + b_j \left( p_j^{(N)} \right)^2 + \frac{c_j}{2} \right] \]

Considering Eq. 2 and non-negative restriction, there exist three situations:

if \( E_j \in (0, \pi E^{(S)} e_j (1+s_j) / e) \), then
\[
\xi_j = \frac{A}{e_j (1+s_j)} - \rho_j e_j + \rho_j \alpha \left( \min \left( 0, \frac{e}{e_j} - 1 \right) \right)
\]

if \( E_j = 0 \), then
\[
\xi_j = \lim_{E_j \to 0^+} \left( \frac{\pi_j (E_j)}{E_j} - \pi_j (0) \right)
\]

if \( E_j = \pi E^{(S)} e_j (1+s_j) / e \), then
\[
\xi_j = \lim_{E_j = \pi E^{(S)} e_j (1+s_j) / e} \left( \frac{\pi_j (E_j)}{E_j} - \pi_j (0) \right)
\]

3.3. Features

According to Eqs. 10-11, generator bids the last one ton allowances will bring \( \min (\rho_e, \rho_j) \) expected profit. Let's define marginal emission revenue of start-generation (SMER) function as
\[
\rho_j^* = \rho_j^* (B_j, s_j, e_j) = \frac{A}{1+s_j} - \frac{B_j}{e_j - \min(0, e-e_j)} \alpha
\]

It can derive some useful features on generator’s demand function:

Lemma 1. In the enhanced market, for those generators whose bidding allowances is above zero, their upper-bound price is ordered as a finite monotonic decreasing sequence according to their operational advantage from strength to weakness.

Proof. First, it needs to prove that SMER can represent generator’s operational advantage. According to the definition of variable \( B_j \), it is the \( j \)-th generator’s expected fuel consumption cost per MWh. This variable is determined by generation facility’s active power, fuel consumption function, and fuel price. Based on the property of fuel consumption function in section Nomenclature, it means \( b_j \geq 0, c_j \geq 0 \) and \( b_j + c_j > 0 \). Since \( a_j \) is generator \( j \)’s fixed fuel consumption propensity, it infers \( a_j > 0 \), \( B_j > 0 \). Therefore, there exist

\[ \frac{\partial B_j}{\partial e_j} < 0 \]

So the negative or positive sign of \( \frac{\partial \rho_j^*}{\partial e_j} \) is determined
by \( A / (1 + s_j) - B_j \) and \( e_j \). Based on generator’s demand function Eq. 11, the critical price point where generator offers positive allowances is \( \rho^{*}_{e_i} = \rho^{*}_c > 0 \), that is to say

(i) \( A / (1 + s_j) - B_j < \left[ e_j - \min \left( 0, e - e_j \right) \right] \rho^{*}_c \Rightarrow \) generator’s bidding price and allowances are zero;

(ii) \( A / (1 + s_j) - B_j \geq \left[ e_j - \min \left( 0, e - e_j \right) \right] \rho^{*}_c \Rightarrow \) generator’s bidding price and allowances are positive

\[ A / (1 + s_j) - B_j > 0 \]

if generator’s bidding price and allowances are positive, and \( e_j \neq e \), then,

\[ \partial \rho^{*}_{e_j} / \partial B_j + \partial \rho^{*}_{e_j} / \partial s_j + \partial \rho^{*}_{e_j} / \partial e_j < 0 \]

\[ i = i_k = \left\{ (i, k) \bigg| 1 = 1, 2, ..., m; k_i = 1, 2, ..., K; \rho^{*}_{c, i} = ... = \rho^{*}_{c, i, k} = \rho^{*}_{i, k} \right\} \]

where \( K_i \) is generator number of the \( i \)-th set.

Assuming that SMER of the \( m \) sets are ordered as

\[ \rho^{*}_{e_1} > \rho^{*}_{e_2} > ... > \rho^{*}_{e_i} > ... > \rho^{*}_{e_m} \]

where \( 1 \leq i \leq m \). And the \( i \)-th \((i \leq i')\) set has bidding allowances above zero, and vice versa. According to the first part proof, this sequence is consistent to SMER from strength to weakness, so critical point of emission marginal profit of generators are order as sequence

\[ \rho^{*}_{e_1} > \rho^{*}_{e_2} > ... > \rho^{*}_{e_i} > ... > \rho^{*}_{e_i} > \rho^{*}_{c} \]

On condition of \( i \leq i' \), there exists

monotony of finite sequence

\[ \min \left( \rho^{*}_{c, i}, \rho^{*}_{c, i} \right), \min \left( \rho^{*}_{e, i}, \rho^{*}_{e, i} \right), \min \left( \rho^{*}_{e, i}, \rho^{*}_{e, i} \right) \]

monotony of finite sequence

\[ \rho^{*}_{e_1} - \rho^{*}_{c, i} > \rho^{*}_{e_2} - \rho^{*}_{c, i} > ... > \rho^{*}_{e_i} - \rho^{*}_{c, i} > \rho^{*}_{e_i} - \rho^{*}_{c, i} \]

\[ 1 \leq K_i \leq n, \quad \sum_{i=1}^{n} K_i = n \]

Lemma 1 implies that, those generators featured as SMER strength tend to bid higher price. In other words, the enhanced market can encourage generators to release their true operational advantage. Therefore it can improve allowances allocation efficiency and reduce CO₂ emission.

Lemma 2. In the enhanced market, for those generators whose bidding allowances are above zero, (a) their bidding allowances are ordered as a finite monotonic increasing sequence in accordance with the sequence of augmented CO₂ intensity \( e_i (1 + s_j) \) from lower to higher; (b) specifically, in the following two conditions, this sequence is inversely ordered contrast to lemma 1 sequence:

(i) each generator’s \( B_j \) and \( e_j \) are equivalent respectively;

(ii) each generator’s \( B_j \) and \( s_j \) are equivalent respectively.

Proof. (a) In Eq. 11, endogenous variables of generator’s bidding allowances are \( e_j \) and \( s_j \). On condition of \( j \in i \leq i' \), there exists

\[ \partial E_j / \partial e_j + \partial E_j / \partial s_j > 0 \]

Therefore, \( \forall j \in i \leq i' \), \( E_j \) is a finite monotonic increasing sequence in accordance with the sequence \( e_j (1 + s_j) \) from lower to higher.

(b) On condition of (i), endogenous variable of generator’s SMER function is \( s_j \), and

\[ 3s(\xi_k) = ... = s(\xi_k) = s(\xi_k) \neq s(r), \forall s(l) \neq s(r) \]

Based on lemma 1’s proof, it infers if \( i \leq i' \), then

\[ d \rho^{*}_{e_i} / ds_i < 0 \]

So lemma 1’s finite monotonic decreasing sequence is still validity according to \( s_j \) from lower to higher.

On the other hand, endogenous variable of Eq. 11 is \( s_j \), if \( j \in i \leq i' \), then \( d E_j(v_j) / ds_j > 0 \).

On condition of (ii), endogenous variable of generator’s
SMER function is $e_i$, and
\[
\exists e(i) = ... = e(i_k) = ... = e(i_r), \quad \forall e(i) \neq e(r), \quad \text{where } i \neq r, 1 \leq i, r \leq i^*.
\]
If $i \leq i^*$ and $e_i \neq e$, then $\rho_{e_i}^* / \partial e_i < 0$.
If $i \leq i^*$ and $e_i = e$, then $\rho_{e_i}^* = \rho_e^*(e) = [A / (1 + s_i) - B_i] / e$.

So lemma 1’s finite monotonic decreasing sequence is also validity according to $e_i$ from lower to higher. Meanwhile, endogenous variable of Eq. 11 is $e_j$.

If $j \in i \leq i^*$, then $dE_j(s_j) / de_j > 0$.
Considering (a)’s proof, this means
If $i \leq i^*$, then $dE_j(e_i, s_j) / de_i + dE_i(e_i, s_i) / ds_i > 0$.

So both condition (i) and (ii), there exists
\[
E^{(i)}(\rho_e) = \sum_{j=1}^{K} \sum_{k=1}^{K} E_{i,j}, \quad \text{if} \quad \rho_e \leq \rho_i \leq \min(\overline{\rho}_i, \rho^*_i).
\]

4.2. Market Supply Function and Equilibrium

The market supply function $E^{(S)}$ is provided by environment regulator. It is coordinately determined by factors of: electricity demand, environmental capacity and CO$_2$ intensity, which is almost price inelastic. Let demand function equal supply function, market equilibrium $E^{(*)}$ is reached, meaning that environment regulator can allocate initial allowances among winners and find market price for allowances.

Because of jump property in demand function, there has a range of equilibrium price. Let’s denote it emission critical price range (ECPR) which is defined as, (i) the covered price range that demand line is identical to line $E^{(*)}$ or, (ii) the covered price range by line $E^{(*)}$’s immediate-up demand line on the $E - \rho_e$ plane. From environment benefit aspect, environment regulator’s dominant strategy is taking ECPR upper-bound as market clearing price (MCP). By increasing price in the reasonably range, it can encourage generator to reduce CO$_2$ emission and invest in CCS. Concretely, carbon allowances pricing is settled by

\[
E^{(*)} = \min \left( \overline{\rho}_e, \rho_e^*, r \right), \quad \text{if} \quad E^{(*)} = E^{(S)} = \min \left( E^{(S)} \sum_{j=1}^{K} \sum_{k=1}^{K} E_{i,j} \right);
\]

\[
\rho_e^* = \begin{cases} \text{and if } 1 \leq i \leq i^*, & \sum_{j=1}^{K} \sum_{k=1}^{K} E_{i,j} < E^{(*)} \leq \sum_{j=1}^{K} \sum_{k=1}^{K} E_{i,j} \\
\min \left( \overline{\rho}_e, \rho_e^*, r \right), & \text{if} \quad E^{(*)} = \sum_{j=1}^{K} \sum_{k=1}^{K} E_{i,j} \end{cases}
\]

Let’s sign ECPR as $\sigma^{(r,i^*)}$, then

0 < $E_1 < E_2 < ... < E_i < ... < E_i^*$

Lemma 2 implies that, those generators featured as stronger operational advantage prefer to bid less carbon allowances. Concerning lemma 1, their allowances demand will be met prior to other generators. Hence, the enhanced market can improve allowances allocation efficiency.

4. Winner Determination

4.1. Market Demand Function

Generators deliver their demand function to environment regulator in a sealed combinatorial auction way. Through piecewise aggregation of individual demand function, environment regulator forms the following market demand function.

\[
E^{(i)}(\rho_e) = \sum_{j=1}^{K} \sum_{k=1}^{K} E_{i,j}, \quad \text{if} \quad \rho_e \leq \rho_i \leq \min(\overline{\rho}_i, \rho^*_i), 1 \leq i \leq i^*.
\]

Concerning lemma 1, their allowances demand will be met operational advantage prefer to bid less carbon allowances. Hence, the enhanced market can improve allowances allocation efficiency.
The MCP pricing is a harmonized result of both auctioning and coordinated regulation. Under ECPR upper-bound pricing mechanism, generators featured as lower CO₂ intensity will earn more expected profits. So it not only can forbid carbon price vibrating too much, but also can promote generator to mitigate CO₂ by controlling self-consumed electricity, expected fuel consumption cost and, CO₂ intensity. In other words, it can simultaneously encourage generator to reduce CO₂ emission and improve operational advantage.

\[
\sigma_{[i^r, j^r]} = \begin{cases} 
\rho_e \mid \min \left( \overline{\rho} \rho, \rho_e \right) < \rho_e \leq \rho_e^* & \text{if } E^* = E^{(5)} \\
\rho_e \mid \rho_e < \rho_e \leq \rho_e^* & \text{if } E^* = \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} 
\end{cases}
\]

(15)

4.3. Allowances Allocation

When environment regulator allocates equilibrium allowances among winners, it sets the allocation policy as, (i) complete meet those generators who still have positive bidding allowances where price is higher than ECPR upper-bound; (ii) average allocate the spare allowances among those generators who have positive bidding allowances where price is at ECPR interval but no bidding allowances where price is higher than ECPR upper-bound. The spare allowances is calculated as

\[
(E^{(r)}_{\text{remain}}) = \min \left( E^{(5)}, \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} \right) - \min \left( E^{(5)}, \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} \right)
\]

(16)

Let’s call it as ECPR allocation rule. As seen in section 3.3, those who have positive bidding allowances where price is higher than ECPR upper-bound own stronger operational advantage. So this rule also has an incentive to motivate generator to improve operational advantage by controlling CO₂ intensity. Theorem 1 proved that, the market equilibrium has some useful features, not only for generator reducing CO₂ emission but also for regulator pursuing social welfare maximization.

Theorem 1. In the enhanced market, concerning those generators offering a positive demand function, (a) those who have bidding allowances where price is above ECPR upper-bound, their allocated allowances is the maximum determined by market power regulatory rule; those who have bidding allowances where price is at ECPR interval but no bidding allowances where price is above ECPR upper-bound allocate the spare allowances averagely; those who have no bidding allowances where price is no less than ECPR upper-bound, their allocated allowances is zero. (b) if bid price upper-bound of all generators is isozant, then, each generator allocated allowances is either the maximum determined by market power or allocating equilibrium allowances averagely; (c) if bid price upper-bound of all generators are different, then, at most one generator allocated allowances is lower than its maximum demand.

Proof. (a) According to Eq. 15, ECPR immediate-up price range and immediate-down price range are \(\sigma_{[i^r-1,j^r]}\) and \(\sigma_{[i^r+1,j^r]}\) respectively. There exists relation:

\[
\sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} < \min \left( E^{(5)}, \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} + E_{\rho_e} \right) \leq \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} < \min \left( E^{(5)}, \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} + E_{\rho_e} \right)
\]

That is to say,

\[
\sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} < \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} < \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} + E_{\rho_e} \equiv E^{(r)} \leq \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} < \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} + E_{\rho_e}
\]

Meanwhile, there exists,

\[
j \in (i^r - 1) \cup (i^r - 2) \cup ... \cup (1) \equiv \min \left( \overline{\rho} \rho, \rho_e \right) \cup \rho_e \cdot \rho_e \cup \rho_e \cdot \rho_e < \sum_{i=1}^{K} \sum_{j=1}^{E_{h_i}} + E_{\rho_e}
\]

\[
\sigma_{[i^r-1,j^r]} \cup \sigma_{[i^r-2,j^r]} \cup ... \cup \sigma_{[i,j^r]}
\]
\[ j \in (i^*+1)_{k^{(m)}_{i^{(m)}}} \cup (i^*+2)_{k^{(m)}_{i^{(m)}}} \cup \ldots \cup (i^*)_{k^{(m)}_{i^{(m)}}} \]  

\[
\min(\bar{\rho}_e, \rho^*_e) \cup \min(\bar{\rho}_e, \rho^*_e) \cup \ldots \cup \min(\bar{\rho}_e, \rho^*_e) \subseteq \sigma_{(i^*+1,j)} \cup \sigma_{(i^*+2,j)} \cup \ldots \cup \sigma_{(i^*,j)}
\]

\[
j \in (i^*)_{k^{(m)}_{i^{(m)}}} \Leftrightarrow \min(\bar{\rho}_e, \rho^*_e) \subseteq \sigma_{(i^*,j)}
\]

Based on ECPR allocation rule, if generator \( j \in (i^*+1)_{k^{(m)}_{i^{(m)}}} \cup (i^*+2)_{k^{(m)}_{i^{(m)}}} \cup \ldots \cup (i^*)_{k^{(m)}_{i^{(m)}}} \), then allocated allowances is \( E^*_j = 0 \).

(b) Under this condition, the market demand function Eq. 13 is simplified as

\[
E^*(\rho_e) = \sum_{k=1}^{K_i} E_{j_k}, \quad \text{if } \rho_e \leq \bar{\rho}_e \leq \min(\bar{\rho}_e, \rho^*_e) = \min(\bar{\rho}_e, \rho^*_e)
\]

According to Eq. 15, there exists

\[
\sigma_{(i^*,j')} = \sigma_{(i^*,j')} = \{\rho_e \mid \rho_e \leq \rho_e \leq \rho^* \}
\]

where \( \sum_{j=1}^{K_i} E_{j_k} \). According to ECPR rule, allocated allowances for those generators who have positive bidding allowances is

\[
E^*_j = \min\left(\frac{E^*(\rho_e) - \sum_{i=1}^{K_i} E_{j_i}}{K_i} \right) \leq \tau E^*(\rho_e) \left(1 + s_j\right) / e
\]  

if generator \( j \in (i^*)_{k^{(m)}_{i^{(m)}}} \), then allocated allowances is \( E^*_j = 0 \).

Under condition (c), lemma 1’s finite decreasing sequence is strictly monotonic. That means, for \( \forall 1 \leq i \leq i^* \), there exist

\[
\min(\bar{\rho}_e, \rho^*_e) \neq \ldots \neq \min(\bar{\rho}_e, \rho^*_e) \neq \ldots \neq \min(\bar{\rho}_e, \rho^*_e)
\]

where \( 1 \leq k_i \leq K_j, 1 \leq K_j \leq n \).

This curve has \( i^* \) non-zero points on vertical axis.

Considering index function in section 3.3, these points are ordered as \( 1 \leq i^* \leq i^* \leq n \).

Under condition of (c), lemma 1’s finite decreasing sequence is strictly monotonic. That means, for \( \forall 1 \leq i \leq i^* \), there exist

\[
\min(\bar{\rho}_e, \rho^*_e) \neq \ldots \neq \min(\bar{\rho}_e, \rho^*_e) \neq \ldots \neq \min(\bar{\rho}_e, \rho^*_e)
\]

where \( 1 \leq k_i \leq K_j, 1 \leq K_j \leq n \).
the sequence

\[ 0 < E_{k_i} < \ldots < \sum_{k_i} E_{k_i} < \ldots < \sum_{k_i} E_{k_i} < \ldots < \sum_{k_i} E_{k_i} < \ldots \]

has only \( i^* \) sections, where \( 1 \leq \sum_{j=1}^{i^*} K_j = i^* \).

So there exists \( K_1 = K_2 = \ldots = K_{i^*} = 1 \). By applying result (a) to this condition, therein, conclusion (c) is correct.

According to above analysis, ECPR pricing and allocation rule of the enhanced market may motivate generators unwilling to rise up or press down bidding price. So all individual demand functions determined by Eq. 11 reach a Nash equilibrium.

5. Comparison and Verification

To test validity of the enhanced market, this section compares it with EU ETS second period mechanism. Under this mechanism, initial allowances is allocated almost free of charge. Suppose carbon allowances is allocated averagely, total supplied electricity under restriction of allowances cap can be calculated as

\[ \text{Supplied electricity under free allocation} = \sum_{j=1}^{n} \frac{E_j^{(S)}}{ne_j (1 + s_j)} \quad (17) \]

From lemma 1 and 2, generators featured as lower CO\(_2\) intensity and self-consumed electricity rate not only have lower allowances demand, but also have a priority to be met. So under the enhanced market, the same issued allowances can bring more supplied electricity. Equivalently, the same dispatched electricity will cause less CO\(_2\) emission. The saved allowances can be calculated by

\[ \text{Saved allowances} = E^{(S)} + \frac{\left( H^{(D)} (1 + s) \right) \sum_{j=1}^{n} \frac{E_j^{(S)}}{ne_j (1 + s_j)}}{\sum_{j=1}^{n} \frac{1}{ne_j (1 + s_j)}}.E^{(*)} \quad (18) \]

6. Simulations

This section presents a numerical simulation to show how the enhanced market works well. Assume market has 8 generators and their operational information is given by table 1 and 2. Table 1 is generator’s fuel consumption function (unit: ton/hour), and table 2 is generator’s upper- and lower-bound active power (unit: MW), emission intensity (unit: ton/MWh), coal price (unit: CNY/ton), and self-consumed electricity rate.

<table>
<thead>
<tr>
<th>Table 1. Generator’s fuel consumption function.</th>
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<tr>
<td>Generator</td>
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</tr>
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<td>G1</td>
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<td>G2</td>
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<td>G3</td>
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<td>G4</td>
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<td>G6</td>
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<td>G7</td>
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<td>G8</td>
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</tbody>
</table>

Electricity regulator released information is as follow, forecasted load demand is 3000000 MWh, grid line loss rate is 0.06, electricity price is between 200 and 400 CNY/MWh, and market power rate is less than 0.3.

Based on electricity regulatory information, environment regulator released information is given below, supplied allowances is 1590000 ton, carbon price is between 10 and 50 CNY/ton, yardstick emission intensity is 0.5 ton/MWh, default emissions penalty rate is 0.2, and the second emission transaction charge rate is 0.1.

<table>
<thead>
<tr>
<th>Table 2. Generator’s technical restrictions, CO(_2) intensity, self-consumed electricity rate, and coal price.</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper-bound power</td>
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<td>-------------------</td>
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<tr>
<td>G1</td>
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</tbody>
</table>

Figure 2 reveal that SMER has a negative relationship to expected fuel cost per unit electricity. The same is also true for SMER and CO\(_2\) intensity (see Figure 3). This is well fitted to theoretical analysis in section 3.3. So SMER can indeed represent generator’s operational advantage.
Figure 2. Relationship between SMER and expected fuel cost per unit electricity.

Figure 3. Relationship between SMER and emission intensity.

Figure 4. Generator’s demand curve.

Figure 5. Market demand function and equilibrium.

Comparing to EU ETS second period mechanism, the enhanced market saves 456950 ton allowances, meaning 28.74% of CO₂ emission is avoided. So the enhanced market can realize CO₂ mitigation and improve market efficiency.

7. Conclusions

To motivate generator to reduce CO₂ emission on condition of electricity supply satisfying national economy development, this paper proposes an enhanced carbon market for coal-fired power industry. It refers to main features of existing carbon market, such as allowances cap, default emission penalty and allowances trade. And it is improved by combinatorial auction, lower- and upper-price boundary, electricity-environment coordinated regulation.

It operates as, first, electricity regulator releases regulatory information on load demand, grid line loss rate, electricity price boundary and market power restriction. Coordinately, environment regulator releases regulatory information on allowances cap, carbon price boundary, yardstick emission intensity, default emission penalty rate and, second transaction charge rate. Utilizing available information, generators bid for carbon allowances by delivering a demand function to market organizer. Market organizer determines MCP price, allocates initial allowances among winners, and organizes allowances second trade.
This article models decision making of generator as a stochastic math programming, which provides its demand function for CO₂ allowances. By adding up individual demand curve and matching with total allowances supplied, the market equilibrium is obtained. In the enhanced market, both bidding price and allowances of generators are respectively ordered according to their operational advantage. These features are useful to improve market efficiency and encourage generator to invest in CCS technology. Comparing with existing cap-and-trade system, it can motivate generator to reduce CO₂ emission. Numerical simulations give intuitive evidences on the validity of enhanced carbon market.

**Nomenclature**

\[ \pi_j : \text{expected profit (} j = 1,...,n \text{)} ; \text{unit: (Yuan)} ; \]
\[ W^{(D)} : \text{load demand; unit: (MWh)} ; \]
\[ F_j(P_j) : \text{fuel consumption function. Generally, it is a convex and increase function. This paper chooses} \]
\[ F_j(P_j) = a_j + b_j P_j + c_j P_j^2 , \text{unit: (ton / hour)} ; \]
\[ P_{\alpha}, P_{\beta}, P_{\delta}, P_{\epsilon}, P_{\rho} : \text{bidding price, marginal emission revenue of start-generation, market clearing price, electricity price, and coal price. Variables under-lined (up-lined) are lower-bound (upper-bound). unit: (Yuan / MWh), (Yuan / ton)} ; \]
\[ E_j, E^{(S)}, e_j, e : \text{bidding allowances, allowances cap, emission intensity, and yardstick emission intensity. unit: (ton), (ton / MWh)} ; \]
\[ P_j, P_{\alpha}, P_{\beta}^{(N)} : \text{active power, lower-bound active power, nameplate (i.e. upper-bound) active power. unit: (MW)} ; \]
\[ \tau : \text{market share (i.e. market power restriction);} \]
\[ m_j : \text{emission intensity slack factor.} \]
\[ \alpha, \beta : \text{default emission penalty rate, second transaction charge rate, generally,} 0 < \alpha, \beta \leq 1 ; \]
\[ s_j, s : \text{self-consumed electricity rate, grid line loss rate.} \]
\[ s_j \equiv W_j^{(Self)}/W_j, s \equiv W_j^{(Loss)} / W_j^{(D)} , \text{where} W_j^{(Self)} \text{ and} W_j^{(Loss)} \text{ each denotes self-consumed electricity and grid line loss electricity.} \]

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**References**


