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The Physics of Mass Gap Problem in the General Field Theory Framework

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Abstract: We develop the gauge theory introduced by Ning Wu with two Yang-Mills fields adjusted to make the mass term invariant. In the specific representation there arise quantum massive and classical massless no-Abelian vector modes and the gauge interaction terms. The suggested model will return into two different Yang-Mills gauge field models. Next, we focus on calculating `the meet of the propagators' of those quantum massive and classical massless vector fields with respects to the double Yang-Mills limit. We demonstrate that our proposed version of the Quantum Chromodynamics (QCD) predicts mass gap $\Delta > 0$ for the compact simple gauge group SU (3). This provides a solution to the second part of the Yang-Mills problem.

Keywords: Gauge field Theories, Quantum Chromodynamics, Yang-Mills Problem

1. Introduction

The laws of quantum physics are for the world of elementary particles what Newton's laws of classical mechanics are for the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles by using geometrical structures. Since 1954, the Yang-Mills (YM) theory [1] has been the foundation of contemporary elementary particle theory. Although the predictions of the YM theory have been tested in many experiments [13], its mathematical foundations remain unclear. The success of the YM theory in describing the strong interactions between elementary particles depends on a subtle quantum-mechanical property, termed the *mass gap*: quantum particles have positive masses even though classical waves travel at the speed of light. The mass gap has been discovered experimentally and confirmed through computer simulations, but is yet to be understood theoretically [2], [3], [4]. The theoretical foundations of the YM theory and the mass gap may require the introduction of new fundamental ideas in physics and mathematics [1]. As explained by Arthur Jaffe and Edward Witten in the Caly Mathematical Institute (CMI) problem description [2], the YM theory is a generalization of Maxwell's theory of electromagnetism, in which the basic dynamical variable is a connection on a G-bundle over four-dimensional space-time.

The YM theory is the key ingredient in the Standard Model of elementary particles and their interactions. A solution to the YM problem, therefore, would both place the YM theory on a firm mathematical footing and demonstrate a key feature of the physics of strong interactions.

The foundation of the YM theory and the mass gap problem could be formulated as follows: Prove that, for any compact simple gauge group G, there exists a quantum YM theory of R^4, and that this theory predicts a mass gap $\Delta > 0$. Or, more explicitly, given a simple Lie group G (e.g. SU (2) or SU (3)), show that

A) there exists a full renormalized quantum version of YM theory on R^4 based on this group;
B) there is a number $\Delta > 0$ such that every state in the theory (except the vacuum) has energy at least Δ. In other words, there are no massless particles predicted by the theory (except the vacuum state)

Assuming that the quantum chromodynamics (QCD) is the valid theory [1], the first part of CMI problem description, the problem of the foundation of the YM theory, is a technical rather than a fundamental physical problem. The goal of this paper is to investigate the possibility of a solution to the second part of the official (CMI) problem description, namely the mass gap problem, by utilizing the General Field Theory (GFT) formulation of QCD ([5], [6], [7], [8], [9], [10], [11],[12]).

2. The Mass Gap Problem

In the standard QCD model, processes that probe the *short-distance* structure of hadrons predict that quarks inside the hadrons interact weakly [13], [14], [15]. Since coupling, g, is small, the classical QCD analysis is a good first approximation for these processes of interaction [13], [14], [15]. However, for YM theories in general, it is a general requirement of the renormalization group equations of the quantum field that coupling, g_s increases in reverse law to the hadrons' momentum transfer, until momentum transfer becomes equal to the vector boson. Since spontaneous symmetry-breaking via the Higgs mechanism to give the gluons mass is not present in QCD ([16], [17], [18], [19], [20], [21], [22], [23]), QCD contains no mechanism to stop the increase of coupling, g_s. In consequence, quantum effects become more and more dominant as distances increase. Analysis of the behavior of QCD at *long distances*, which includes deriving the hadrons spectrum, requires foundation of the full YM quantum theory. This analysis is proving to be very difficult ([2], [3], [4]).

[1] Note: Lattice QCD is a non-perturbative approach to solving the QCD theory [27], [28]. Analytic solutions in low-energy QCD are hard due to the nonlinear nature of the strong force. This formulation of QCD in discrete space-time introduces a momentum cut off at the order 1/s, where s is the lattice spacing [29], [30].

The mass gap problem can be expressed as follows:

Why are nuclear forces short-range? Why there are no massless gauge particles, even though current experiments suggest that gluons have no mass?

This contradiction is the physical basis of the mass gap problem. More explicitly, let us denote this energy minimum (vacuum energy) of YM Hamiltonian H' by E_m, and let us denote the lowest energy state by φ_m. By shifting original Hamiltonian H by $-E_m$, the new Hamiltonian $H' = H - E_m$ has its minimum at $E = 0$ (massless state) and the first state φ_m is the vacuum vector. We now notice that in space-time the spectrum of Hamiltonian is not supported in region $(0, \Delta)$, with $\Delta > 0$.

3. The General Field Theory Framework

As we have seen, the Higgs mechanism ([16], [17], [18], [19], [20], [21], [22], [23]) is excluded in the QCD theory (gluons are massless because the QCD Lagrangian has no spinless fields and, therefore, no obvious possibility for spontaneous symmetry-breaking). It follows that the only possible mechanism that can introduce the mass term in the gluon field while remaining consistent with the occurrence of mass gap is the Wu mass generator mechanism ([5-12],[41-46]). Here, the Wu mass generator mechanism introduces mass terms without violating the SU (3) gauge symmetry as long as the second gauge field is a purely quantum phenomenon [42].

The quark field is denoted as

$$\psi_a(x), (a = 1,2,3) \tag{1}$$

where α is color index and the flavor index is omitted.

Ψ is defined as follows:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \tag{2}$$

All ψα form the fundamental representative space of SU (3) c.

In this propose version of QCD, the gauge fields $A_{1\mu}^a$ and $A_{2\mu}^a$ are introduced .Gauge field $A_{2\mu}^a$ is introduced to ensure the local gauge invariance of the theory. The generation of gauge field $A_{1\mu}^a$ is a purely quantum phenomenon: $A_{1\mu}^a$ is generated through non-smoothness of the scalar phase of the fundamental spinor fields [42], [47].

From the viewpoint of the gauge field $A_{1\mu}^a$ generation described here, the gauge principle is an "automatic" consequence of the non-smoothness of the field trajectory in the Feynman path integral [42], [47]. The Lagrangian of the model is:

$$\begin{aligned} \Im = -\bar{\psi}[\gamma^5(\partial_\mu - igA_{1\mu}) + m]\psi - \frac{1}{4K}Tr(A_1^{\mu\nu}A_{1\mu\nu}) \\ -\frac{1}{4K}Tr(A_2^{\mu\nu}A_{2\mu\nu}) - \frac{m^2}{2K}Tr[(\cos aA_1^\mu \sin aA_2^\mu)(\cos aA_{1\mu} + \sin aA_{2\mu})] \end{aligned} \tag{3}$$

where

$$A_{1\mu\nu} = \partial_\mu A_{1\nu} - \partial_\nu A_{1\mu} - ig[A_{1\mu}, A_{1\nu}],$$

$$A_{2\mu\nu} = \partial_\mu A_{2\nu} - \partial_\nu A_{2\mu} + igta[A_{2\mu}, A_{2\nu}] \tag{4}$$

This Lagrangian can be proved to have strict SU (3) c gauge symmetry. Since A_μ and $A_{2\mu}$ are not eigenvectors of mass matrix, we can apply the following transformations:

$$\begin{aligned} G_{1\mu} &= \cos aA_{1\mu} + \sin aA_{2\mu} \\ G_{2\mu} &= -\sin aA_{1\mu} + \cos aA_{2\mu} \end{aligned} \tag{5}$$

After these, the Lagrangian given by equation (3) changes into

$$\Im = -\bar{\psi}[\gamma^5(\partial_\mu - ig\cos aG_{1\mu} + ig\sin aG_{2\mu}) + m]\psi - \frac{1}{4}G_{10}^{\mu\nu}G_{10\mu\nu}^i \\ -\frac{1}{4}G_{20}^{\mu\nu}G_{20\mu\nu}^i - \frac{m^2}{2}G_1^{i\mu}G_{1\mu}^i + \Im_{gI}, \tag{6}$$

$$\Im_{Ig} = ig_2\bar{\psi}\gamma^\mu(\cos\theta_{wu}C_\mu - \sin\theta_{wu}F\mu)\psi - \frac{\cos 2\theta_{wu}}{2\cos\theta_{wu}}g_2 f^{ijk}C_0^{i\mu\nu}C_\mu^i C_\nu^k \\ +\frac{\sin\theta_{wu}}{2}g_2 f^{ijk}F_0^{i\mu\nu}F_\nu^j F_\nu^k - \frac{\sin\theta_{wu}}{2}g_2 f^{ijk}F_0^{i\mu\nu}C_\mu^j C_\nu^k + g_2\sin\theta_{wu}f^{ijk}C_0^{i\mu\nu}C_\mu^j F_\nu^k \\ -\frac{1-\frac{3}{4}\sin^2 2\theta_{wu}}{4\cos^2\theta_{wu}}g_2^2 f^{ijk}f^{ilm}C_\mu^j C_\nu^k C^{l\mu}C^{m\nu} - \frac{\sin^2\theta_{wu}}{2}g_2^2 f^{ijk}f^{ilm}F_\mu^j F_\nu^k F^{lm}F^{m\nu} \\ +g_2^2 tc\theta_{wu}\sin\theta_{wu}f^{ijk}f^{ilm}C_\mu^j C_\nu^k C^{lm}F^{m\nu} \\ -\frac{\sin^2\theta_{wu}}{2}g_2^2 f^{ijk}f^{ilm}(C_\mu^j C_\nu^k F^{l\mu}F^{m\nu} + C_\mu^j F_\nu^k F^{l\mu}C^{\nu m} + C_\mu^j F_\nu^k C^{l\mu}F^{m\nu}) \tag{6a}$$

Where

$$G_{m0\mu\nu} = \partial_\mu G^i{}_{m\nu} - \partial_\nu G^i{}_{m\mu}, (m = 1,2), \; G_{m\mu} = G_{m\mu}^i\lambda_i / 2 \tag{7}$$

The Lagrangian $\Im_{gI}$ only contains interaction terms of gauge fields. $\Im_{gI}$ includes a quantum massive gluon field, G_1, with mass m ,and massless gluon field, G_2 .

Since there exist two sets of gluons, there may exist three sets of glueballs in mass spectrum, (g1g1), (g1g2) and (g2g2), with the same spin-parity but different masses (see [12]).

Transformation (5) is pure algebraic operations which do not affect the gauge symmetry of the Lagrangian [5-12]. They can, therefore, be regarded as redefinitions of gauge fields. The local gauge symmetry of the Lagrangian is still strictly preserved after field transformations. In other words, the symmetry of the Lagrangian before transformations is absolutely the same with the symmetry of the Lagrangian after transformations. We do not introduce any kind of symmetry breaking in this paper.

3.1. Renormalization of the Model

In order to quantize the suggested gauge field theory of nuclear forces in the path integral formulation, we first have to select gauge conditions [9], [42]. To fix the degree of freedom of the gauge transformation, we must select two gauge conditions simultaneously: one for the quantum massive gluons $G_{1\mu}$ and another for the massless gluons $G_{2\mu}$ [9], [42] For instance, if we select temporal gauge condition for massless gluons $G_{2\mu}$,

$$G_2 = 0, \tag{7a}$$

there still exists a remainder gauge transformation degree of freedom, because the temporal gauge condition is unchanged under the following local gauge transformation:

$$G_2 \to UG_2U^{-1} + (1/ig\sin\theta)U\partial_\mu U^{-1}, \tag{7b}$$

where

$$\partial_t U = 0, \;\; U = U(\vec{x}). \tag{7c}$$

In order to make this remainder gauge transformation degree of freedom completely fixed, we have to select another gauge condition for gluons $G_{1\mu}$. For instance, we can select the following gauge condition for gluons $G_{1\mu}$,

$$\partial^\mu G_{1\mu} = 0. \tag{7d}$$

If we select two gauge conditions simultaneously, when we quantize the theory in path integral formulation there will be two gauge fixing terms in the effective Lagrangian. The effective Lagrangian can then be written as:

$$\Im_{eff} = \Im - \frac{1}{2a_1}f_1^a f_1^a - \frac{1}{2a_2}f_2^a f_2^a + \bar{\eta}_1 M_{f_1}\eta_1 + \bar{\eta}_2 M_{f_2}\eta_2, \tag{7e}$$

where

$$f_1^a = f_1^a(G_{2\mu}), \;\; f_2^a = f_2^a(G_{1\mu}). \tag{7δ}$$

If we select

$$f_2^a = \partial^\mu G_{1\mu}^\alpha, \tag{7ε}$$

then the propagator for quantum massive gluons $G_{1\mu}$ is:

$$-i\tilde{\Delta}_{F\mu\nu}^{\alpha\beta}(k) = -i\delta^{\alpha\beta} / (k^2 + m_g^2 - i\varepsilon) \times \left[g_{\mu\nu} - (1-a_1)k_\mu k_\nu / (k^2 + a_2 m_g^2)\right]. \tag{7μ}$$

If we let k approach infinity, then

$$\tilde{\Delta}_{F\mu\nu}^{\alpha\beta}(k) = \frac{1}{k^2}. \tag{7θ}$$

In this case, and according to the power-counting law, the gauge field theory of nuclear forces with quantum massive gluons suggested in this paper is a kind of renormalizable theory [9], [42]. A strict proof on the renormalizability of the gauge field theory can be found in [9].

3.2. The Two Different Kinds of Yang-Mills QCD Limit

In a proper limit, GFT [5-12] gauge field theory can return to YM gauge field theory. There are two kinds of YM limits. The first kind of YM limit corresponds to very small parameter a in the equation (5). Let:

$$a \to 0 \tag{7ι}$$

Then

$$\cos\theta = 1, \sin\theta = 0 \tag{7ξ}$$

$$G_{1\mu} = A_\mu, G_{2\mu} = B_\mu \tag{7φ}$$

In this case, the Lagrangian density becomes

$$\begin{aligned} \Im &= -\bar{\psi}\left[\gamma^\mu\left(\partial_\mu - igG^i_{1\mu}T^i\right) + m\right]\psi \\ &-\frac{1}{4}C^{i\mu\nu}C^i_{\mu\nu} - \frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{m^2}{2}C^{i\mu}C^i_\mu \end{aligned} \tag{7λ}$$

We could see that, only quantum massive gauge field directly interacts with matter field. This limit corresponds to the case that gauge interactions are mainly transmitted by quantum massive gauge field. But if a strictly vanishes, the Lagrangian does not have gauge symmetry and the theory is not renormalizable. The second kind of YM limit corresponds to very large parameter a in the equation (5). Let:

$$a \to \infty \tag{7o}$$

then

$$\cos\theta = 0, \sin\theta = 1 \tag{7κ}$$

$$G_{1\mu} = B_\mu, G_{2\mu} = -A_\mu \tag{7σ}$$

Then, the Lagrangian density becomes

$$\begin{aligned} \Im &= -\bar{\psi}\left[\gamma^\mu\left(\partial_\mu + igG^i_{2\mu}T^i\right) + m\right]\psi \\ &-\frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{1}{4}C^{i\mu\nu}C^i_{\mu\nu} - \frac{m^2}{2}C^{i\mu}C^i_\mu \end{aligned} \tag{7υ}$$

In this case, only massless gauge field directly interacts with matter fields. This limit corresponds to the case that gauge interactions are mainly transmitted by massless gauge field. In the particles' interaction model, the parameter a should be finite,

$$0 < a < \infty \tag{7π}$$

In this case, both quantum massive gauge field and massless gauge field directly interact with matter fields, and gauge interactions are transmitted by both of them.

4. The Physics of Mass Gap Problem in the General Field Theory Framework

In Quantum Field Theory (QFT) the number of particles is not constant, due to the particles' creation and annihilation. Therefore, while we can define the mass-state of the system as the mass-state of all particles or equivalent, we cannot define the mass-state of a specific particle. In propose version of QCD, nevertheless, the mass-state of the system has non-zero mass. A possible interpretation of the existence of both quantum massive and massless gluon states is that Wu's mass generator mechanism introduces the phenomenon of mass gap. This is achieved in the following steps: First, in the proposed QCD version the non-zero gluon mass corresponds to the massive vector propagator [24] and the zero gluon mass corresponds to the massless vector propagator of the short-distance QCD [13], [14], [15]. For the given wave vectors k_1 and k_2 the propagators of massless and quantum massive gluons with respect to the two different YM gauge field models are given by:

$$\Delta^{\alpha\beta}_{2\mu\nu}(k_2) = -i\delta^{\alpha\beta}\left(\frac{g_{2\mu\nu}}{k_2^2 + i\varepsilon}\right) \tag{8}$$

$$\Delta^{\alpha\beta}_{1\mu\nu}(k_1) = \frac{-i\delta^{\alpha\beta}}{k_1^2 + M^2 - i\varepsilon}\left(g_{1\mu\nu} + \frac{k_{1\mu}k_{1\nu}}{M^2}\right) \tag{9}$$

where [2]

$$1\mu \neq 2\mu, 1\nu \neq 2\nu, \tag{10a}$$

$$g_{1\mu\nu} = (e_{1\mu}, e_{1\nu}), \; g_{2\mu\nu} = (e_{2\mu}, e_{2\nu}), \tag{10b}$$

$$\frac{\partial}{\partial x^{1\mu}} = e_{1\mu}, \quad \frac{\partial}{\partial x^{2\mu}} = e_{2\mu} \tag{10c}$$

$g_{1\mu\nu}, g_{2\mu\nu}$ are metrics of the space-time regions and $e_{2\mu}$, $e_{2\mu}$ are vectors of the basis induced by the selection of local coordinates $x^{1\mu}$, $x^{2\mu}$ on the neighborhood of the given points corresponds to the two different YM gauge field models.

Furthermore, due to equation (10a), the wave vectors $k_{1\mu}$ and $k_{2\mu}$ lie in different regions in the momentum space in their own right. Here, we express the meet of the propagators $\Delta^{\alpha\beta}_{1\mu\nu}(k_1)$ and $\Delta^{\alpha\beta}_{2\mu\nu}(k_2)$ of the two different YM gauge field models in the momentum space, in terms of its hypersurface of the present ([25]) at the origin of the propagators paths as follows

$$\begin{aligned} &\delta^{1\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{1\mu\nu}(k_1) \wedge \delta^{2\mu\nu}_{\alpha\beta}\Delta^{\alpha\beta}_{2\mu\nu}(k_2) = \\ &\liminf_{(k_1,k_2)\to S'} \{\min_{i\varepsilon\to 0}(\Delta_1(k_1), \Delta_2(k_2))\} \end{aligned} \tag{11a}$$

where

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(k_1)=\Delta_1(k_1)=\frac{1}{k_1^2+m^2-i\varepsilon}, \quad (11b)$$

$$\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(k_2)=\Delta_2(k_2)=\frac{1}{k_2^2-i\varepsilon} \quad (11c)$$

are functions in the momentum space. The subset $S'=S\setminus r_1$ of the hypersurface of the present is disconnected, since it is the disjoint union of the two propagator's half-planes of the present that corresponds to the different 1μ and 2μ indices of two different YM gauge field models.

[2] Note: In the proposed version of QCD, the letters $1\mu,1\nu$ and $2\mu,2\nu$ denote the different space-time indices of the massive and massless propagators corresponding to different YM gauge field models

$$U_1^+\cap U_2^-=\varnothing$$

and

$$S'=S\setminus r_1=U_1^+\cup U_2^-=(-\infty,+\infty)\setminus\{0\}, \quad (11d)$$

$$U_1^+=\{S'\cap\{(k_{11},k_{12})\in R^2: ak_{11}+bk_{12}+c>0\}=(0,+\infty), \quad (11e)$$

$$U_1^-=\{S'\cap\{(k_{21},k_{22})\in R^2: ak_{21}+bk_{22}+c<0\}=(-\infty,0) \quad (11f)$$

the $r_1: ak_x+bk_y+c=0$ is line such that the points (k_{11},k_{12}) and (k_{21},k_{22}) of S lie on different sides of r_1 and U_1^+,U_2^- are the propagator's half-planes of the present.

Substituting equations (11d), (11c), (11d) to equation (11a), we calculate the meet of the $\Delta_{1\mu\nu}^{\alpha\beta}(k_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(k_2)$ propagator in the momentum space

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(k_1)\wedge\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(k_2)=0 \quad (11g)$$

Equation (11g) shows that the meet of the propagators $\Delta_{1\mu\nu}^{\alpha\beta}(k_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(k_2)$ of the two different YM gauge field models is zero, because the propagator's half-planes of the present are disjoint in the momentum space. After some calculations, the Fourier transformation to the position space of equation (11g) is given by

$$\delta^{(4)}(x_2-y_2)\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1-y_1)\wedge\delta^{(4)}(x_1-y_1)\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2-y_2)=0 \quad (12)$$

where

$$\delta^{(4)}(x_1-y_1)=\int_{-\infty}^{+\infty}\frac{d^4k_1}{(2\pi)^4}e^{ik_1(x_1-y_1)},\ \delta^{(4)}(x_2-y_2)=\int_{-\infty}^{+\infty}\frac{d^4k_2}{(2\pi)^4}e^{ik_2(x_2-y_2)} \quad (13)$$

$$\Delta_{2\mu\nu}^{\alpha\beta}(x_2-y_2)=\lim_{\varepsilon\to0^+}\int_{-\infty}^{+\infty}\frac{d^4k_2}{(2\pi)^4}\frac{e^{ik_2(x_2-y_2)}}{k_2^2+i\varepsilon}g_{2\mu\nu}(-i\delta^{\alpha\beta}), \quad (14)$$

$$\Delta_{1\mu\nu}^{\alpha\beta}(x_1-y_1)=\lim_{\varepsilon\to0^+}\int_{-\infty}^{+\infty}\frac{d^4k_1}{(2\pi)^4}\frac{e^{ik_1(x_1-y_1)}}{k_1^2+M^2-i\varepsilon}(-i\delta^{\alpha\beta})\left(g_{1\mu\nu}+\frac{k_{1\mu}k_{1\nu}}{M^2}\right) \quad (15)$$

$\delta^{(4)}(x_1-y_1)$, $\delta^{(4)}(x_2-y_2)$ are the Dirac delta functions and $\Delta_{1\mu\nu}^{\alpha\beta}(x_1-y_1)$, $\Delta_{2\mu\nu}^{\alpha\beta}(x_2-y_2)$ are the two gluon propagators in the position space.

The integral of equation (12) over d^4x_1 , d^4x_2 is given as follows:

$$\iint\delta^{(4)}(x_2-y_2)\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1-y_1)d^4x_2d^4x_1\wedge\iint\delta^{(4)}(x_1-y_1)\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2-y_2)d^4x_1d^4x_2=0 \quad (16)$$

Here, we integrate equation (16) in the following steps. First, by fixed y_1,y_2 , the equation (16) becomes

$$\int\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1-b_1)d^4x_1\wedge\int\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2-b_2)d^4x_2=0 \quad (17)$$

Next, by considering an insulating color source, in Feynman gauge, the propagators magnitudes are constant in the position space. Therefore the integral of the equation (12) over d^4x_1 , d^4x_2 , gives

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1-b_1)V_4^{(1)}\wedge\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2-b_2)V_4^{(2)}=0 \quad (18)$$

where

$$V_4^{(1)}=\int d^3x_1\int dt1\ ,\ V_4^{(2)}=\int d^3x_2\int dt1 \quad (19)$$

$V_4^{(1)}$, $V_4^{(2)}$ are the space-time volumes.

Equation (18) is written in style of (11g) as follows

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1)\wedge\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2)=$$

$$\liminf_{(x_1,x_2)\to S'}\{\min(\Delta_1(x_1),\Delta_2(x_2))\}=0 \quad (20a)$$

where

$$\delta_{\alpha\beta}^{1\mu\nu}\Delta_{1\mu\nu}^{\alpha\beta}(x_1)=\Delta_1(x_1)=\frac{e^{-mx_1}}{x_1}, \quad (20b)$$

$$\delta_{\alpha\beta}^{2\mu\nu}\Delta_{2\mu\nu}^{\alpha\beta}(x_2)=\Delta_2(x_2)=\frac{1}{x_2} \quad (20c)$$

are functions in the position space. The subset $\tilde{S}'=\tilde{S}\setminus r_2$ of the hypersurface of the present is disconnected, since it is the disjoint union of the two propagator's half-planes of the present that corresponds to the different 1μ and 2μ indices of two different YM gauge field models.

$$\tilde{U}_1^+\cap\tilde{U}_2^-=\varnothing \text{ and } \tilde{S}'=\tilde{S}\setminus r_2=\tilde{U}_1^+\cup\tilde{U}_2^-=(-\infty,+\infty)\setminus\{0\} \quad (20d)$$

$$\tilde{U}_1^+=\{\tilde{S}'\cap\{(x_{11},x_{12})\in R^2:ax_{11}+bx_{12}+c>0\}=(0,+\infty), \quad (20e)$$

$$\tilde{U}_1^-=\{\tilde{S}'\cap\{(x_{21},x_{22})\in R^2:ax_{21}+bx_{22}+c<0\}=(-\infty,0) \quad (20f)$$

the $r_2:ax+by+c=0$ is line such that the points (x_{11},x_{12}) and (x_{21},x_{22}) of $\tilde{S}$ lie on different sides of r_2 and $\tilde{U}_1^+,\tilde{U}_2^-$ are the propagator's half-planes of the present.

Equation (20a) shows that the propagator's half-planes of the present are disjoint in the position space, because the meet of $\Delta_{1\mu\nu}^{\alpha\beta}(x_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(x_2)$ of the two different YM gauge field models is zero. $\Delta_{1\mu\nu}^{\alpha\beta}(x_1-b_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(x_2-b_2)$ lie outside each other's light cone [25]; these propagators, therefore, are not casually connected [26]. Propagators $\Delta_{1\mu\nu}^{\alpha\beta}(x_1-b_1)$ and $\Delta_{2\mu\nu}^{\alpha\beta}(x_2-b_2)$ are separated from each other in a (space-like) direction of distance

$$S^2=|\eta_{1\mu\nu}x^{1\mu}x^{1\nu}-\eta_{2\mu\nu}x^{2\mu}x^{2\nu}| \quad (20g)$$

Where $\eta_{1\mu\nu}$, $\eta_{2\mu\nu}$ are the metrics of the disjoint space-time regions of the two different YM gauge field models. The space-like region S^2 corresponds to an energy scale ($m_g^2c^4=\hbar^2/S^2$). Here, the gluon mass is complex number $\mu_g+i\mu_g$, the imaginary part $i\mu_g$ corresponds to the space-like region between the two YM light cone limits and the real part μ_g is obtained by the mass definition within the space-time regions of the two different YM gauge field models, which must be real. The m_g is the absolute value of the gluon mass $m_g = (\mu_g+i\mu_g)(\mu_g-i\mu_g)$ that is constrained to the above space-like portion of space-time [37],[38],[39],[40]. This massive gluon either travels forward in time with complex mass $\mu_g+i\mu_g$ or travels backward in time with complex conjugate mass $\mu_g-i\mu_g$ [37], [38].

Next, a basic principle in nuclear physics which states that the combined operations of charge conjugation (C), time reversal (T), space inversion (P) in any order is an exact symmetry of the strong force [31], [32]. The CPT symmetry is conserved only if our theory respects the Lorentz invariant and the microcasuality principle [33], [34].

Gluon of complex mass exchanged between quarks affect our massless YM light cone limit, respecting the microcasuality principle by the following postulates:

A) The proposed gluons of complex mass $\mu_g+i\mu_g$ lie outside of our massless YM light cone limit. Therefore such gluons are not an observable mass state for an observer that lies inside our massless YM light cone limit.

B) In the low scale of energies quarks and gluons will arrange themselves into a strong bound state in where at least two quarks and gluons must lie outside each other's YM light cone limit. Furthermore, for a multi quarks and gluons states that lie inside our massless YM light cone limit, there exists at least one quark and gluon state that lies inside the massive YM light cone limit.

A gluon of complex mass which is exchanged between quarks is equivalent to the massive Abelian gluon Δ_μ by postulate (B), whose absolute value of mass m_g is not an observable mass state in our massless YM light cone limit by the postulate (A).

The postulates predict the quantum massive gap Abelian gluon Δμ whose absolute value of mass affects our massless YM limit in a way that makes nuclear forces short-range at low scale of energies [1], [2].

The Lagrangian density (7υ) becomes

$$\Im=-\bar{\psi}\left[\gamma^\mu\left(\partial_\mu+igG_{2\mu}^iT^i+g_\delta\Delta_{1\mu}^0T^0\right)+m\right]\psi-\frac{1}{4}G^{i20\mu\nu}G_{20\mu\nu}^i-\frac{1}{4}\Delta^{0\mu\nu}\Delta_{\mu\nu}^0-\frac{\Delta^2}{2}\Delta^{0\mu}\Delta_\mu^0 \quad (20\upsilon)$$

where Δμ quantum massive gap Abelian gluon, G2 massless gluon and (Δ) is the mass gap: Δ=(μg+iμg)(μg-iμg)= mg. Where g and gδ are the massless gluon coupling and quantum massive gap Abelian gluon coupling respectively.

Here, the quantum massive gap Abelian gluon is defined in the Abelian subgroup U (1) Δ of the SU(3)c gauge group for details (see [48]).

For the Lagrangian density (20υ) we construct the Hamiltonian of the system for details (see [49]).

Equations (11g) and (20a) predict that the Hamiltonian is not supported in an space-like region S_Δ^2 that corresponds to an energy scale ($\Delta^2c^4=\hbar^2/S_\Delta^2$). Therefore, the Hamiltonian in space-time is bounded by Δ.

Finally, in the proposed version of QCD, the vacuum state has zero energy (E=0), corresponding to G2 (massless gluon). The excitation state above the vacuum energy has non-zero energy Em, corresponding to G1 (quantum massive gluon). The latter predicts that the Hamiltonian is not supported in the region $(0,\Delta)$ with $\Delta>0$ and is bounded below by Δ .

5. Discussion

Hadron colliders, such as the Tevatron, can place the strongest limits on the new colored particles (squarks and gluinos in supersymmetry or KK excitations of quarks and gluons in models with universal extra dimensions [35–36]. Typically, limits of~ 200 GeV are obtained for new colored particles. Here, the predicted gluon mass gap state Δ is not close to the gluon mass ~ 200 GeV , thus gluon mass gap is not subject to the Hadron collider's limits [35-36].

In the Lorentz gauge, the free Lagrangian of QCD field theory (equation (6), with $\Im_{gI} = 0$) yields to the following field equations:

$$\frac{1}{c^2}\frac{\partial^2 G_{1\mu}}{\partial t^2} = \nabla^2 G_{1\mu}(r) - \frac{1}{L^2}G_{1\mu}(r), \qquad \frac{1}{c^2}\frac{\partial^2 G_{2\mu}}{\partial t^2} = \nabla^2 G_{2\mu}(r) \tag{30}$$

L the length of the system given by

$$L = \frac{\hbar}{mc} \tag{31}$$

Where $\hbar$ the Planck constant, c is the speed of light, $\Delta=(\mu_g+i\mu_g)(\mu_g-\mu_g)= m_g$ the mass gap of the system. If the source is a point color at the origin, only the time-components $C_{02} = \Phi_2$, $C_{01} = \Phi_1$ are nonvanishing. The short-distance solution of the eq (30) that corresponds to the massless gluons is given as follows:

$$G_{20}(r) = \frac{a_s}{4\pi\,|\,r\,|} \tag{32}$$

where a_s is the quark-gluon coupling.

Gluons that respect the solution (32) are mediators of the long-range nuclear force at high energy scale. The long-distance solution of eq (30) corresponds to the massive gluons and is given as follows:

$$G_{10}(r) = \frac{a_s}{4\pi\,|\,r\,|}e^{-|r|/L} \tag{33}$$

Gluons that respect the solution (33) are mediators of the short-range nuclear force at low energy scale. Here, the massive and massless gluons lie outside each other's light cone, because of eq (20a). The term (31) in eq (30) and (33) reveals as a mass gap. Supposing that the range of the strong force is about the radius of the proton, equation (31) predicts mass gap of about 1GeV.

We find that, in the propose version of QCD, free system's energies are indeed bounded in space-time. Consequently, in space-time, the YM Hamiltonian H_{ym} has spectrum bounded below. Let us denote this energy minimum of the YM Hamiltonian H_{ym} by E_m , and let us denote the lowest energy state by φ_m . By shifting the original Hamiltonian H by $-E_m$, the new Hamiltonian $H_G = H - E_m$ has its minimum at $E = 0$. The first state φ_m is the vacuum vector. We now notice that, in space-time, the spectrum of the YM Hamiltonian is not supported in region $(0,\Delta)$, with $\Delta > 0$.

According to the YM theory, therefore, over compact gauge groups SU (3) there is always a vacuum vector φ_m and a mass gap Δ. Wu's mass generator mechanism can always be applied to any YM type of action. Thus, any YM theory accounts for a mass gap.

The proposed model is a massive theory without a symmetry breaking mechanism and is therefore a model with a quantum mass gap. Thus, the massless YM theory is altered with a quantum massive model and the original problem could be solved.

6. Conclusion

The general gauge theory with quantum massive and classical massless no-Abelian vector modes and the gauge interaction terms is developed. The suggested gauge field model will return to the two different YM gauge field models. `The meet of the propagators' of those quantum massive and classical massless vector fields with respects to the double Yang-Mills limit is calculated.

We have shown that the proposed gauge model can introduce mass terms while being consistent with the existing mass gap Δ. In this model the mass gap is introduced by Wu's mass generator mechanism.

Since the gluon quantum field G_1 is massive, whereas the gluon field G_2 is massless, we conclude that system's free energies in the proposed version of QCD are bounded in space-time.

We also notice that, in space-time, the spectrum of the YM Hamiltonian is not supported in region $(0,\Delta)$ with $\Delta > 0$. Wu's mass generator mechanism can always be applied to any YM type of action. Therefore any YM theory accounts for a mass gap.

The proposed model is a massive theory without a symmetry breaking mechanism and is therefore a model with a quantum mass gap. Thus, the massless YM theory is altered with a quantum massive model and the original problem could be solved.

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