Optimization of closed-loop supply chain problem for calculation logistics cost accounting

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Abstract: This paper aims to build closed loop supply chain model (CLSCM) and propose a multi-objective genetic algorithm. This paper designs the method of calculation for a solution using optimization algorithms with the priority-based genetic algorithm (priGA), and Adaptive Weight Approach (AWA). In this paper, we present a multi-objective closed loop supply chain model in integrated logistics system. We formulated a mathematical model with two objectives functions: (1) minimize transportation cost, open cost, inventory cost, purchase cost, disposal cost and saving cost of integrated facilities of CLSCM, (2) minimize the delivery time tardiness in all periods. Finally, a simulation is investigated to demonstrate the applicability of the proposed multi-objective closed loop supply chain model (CLSCM) and solution approaches.

Keywords: Closed-Loop Supply Chain, Logistics Cost Accounting, Genetic Algorithm

1. Introduction

Reverse Logistics, which is the logistics activity covering over the produce recovery, recycling, waste disposal and etc., has received considerable attention due to the following two reasons. First, the seriousness of environmental problem has been embossed in corporate logistics activity and the environmental logistics problem has been international issue by Government Resolutions and etc. Second, the resources have been exhausted all over the world.

In the recent years due to the ever-increasing development of the competitive environment, the researchers have paid attention to the supply chain network problem. This problem has a key importance role in long-term decisions/performance and requires to be optimized for efficiency of the whole supply chain [1].

In the following, we introduce research papers associated with closed-loop supply chain model (CLSCM).

Ma and Wang [2] consider a closed-loop supply chain (CLSC) with product recovery, which is composed of one manufacturer and one retailer. The retailer is in charge of collecting and the manufacturer is responsible for product recovery. The system can be regarded as a coupling dynamics of the forward and reverse supply chain.

Chuang et al. [3] study closed-loop supply chain models for a high-tech product which is featured with a short life-cycle and volatile demand. They focus on the manufacturer's choice of three alternative reverse channel structures (i.e., manufacturer collecting, retailer collecting, and third-party firm collecting) for collecting the used product from consumers for remanufacturing: (1) the manufacturer collects the used product directly; (2) the retailer collects the used product for the manufacturer; and (3) the manufacturer sub contracts the used product collection to a third-party firm.

Lee et al. [4] is also proposed multi-objective closed-loop supply chain model. However, they only consider a limited logistics cost (e.g., transportation cost and open cost). We propose the improved model considering minimization of total cost (e.g., transportation cost, open cost, inventory cost, purchase cost, disposal cost and saving cost of integrated facilities) and minimization of total delivery tardiness.

This paper is organized as follows: in Section 2, the mathematical model of the multi-objective reverse logistics network is introduced; in Section 3, the priority-based encoding method and adaptive weight approach (AWA) are explained in order to solve this problem; in Section 4, numerical experiments are presented to demonstrate the efficiency of the proposed method; finally, in Section 5, concluding remarks are outlined.
2. Problem Definition

2.1. Mathematical Model of CLSC

The model is a multi-objective problem considered the multi echelon, multi period, and multi product in closed-loop supply chain. We formulated the CLSCM as a multi-objective 0-1 mixed integer linear programming model.

The following assumptions are made in the development of the model:
A1. Only one product is treated in closed loop supply chain model.
A2. The inventory factor is existed over finite planning horizons.
A3. The requirement by manufacturer and the quantity of collected products is known in advance.
A4. The maximum capacities about echelons are known.
A5. All of inventory holding costs of processing centers are same.
A6. In the case of transportation from processing center to manufacturer, the lot size is 100 and the lead time is not considered.

The parameters, decision variables, objective functions, and restrictions in this closed-loop supply chain model are as follows.

(1) Indices
- $i$: index of manufacturer ($i=1, 2, \ldots, I$)
- $j$: index of distribution center ($j=1, 2, \ldots, J$)
- $k$: index of retailer ($k=1, 2, \ldots, K$)
- $l$: index of customer ($l=1, 2, \ldots, L$)
- $k'$: index for returning center ($k'=1, 2, \ldots, K'$)
- $m$: index of processing center ($m=1, 2, \ldots, M$)
- $t$: index of time period ($t=1, 2, \ldots, T$)

(2) Parameters
- $I$: number of manufacturers
- $J$: number of distribution centers
- $K$: number of retailers
- $L$: number of customers
- $K'$: number of returning centers
- $M$: number of processing centers
- $N$: disposal center
- $S$: supplier
- $T$: planning horizons
- $a_i$: capacity of manufacturer $i$
- $b_j$: capacity of distribution center $j$
- $u_k$: capacity of retailer $k$
- $u_k'$: capacity of returning center $k'$
- $u_m$: capacity of processing center $m$
- $d_{kj}$: demand of manufacturer $i$
- $c_{ij}$: unit cost of transportation from manufacturer $i$ to distribution center $j$
- $c_{jk}$: unit cost of transportation from distribution center $j$ to retailer $k$
- $c_{lk}$: unit cost of transportation from retailer $k$ to customer $l$
- $c_{lk'}$: unit cost of transportation from customer $l$ to returning center $k'$
- $c_{kl'}$: unit cost of transportation from returning center $k'$ to processing center $m$
- $c_{mk}$: unit cost of transportation from processing center $m$ to disposal $N$
- $c_{mk'}$: unit cost of transportation from processing center $m$ to manufacturer $i$
- $c_{sk}$: unit cost of transportation from supplier $S$ to manufacturer $i$
- $c_{sk'}$: open cost of distribution center $j$
- $c_{sk'}$: open cost of retailer $k$
- $c_{kl'}$: open cost of returning center $k'$
- $c_{jk'}$: unit holding cost of inventory per period at distribution center $j$
- $c_{jk'}$: unit holding cost of inventory per period at retailer $k$
- $c_{jk'}$: unit holding cost of inventory per period at returning center $k'$
- $c_{mk'}$: unit holding cost of inventory per period at processing center $m$
- $r_{N}$: disposal rate
- $d_{ij}$: delivery time from returning center $i$ to processing center $j$
- $d_{jm}$: delivery time from processing center $j$ to manufacturer $M$
- $p_i$: processing time for reusable product in processing center $i$
- $t_e$: expected delivery time by customers
- $f_2$: (3) Decision variables
- $x_{ij}(t)$: amount shipped from manufacturer $i$ to distribution center $j$ in period $t$
- $x_{jk}(t)$: amount shipped from distribution center $j$ to retailer $k$ in period $t$
- $x_{lk}(t)$: amount shipped from retailer $k$ to customer $l$ in period $t$
- $x_{jk'}(t)$: amount shipped from returning center $k'$ to processing center $m$ in period $t$
- $x_{mk}(t)$: amount shipped from processing center $m$ to disposal $N$ in period $t$
- $x_{mk'}(t)$: amount shipped from processing center $m$ to manufacturer $i$ in period $t$
- $y_{ij}(t)$: inventory amount at distribution center $j$ in period $t$
- $y_{ik}(t)$: inventory amount at processing center $m$ in period $t$
- $y_{ik'}(t)$: inventory amount at returning center $k'$ in period $t$
- $y_{mk}(t)$: inventory amount at processing center $m$ in period $t$

2.2. Mathematical Formulation

The first objective function, $f_1$, consists of the total cost.

\[
\text{Minimize } \{f_1, f_2\} \quad (1)
\]

Min

\[
f_1 = TC + OC + IC + PC + RTC + ROC + RIC + DC - SC \quad (1)\]

The cost components in the objective function $F_1$ can be calculated by using the following relations:

Forward logistics transportation costs
\[ TC = \sum_{j=1}^{J} \sum_{i=1}^{I} c_{ij}^p x_{ij}(t) + \sum_{j=1}^{J} \sum_{k=1}^{K} c_{jk}^p x_{jk}(t) + \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl}^p x_{kl}(t) \]

Forward logistics open costs

\[ OC = \sum_{j=1}^{J} \sum_{i=1}^{I} c_{ij}^o z_{ij}(t) + \sum_{k=1}^{K} \sum_{i=1}^{I} c_{ik}^o z_{ik}(t) \]

Forward logistics inventory costs

\[ IC = \sum_{j=1}^{J} \sum_{i=1}^{I} c_{ij}^m y_{ij}(t) + \sum_{k=1}^{K} \sum_{i=1}^{I} c_{ik}^m y_{ik}(t) \]

Forward logistics purchase costs

\[ PC = \sum_{j=1}^{J} \sum_{i=1}^{I} c_{ij}^b x_{ij}(t) \]

Reverse logistics transportation costs

\[ RTC = \sum_{j=1}^{J} \sum_{k=1}^{K} c_{jk}^r x_{jk}(t) + \sum_{k=1}^{K} \sum_{m=1}^{M} c_{km}^r y_{km}(t) + \sum_{l=1}^{L} \sum_{m=1}^{M} c_{lm}^r y_{lm}(t) \]

Reverse logistics open costs

\[ ROC = \sum_{j=1}^{J} \sum_{k=1}^{K} c_{jk}^o z_{jk}(t) \]

Reverse logistics inventory costs

\[ RIC = \sum_{j=1}^{J} \sum_{k=1}^{K} c_{jk}^m y_{jk}(t) + \sum_{k=1}^{K} \sum_{m=1}^{M} c_{km}^m y_{km}(t) \]

Reverse logistics disposal costs

\[ DC = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}^r x_{ij}(t) \]

Saving cost from integrating retailer/returning center

\[ SC = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}^s z_{ij}(t) \]

The second objective function, \( f_2 \) is total delivery tardiness.

\[ f_2 = \sum_{j=1}^{J} \sum_{i=1}^{I} d_{ij} x_{ij}(t) + \sum_{j=1}^{J} (d_{j} + p_j) x_{j}(t) - t_d y(t) \]  

\[ f_2 = \sum_{j=1}^{J} \sum_{i=1}^{I} d_{ij} x_{ij}(t) + \sum_{j=1}^{J} (d_{j} + p_j) x_{j}(t) - t_d y(t) \]

Subject to

- open cost

\[ 1 - z_{j}(t-1) = z_{j}(t) \quad \forall j \in J, t \in T \]  

\[ 1 - z_{k}(t-1) = z_{k}(t) \quad \forall k \in K, t \in T \]

- inventory costs

\[ y_{j}(t) = x_{j}(t) + x_{j}(t-1) \quad \forall j \in J, t \in T \]

\[ y_{k}(t) = x_{k}(t) + x_{k}(t-1) \quad \forall k \in K, t \in T \]

\[ y_{m}(t) = x_{m}(t) + x_{m}(t-1) - x_{m}(t) \quad \forall m \in M, t \in T \]

- disposal costs

\[ \sum_{j=1}^{J} x_{j}(t) = \sum_{j=1}^{J} x_{j}(t) y_{j}(t) \quad \forall t \in T \]

- capacity constraints

\[ \sum_{j=1}^{J} x_{j}(t) + y_{j}(t-1) \leq b_{j} z_{j}(t) \quad \forall j \in J, t \in T \]

\[ \sum_{k=1}^{K} x_{k}(t) + y_{k}(t-1) + \sum_{m=1}^{M} y_{m}(t) + y_{m}(t-1) \leq a_{k} z_{k}(t) + u_{k} z_{k}(t) \quad \forall k \in K, k \in K', t \in T \]

- demand constraints

\[ x_{m}(t) + x_{m}(t) + x_{m}(t) = d_{j} \quad \forall t \in T \]

\[ \sum_{j=1}^{J} x_{j}(t) \leq d_{j}(t) \quad \forall t \]

- non-negativity constraints

\[ x_{j}(t), x_{j}(t), x_{j}(t), x_{j}(t), x_{j}(t), x_{j}(t), x_{j}(t), x_{j}(t) \geq 0 \quad \forall i \in I, j \in J, k \in K, l \in L, m \in M, t \in T \]

- binary constraints

\[ z_{j}(t) = \{0,1\} \quad \forall j \in J, t \in T \]

\[ z_{k}(t) = \{0,1\} \quad \forall k \in K, t \in T \]

\[ z_{m}(t) = \{0,1\} \quad \forall k \in K', t \in T \]

\[ \sum_{j=1}^{J} x_{j}(t) + y_{j}(t-1) \leq b_{j} z_{j(t)} \quad \forall j, t \]

\[ y_{j}(t) = \sum_{j=1}^{J} \sum_{j=1}^{J} x_{j}(t) - x_{j}(t) = y_{j}(t) \quad \forall j, t \]

\[ x_{j}(t), x_{j}(t), y_{j}(t) \geq 0, \forall i, j, t \]

3. Optimization of the Closed-Loop Supply Chain with the Genetic Algorithm

3.1. Priority-Based Encoding Method

For a transportation problem, a chromosome consists of priorities of sources and depots to obtain transportation tree
and its length is equal to total number of sources \( m \) and depots \( n \), i.e. \( m+n \). The transportation tree corresponding with a given chromosome is generated by sequential arc appending between sources and depots [5]. At each step, only one arc is added to tree selecting a source (depot) with the highest priority and connecting it to a depot (source) considering minimum cost [6].

3.2. Adaptive Weight Approach

While we consider multiobjective problem, a key issue is to determine the weight of each objective. Gen et al. [7] proposed an Adaptive Weight Approach (AWA) that utilizes some useful information from the current population to re-adjust weights for obtaining a search pressure toward a positive ideal point[8]. In this study, we are using the following objectives:

1. Minimization of the total cost \( (c_1) \)
2. Minimization of the delivery tardiness. \( (c_2) \)

\[
\begin{align*}
\max \{ & f_1 = \frac{1}{f_1(v_i)} = \frac{1}{c_1}, \quad f_2 = \frac{1}{f_2(v_i)} = \frac{1}{c_2} \} \\
\end{align*}
\]

(20)

For the solutions at each generation, \( z_q^{\text{max}} \) and \( z_q^{\text{min}} \) are the maximal and minimal values for the \( q \)th objective as defined by the following equations:

\[
\begin{align*}
z_q^{\text{max}} &= \max\{f_q(v_i), k = 1, 2, \ldots, \text{popSize}\}, \quad q = 1, 2 \\
z_q^{\text{min}} &= \min\{f_q(v_i), k = 1, 2, \ldots, \text{popSize}\}, \quad q = 1, 2 \\
\end{align*}
\]

(21)

The adaptive weights are calculated as

\[
w_q = \frac{1}{z_q^{\text{max}} - z_q^{\text{min}}}, \quad q = 1, 2
\]

(22)

The weighted-sum objective function for a given chromosome is then given by the following equation

\[
eval(v_i) = \sum_{q=1}^{\text{popSize}} w_q (f_q(v_i) - z_q^{\text{min}}), \quad k = 1, 2, \ldots, \text{popSize}
\]

(23)

4. Simulation

In this section, multiobjective hybrid genetic algorithm and CPLEX software is used to compare the results of small-size problems. All the test problems are solved on a Pentium 4, 3.20GHz clock pulse with 1GB memory. The data in test problems were also randomly generated to provide realistic scenarios. The 3 test problems were combined, as shown in Table 1.

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>Period</th>
<th>returning centers</th>
<th>processing centers</th>
<th>No. of constraints</th>
<th>No. of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>264</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>852</td>
<td>404</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>20</td>
<td>15</td>
<td>2988</td>
<td>1452</td>
</tr>
</tbody>
</table>

We ran the procedure for 20 times for each problem considering following parameters:

- Population size, \( \text{popSize}=100 \);
- Maximum generations, \( \text{maxGen}=1000 \);
- Crossover probability, \( p_c=0.7 \);
- Mutation probability, \( p_m=0.3 \).

4.1. Numerical Results

In this paper, we compared percentage gap of CPLEX, Priority-based encoding method with Adaptive Weight Approach (pri-awGA) and multiobjective hybrid genetic algorithm (mo-hGA).

In order to compare the non-dominated solutions of the two methods, the value of the second objective function \( (f_2) \) of each solution obtained by mo-hGA is insersted into the model formulation as a new constraint.

\[
\text{gap}(%)=100\left( \frac{\text{f}_2 - \text{CPLEX f}_2}{\text{CPLEX f}_1} \right)
\]

(24)

In table 2, we explain the simulation results for 3 test problems with 20 instances in each. We use the percentage gap between optimum solution and heuristic solutions, which are pri-awGA and mo-hGA. And we also compare CPLEX \( f_1 \) and GAs \( f_i \) and Pareto solutions \( (f_1, f_2) \) at the same time.

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>CPLEX ( f_1 )</th>
<th>pri-awGA ( f_1 )</th>
<th>mo-hGA ( f_1 )</th>
<th>Optimalaty Gaps (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_1 )</td>
<td>( f_1 )</td>
<td>( f_1 )</td>
<td>( f_1 )</td>
</tr>
<tr>
<td>1</td>
<td>201020</td>
<td>201020</td>
<td>201020</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>290866</td>
<td>290866</td>
<td>290866</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>664392</td>
<td>651655</td>
<td>651655</td>
<td>1.20</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>0.40</td>
</tr>
</tbody>
</table>

When the GAs \( f_i \) are compared with respect to average gap over all 3 problems, the result are same CPLEX, pri-awGA and mo-hGA in problem 1 and 2. On the other hand, the average gap in pri-awGA and mo-hGA are 1.20\% over CPLEX in problem 3.

When the Pareto solutions \( (f_1, f_2) \) are compared with respect to average gap over all 3 problems, it is seen that the mo-hGA exhibits the best performance with the average gap of 0.95\%. While the average gap for pri-awGA is 2.00\%, which is a small value.
The comparison is first done according to the computation time and it is seen that the computation time needed for mo-hGA is less than pri-awGA in all problems. Next, according to the number of Pareto solutions, both methods found the same number for problem 1, and they are slightly same for problem 2. But for problem 3, the number of Pareto solutions found by pri-awGA are less than that found by mo-hGA. Finally the improvement rate according to each time period are shown in the last column of Table 3. When comparing number of Pareto solutions, the results are same in problem 1 and 2. On the other hand, the result using mo-hGA is better in problem 3.

Fig. 1–3 represents Pareto solutions obtained from CPLEX, pri-awGA and mo-hGA for test problems. In this figure, the corresponding solutions on CPLEX and pri-awGA and mo-hGA Pareto optimal solutions with the same $f_2$ values are labeled with the same letters.

<table>
<thead>
<tr>
<th>No.</th>
<th>Time period</th>
<th>Computational time[sec]</th>
<th>No. of Pareto solutions [$S_t$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>pri-awGA</td>
<td>mo-hGA</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$t=1$</td>
<td>4,537</td>
<td>3,546</td>
</tr>
<tr>
<td></td>
<td>$t=2$</td>
<td>4,596</td>
<td>3,596</td>
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<td>$t=3$</td>
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<td>3,656</td>
</tr>
<tr>
<td></td>
<td>$t=4$</td>
<td>4,696</td>
<td>3,696</td>
</tr>
<tr>
<td>2</td>
<td>$t=1$</td>
<td>5,347</td>
<td>4,346</td>
</tr>
<tr>
<td></td>
<td>$t=2$</td>
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<td>4,396</td>
</tr>
<tr>
<td></td>
<td>$t=3$</td>
<td>5,446</td>
<td>4,456</td>
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<td>5,296</td>
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<tr>
<td></td>
<td>$t=3$</td>
<td>6,346</td>
<td>5,346</td>
</tr>
<tr>
<td></td>
<td>$t=4$</td>
<td>6,396</td>
<td>5,396</td>
</tr>
</tbody>
</table>

Table 3. The experimental results of pri-awGA and mo-hGA

5. Conclusion

In this paper, we presented 0-1 mixed-integer linear programming model for multi-objective optimization of CLSCM and a genetic algorithm approach.

We propose the improved model considering minimization of total cost (e.g., transportation cost, open cost, inventory cost, purchase cost, disposal cost and saving cost of integrated facilities) and minimization of total delivery tardiness.

Finally, through the comparison of percentage gap of CPLEX, Adaptive Weight Approach (pri-awGA) and multi-objective hybrid genetic algorithm (mo-hGA), the effectiveness of the proposed method was demonstrated.

References


