Analysis and prediction of crack propagation in plates by the enriched free Galerkin method

Bui Manh Tuan1,2,*, Chen Yun Fei1

1School of Mechanical Engineering, Southeast University, Nanjing city, Jiangsu Province, China
2Faculty of Mechanical Engineering, Tuy Hoa Industrial College, Tuy Hoa City, Phu Yen Province, Vietnam

Email address:
buimanhtuan.tic@gmail.com (B. M. Tuan), yunfeichen@seu.edu.cn (Chen Yunfei)

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Abstract: This paper presents a centre and edge crack analysis using meshless methods which is based on moving least squares (MLS) approximation. The unknown displacement function u(x) is approximated by moving least square approximation uh(x). These approximation are constructed by using a weight function which is based a monomial basis function and a set of non-constant coefficients. A subdivision that is similar to finite element method is used to provide a background mesh for numerical integration. An enriched EFG formulation with fracture problems is proposed to improve the solution accuracy for linear elastic fracture problem. The essential boundary conditions are enforced by Lagrange multipliers method. A code has been written in Matlab for the analysis of a crack tip. The obtained results of the developed EFG-code were compared to available experimental data and other numerical (exact methods and finite element method) methods.

Keywords: Crack, Stress Intensity Factor, EFG Method, Moving Least Squares Approximant, Crack Propagation

1. Introduction

There are many numerical methods applied for modeling cracks in mechanical problems such as the Boundary Element Method (BEM) [1], the Finite Element Method (FEM) [2], the extended Finite Element Method (XFEM) [3] and Meshless Methods (MMs). Among these methods, the element free Galerkin method and enriched element free Galerkin method has developed by Belytschko et al. [4,5] and it has been widely applied in fracture mechanics [6-8]. The meshless method works better than the traditional finite element method (FEM) in treatment of arbitrary evolving discontinuity. Due to the elements are independent of each other so the problems of crack analysis is simplified considerably which can be analyzed on a fixed mesh and unnecessary remeshing for crack development issues.

In this paper, we use a meshless method to analyze two-dimensional elastic problem by using Element-Free Galerkin (EFG) method that is based on moving least squares approximation (MLS) to construct the function approximation for the Galerkin weak form. These approximations are constructed by using a weight function that base on a monomial basis function and a set of non-constant coefficients. A subdivision similar to finite element method is used to provide a background mesh for numerical integration. The necessary boundary conditions are implemented by means of Lagrange multipliers. The method of enriched EFG that is enriched basis and nodal refinement is utilized to calculate and simulate crack growth. Therefore, the precision calculation is improved. The continuous crack propagation is modeled as a linear series of crack growth. In there, plate for cracks in the center and edge cracks is presented in this paper. The results obtained for two-dimensional problem with different of the number of nodes, crack length, load and dimensionless size of the support domain in the region of the crack tip compare to element finite method and exact method. In addition, Matlab code of EFG method also is offered in this paper. This Matlab code can be developed for meshfree application software or other meshfree method in the further.

2. MLS Approximations Functions

MLS functions were developed by Lancaster and Salkauskas in the literature [9] to approximate curves and surfaces, and then was used in EFGM method to generate shape functions [1,9]. In EFGM, a field variable u(x) is approximated by MLS approximation, u0h(x) [22] which is given as
\[ u(x) \equiv u^*(x) = p^r(x) a(x) \quad \forall x \in \Omega_x \quad (1) \]

Where \( p(x) \) is a linearly independent basis of \( m \) functions
\[ p^r(x)=\left[p_1(x), p_2(x), \ldots, p_m(x)\right] \quad (2) \]

And \( a(x) \) collects the undetermined parameters of the approximation
\[ a(x)=\left[a_1(x), a_2(x), \ldots, a_m(x)\right]^T \]

The \( a(x) \) parameters are obtained by minimizing a weighted least square sum. The weighted least square sum denoted by \( L(x) \) can be written as follows:
\[ L(x) = \sum_{i=1}^{n} \omega(x-x_i)(u_i(x_i, x) - u_i)^2 \]

Where \( \omega(x-x_i) \) is a weighting function which is nonzero on the influence domain of the node \( x_i \); \( n \) is the number of points in the neighbourhood of \( x_i \), and \( u_i \) is the nodal value of \( u \) at \( x=x_i \). The dimension of the influence domain of each node and the choice of the weighting function are decisive parameters for the approximation by MLS [4].

Minimizing \( L(x) \) in order to the unknown parameters \( a(x) \) results in
\[ A(x)a(x) = B(x)u \quad (4) \]

Or
\[ a(x) = A^{-1}(x)B(x)u \quad (5) \]

With:
\[ A(x) = \sum_{i=1}^{n} w(x-x_i)\{p^r(x_i)\} \]
\[ B(x) = \{w(x-x_1)p(x_1), w(x-x_2)p(x_2), \ldots, w(x-x_n)p(x_n)\} \quad (6) \]

Substituting the result (5) in the initial approximation (1), the MLS approximation is obtained as:
\[ u^r(x) = \sum_{i=1}^{n} \Phi_i(x) u_i = \Phi(x)U \quad (8) \]

Where the shape function is defined by
\[ \Phi_i(x) = \sum_{j=1}^{m} p_j(x) (A^{-1}(x)B(x))_{ij} = p^r A^{-1}B_{ij} \quad (9) \]

Where \( m \) is the order of the polynomial \( p(x) \). To determine the derivatives from the displacement (8), it is necessary to obtain the shape function derivatives. The spatial derivatives of the shape functions are obtained by:

\[ \Phi_{x_i} = (p^r A^T B)_{ij} = p^r A^T B_{ij} \quad (10) \]

Where
\[ B_{ij}(x) = \frac{d}{dx}(x-x_i) p(x_j); \]
\[ A_{ij}(x) = -A^T A_{ij}; \quad (11) \]
\[ A_{ij} = \sum_{j=1}^{m} \omega(x-x_i)p(x_j)p^r(x_j) \]

It should be noted that EFG shape functions do not satisfy the Kronecker delta criterion: \( \Phi_i(x) \neq \delta_{ij} \), so \( u^r(x_i) \neq u_i \), the nodal parameters \( u_i \) are not the nodal values of \( u(x_i) \). Therefore, we use Lagrange multiplier method to enforce the essential boundary conditions.

3. Choice of Support Domain and Weight Function

The support domain usually used circular or rectangular. There is no difference if a circular or rectangular support domain is used in the EFG method [4, 5]. A weight function needs properties as following:
- Compact support, i.e. zero outside the support domain.
- The values of all points in the support domain is positive.
- The value of its is maximum at the current point and decrease when moving outwards.

There are many kinds of function satisfying for these properties. In this paper, we used the quadratic spline function as follow
\[ w(d) = \begin{cases} 1-6\left(\frac{d}{d_w}\right)^2+8\left(\frac{d}{d_w}\right)^3-3\left(\frac{d}{d_w}\right)^4 & \text{if } d < d_w \\ 0 & \text{if } d \geq d_w \end{cases} \quad (12) \]

With: \( d_1 = |x-x_i| \)
\( d_{\text{max}} \) is the radius of influence domain \( x_i \)
\[ d_{\text{int}} = d_{\text{max}}, c_1 \]

Where the scaling parameter \( d_{\text{int}} \) is usually chosen 1.5 ÷ 4 for static analysis. The characteristic dimension parameter \( c_1 \) represents the nodal spacing. If the nodes are uniformly distributed then \( c_1 \) is the distance between two adjacent nodes.

4. Discrete Equations and Integration

In EFGM, the shape functions dissatisfied the Kronecker delta property. Therefore, we have to use Lagrange multiplier to invoke essential boundary. The Lagrange multiplier in [4] as follow as:

\[ \Phi_{x_i} = \frac{d}{dx}(x-x_i) p(x_j); \]
\[ A_{ij}(x) = -A^T A_{ij}; \]
\[ A_{ij} = \sum_{j=1}^{m} \omega(x-x_i)p(x_j)p^r(x_j) \]

It should be noted that EFG shape functions do not satisfy the Kronecker delta criterion: \( \Phi_i(x) \neq \delta_{ij} \), so \( u^r(x_i) \neq u_i \), the nodal parameters \( u_i \) are not the nodal values of \( u(x_i) \). Therefore, we use Lagrange multiplier method to enforce the essential boundary conditions.
\[ L = L + \int_{S_u} \lambda^t (u - \bar{u}) dS_u \]  

(13)

The Lagrange multiplier (\( \lambda \)) can be interpreted as the reaction forces needed to fulfill the displacement conditions at the boundary. The approximation given by:

\[ u(x) \approx u^*(x) = \sum_{i=1}^{n} \Phi_i(x) u_i = \Phi(x) U \]  

(14)

By enforcing essential boundary conditions using Lagrange multiplier approach. Discretization of (14) results in [7].

\[ T = K G U f + G o q \]  

(15)

With K is stiffness matrix.

\[ T = K \int_{\Omega} B^T J B d\Omega \]  

(16)

\[ f_j = \int_{\Omega} \Phi^T J b d\Omega + \int_{S_r} \Phi^T J dS_r \]  

(17)

\[ B_i = \begin{bmatrix} \Phi_{i,x} & 0 \\ 0 & \Phi_{i,y} \\ \Phi_{i,y} & \Phi_{i,x} \end{bmatrix}, \quad N_K = \begin{bmatrix} N_K & 0 \\ 0 & N_K \end{bmatrix} \]  

(18)

\[ G_{ik} = \int_{S_u} -\nabla^T \Phi dS_u, \quad q_k = \int_{S_u} -\nabla^T \bar{u} dS_u \]  

(19)

For plane stress:

\[ D = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} = D_j \]  

(20)

\[ \sigma = \varepsilon . D \]  

(21)

5. The Stress and Displacement near Crack Tip

The analytical solution for the stresses of an infinite plate [10,11]

Mode 1:

\[ \sigma = \frac{K_e}{\sqrt{2 \pi r}} \cos \theta \left[ 1 + \sin \frac{\theta}{2} \sin \frac{\theta}{2} \right] + O(r^s) \]  

(22)

\[ \tau = \frac{K_e}{\sqrt{2 \pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} + O(r^s) \]  

(23)

6. Enrichment Functions

In the EFG method, by adding extensions in Meshless methods to reflect the discontinuous displacement field generated by the crack. For the case of linear elastic fracture mechanics, two sets of functions are used: a Heaviside jump functions to capture the jump across the crack faces and asymptotic Branch functions that span the 2D asymptotic crack tip fields. The enriched approximation for fracture mechanics problems take the form [12-15]

\[ u^*(x) = \sum_{i \in N} \Phi_i(x) u_i + \sum_{i \in N^b} \Phi_i(x) H(x) a_i \]  

(26)

In the equation (26), the first term is the standard approximations functions of EFG methods, the second term reflects discontinuous along the two sides of crack surface, and the third term reflects the crack tip singularity.

Where N is the entire set of particles in the domain, N^b is
the set of particles whose domain of influence is completely bisected by the crack, \( N^c \) is the set of particles whose domain of influence is bisected by the crack tip as shown in Fig.1.

\[ H(f(x)) = \begin{cases} +1 & f(x) > 0 \\ -1 & f(x) < 0 \end{cases} \]  

(27)

(\( B(x) \) is Branch enriched function, the crack tip extension function, and given by [17]:

\[ B^e(x) = \left[ \sqrt{r \cos(\theta/2)} \sqrt{r \sin(\theta/2)} \sqrt{r \sin(\theta)} \sqrt{r \cos(\theta/2)} \sin \theta \right] \]  

(28)

The Branch enrichment is crucial to accurately locate the crack tip in enriched meshfree methods. The crack tip enrichment ensures that the crack is properly closed at the crack tip.

\[ \text{Figure 1. Selection of the support domain near a crack face} \]

Where \( f(x) \) is the signed distance function from the crack line; \( x \) is the sample points, \( x_0 \) is the distance from the surface cracks to the point \( x \) nearest; \( r \) and \( \theta \) are polar coordinates with respect to the crack tip as shown in Fig. 1.

7. Crack Growth Direction

The propagation of crack requires a suitable criterion for a crack growth. The commonly used criteria are: the maximum principal stress criterion [18], the maximum energy release rate criterion [19], and the minimum strain energy density criterion [20].

In this study, we use maximum hoop stress theory, which assumes that the crack may grow in a direction perpendicular to the maximum principal stress [21].

\[ \sigma_{\theta\theta} = \frac{1}{2\sqrt{\pi r}} \cos \theta \left[ \frac{1}{2} K_I \sin \theta + \frac{1}{2} K_{II} (3 \cos \theta - 1) \right] = 0 \]  

(29)

Where \( K_I \) and \( K_{II} \) are the stress intensity factor of modes 1 and 2, respectively. The variables \( r \) and \( \theta \) are as shown in Fig. 2. Therefore, the crack growth direction \( \theta_0 \) for each crack increment is obtained by following condition [22].

\[ K_I \sin \theta_0 + K_{II} (3 \cos \theta_0 - 1) = 0 \]  

(30)

After solving the above equation, we obtain

\[ \theta_0 = 2 \arctan \left[ \frac{1}{4} \left( \frac{K_I}{K_{II}} \right) \pm \sqrt{\left( \frac{K_I}{K_{II}} \right)^2 + 8} \right] \]  

(31)

According to this criterion, the equivalent mode I SIF is

\[ K_e = K_I \cos^3 (\theta_0 / 2) - 3K_{II} \cos^2 (\theta_0 / 2) \sin(\theta_0 / 2) \]  

(32)

When equivalent stress intensity factor \( K_e \) is greater than fracture toughness of the material \( K_c \), cracks began to expand. \( K_c \) is fracture toughness of the material.

The steps of calculations in Matlab code:

- Step 1: Set up the nodal coordinates for a problem domain.
- Step 2: Set up a background mesh for numerical integration.
- Step 3: Determine the Gauss point, weight function and Jacobi.
- Step 4: Determine the domain of influence of each node in the model.
- Step 5: Determine shape functions MLS and shape function derivatives.
- Step 6: Enriched.
- Step 7: Determine stiffness matrix \( K \).
- Step 8: Enforce essential boundary conditions using Lagrange multipliers.
- Step 9: Assembled to form the master stiffness matrix and solve for equations.
- Step 10: Solve for nodal parameters, solve for stresses.
- Step 11: Solve for stress intensity factors and direction of crack propagation.


8. Numerical Simulation

8.1. Rectangular Plate with a Center Crack under Tension Shown in Fig. 3.

The plate has an initial crack length of \( 2a=40cm \), a plate length of \( l=350cm \), a plate width of \( b=175cm \); Elastic modulus \( E = 2x10^7 N/cm^2 \); Poisson’s ratio \( \nu=0.3 \). Fracture
toughness $K_c = 140 \text{N/cm}^{3/2}$; The dimensionless size of support domain $d_{\text{max}} = 1.75$; a 4x4 Gauss quadrature is used and 6x6 nodes around the crack tip are selected for enrichment.

Thanks to the symmetry, only half of the model calculations.

The reference mode I SIF is given by [11]

$$K_{\text{theory}} = \sigma \sqrt{\pi a} \frac{F_1(a/b)}{b}$$

Where $a$ is the crack length, $b$ is the plate width and $F(a/b)$ is an empirical function given as

$$F_1(a/b) = \left[1 - 0.025(a/b)^2 + 0.06(a/b)^3\right] \sqrt{\frac{\pi a}{2b}}$$

$$\text{Error}(\%) = \frac{K_{\text{EFG}} - K_{\text{FEM}}}{K_{\text{EFG}}} . 100\%$$

The results obtained in Table 1 show that the accuracy of the EFG method depend on the smoothing node density. If
the node density is as smoothing as a high precision and vice versa. This article demonstrates that the problem of distributed nodes in EFG method great influence to the accuracy of calculations. To increase the accuracy level, we just simply add the button to the crack tip. Because no grid structure, hence the problem of adding nodes can be implemented and reduce computation time.

Using the results of EFG method compared to the results of element finite method and exact method, the values of error are mostly less than 1% is shown in Fig.7.

8.1.2. Analysis of the Crack Length in the SIF Calculation.

With $a = 15\text{cm}$, the minimum error of approximately 0.4868%.

With $a = 45\text{cm}$, the maximum error is about 1.0982%, the crack length under three different approaches. In Fig. 8 we also see that error of EFG method is small. When the crack length $a = 45\text{cm}$, the maximum error is about 1.0982%, the crack length $a = 15\text{cm}$, the minimum error of approximately 0.4868%.

Table 2. shows the mode I SIFs obtained using different of the crack length

<table>
<thead>
<tr>
<th>$a$(cm)</th>
<th>$\text{Nel}$</th>
<th>$K_{\text{EFG}}$</th>
<th>$K_{\text{theory}}$</th>
<th>$K_{\text{FEM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>40x80</td>
<td>119.5638</td>
<td>118.9573</td>
<td>118.09</td>
</tr>
<tr>
<td>10</td>
<td>40x80</td>
<td>169.3015</td>
<td>168.4756</td>
<td>167.1297</td>
</tr>
<tr>
<td>15</td>
<td>40x80</td>
<td>207.8535</td>
<td>206.8415</td>
<td>203.7212</td>
</tr>
<tr>
<td>20</td>
<td>40x80</td>
<td>241.0183</td>
<td>239.6579</td>
<td>237.8907</td>
</tr>
<tr>
<td>25</td>
<td>40x80</td>
<td>270.8392</td>
<td>269.1344</td>
<td>267.5569</td>
</tr>
<tr>
<td>30</td>
<td>40x80</td>
<td>298.436</td>
<td>296.4354</td>
<td>295.4391</td>
</tr>
<tr>
<td>35</td>
<td>40x80</td>
<td>324.9144</td>
<td>322.2807</td>
<td>322.2666</td>
</tr>
<tr>
<td>40</td>
<td>40x80</td>
<td>350.4446</td>
<td>347.1662</td>
<td>348.433</td>
</tr>
<tr>
<td>45</td>
<td>40x80</td>
<td>375.588</td>
<td>371.463</td>
<td>374.166</td>
</tr>
<tr>
<td>50</td>
<td>40x80</td>
<td>399.6855</td>
<td>395.4695</td>
<td>399.5562</td>
</tr>
</tbody>
</table>

From table 2, we can show that the different of the crack length under three different approaches. In Fig. 8 we also see that error of EFG method is small. When the crack length $a = 45\text{cm}$, the maximum error is about 1.0982%, the crack length $a = 15\text{cm}$, the minimum error of approximately 0.4868%.

8.1.3. Analysis Dimensionless Size of the Support Domain ($d_{\text{max}}$) in the SIF Calculation

Table 3. shows the mode I SIFs obtained using different of $d_{\text{max}}$

<table>
<thead>
<tr>
<th>$d_{\text{max}}$</th>
<th>$\text{Nel}$</th>
<th>$K_{\text{EFG}}$</th>
<th>$K_{\text{theory}}$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>20x40</td>
<td>297.1336</td>
<td>306.9168</td>
<td>-3.2907</td>
</tr>
<tr>
<td>1.7</td>
<td>20x40</td>
<td>300.1515</td>
<td>306.9168</td>
<td>-2.2539</td>
</tr>
<tr>
<td>1.8</td>
<td>20x40</td>
<td>298.9854</td>
<td>306.9168</td>
<td>-2.6527</td>
</tr>
<tr>
<td>1.9</td>
<td>20x40</td>
<td>297.4066</td>
<td>306.9168</td>
<td>-3.1977</td>
</tr>
<tr>
<td>2.0</td>
<td>20x40</td>
<td>299.0257</td>
<td>306.9168</td>
<td>-2.6389</td>
</tr>
<tr>
<td>2.1</td>
<td>20x40</td>
<td>301.3948</td>
<td>306.9168</td>
<td>-1.8321</td>
</tr>
<tr>
<td>2.2</td>
<td>20x40</td>
<td>306.14</td>
<td>306.9168</td>
<td>-0.2537</td>
</tr>
<tr>
<td>2.3</td>
<td>20x40</td>
<td>317.113</td>
<td>306.9168</td>
<td>3.2153</td>
</tr>
</tbody>
</table>

Figure 8. SIF% error of different crack length

8.1.4. Analysis of the Tension in the SIF Calculation: $q=5*i$ (N/cm) $(i=1:10)$

Figure 9. SIF% error of different $d_{\text{max}}$

From Fig. 10, we find that the calculation error is independent of the distribution of the load $q$. All computational errors always equal 0.8280%.

Table 4. shows the mode I SIFs obtained using different of load

<table>
<thead>
<tr>
<th>$q$(N/cm)</th>
<th>$\text{Nel}$</th>
<th>$K_{\text{EFG}}$</th>
<th>$K_{\text{theory}}$</th>
<th>$K_{\text{FEM}}$</th>
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<tbody>
<tr>
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<td>20x40</td>
<td>40.2263</td>
<td>39.9429</td>
<td>39.3728</td>
</tr>
<tr>
<td>10</td>
<td>20x40</td>
<td>80.4525</td>
<td>79.8859</td>
<td>78.745792</td>
</tr>
<tr>
<td>15</td>
<td>20x40</td>
<td>120.6788</td>
<td>119.8289</td>
<td>118.1186</td>
</tr>
<tr>
<td>20</td>
<td>20x40</td>
<td>160.9051</td>
<td>159.7719</td>
<td>157.4915</td>
</tr>
<tr>
<td>25</td>
<td>20x40</td>
<td>201.3134</td>
<td>199.7149</td>
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<tr>
<td>30</td>
<td>20x40</td>
<td>241.3576</td>
<td>239.6579</td>
<td>236.2373</td>
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</tr>
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<td>20x40</td>
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<td>314.9831</td>
</tr>
<tr>
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<td>20x40</td>
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<td>354.3561</td>
</tr>
<tr>
<td>50</td>
<td>20x40</td>
<td>402.2627</td>
<td>399.4298</td>
<td>393.7289</td>
</tr>
</tbody>
</table>

Figure 10. SIF% error of different load
8.2. Rectangular Plate with a Single Edge Crack under Tension Shown in Fig. 5.

![Figure 11. Single Edge Crack Computational model (mode2)](image)

The plate has an initial crack length of $a=32\text{cm}$, a plate length of $H=350\text{cm}$, a plate width of $b=175\text{cm}$; Elastic modulus $E=2\times10^7\text{N/cm}^2$; Poisson's ratio $\nu=0.3$. Fracture toughness: $K_c=140\text{N/cm}^{3/2}$; The dimensionless size of support domain $d_{\text{max}}=1.75$; A Gauss integration of 4x4 orders is used, without local refinement at the crack tip.

The reference mode II SIF is given by \[ K_{\text{II}} = \sigma \sqrt{\pi a} F_{\text{II}} \left( \frac{a}{b} \right) \]

Where $a$ is the crack length, $b$ is the plate width and $F_{\text{II}}(a/b)$ is an empirical function given as

\[
F_{\text{II}} \left( \frac{a}{b} \right) = \begin{cases} 
1.122 - 0.231 \left( \frac{a}{b} \right) + 10.55 \left( \frac{a}{b} \right)^2 \\
- 21.718 \left( \frac{a}{b} \right)^3 + 30.382 \left( \frac{a}{b} \right)^4 
\end{cases}
\]

8.2.1. Analysis of the Number of Nodes in the SIF $K_{\text{II}}$ Calculation

<table>
<thead>
<tr>
<th>$a$ (cm)</th>
<th>$N_{\text{el}}$</th>
<th>$K_{\text{II}} \text{EFG}$</th>
<th>$K_{\text{II}} \text{Theory}$</th>
<th>$K_{\text{II}} \text{FEM}$ [23]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>65x130</td>
<td>190.7816</td>
<td>191.6101</td>
<td>194.8391</td>
</tr>
<tr>
<td>20</td>
<td>65x130</td>
<td>285.7325</td>
<td>286.8245</td>
<td>290.5711</td>
</tr>
<tr>
<td>30</td>
<td>65x130</td>
<td>378.9788</td>
<td>381.316</td>
<td>384.4031</td>
</tr>
<tr>
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<td>65x130</td>
<td>482.2136</td>
<td>485.6039</td>
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</tr>
<tr>
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<td>65x130</td>
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<td>600.5331</td>
<td>609.3197</td>
</tr>
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<td>65x130</td>
<td>811.8003</td>
<td>817.2502</td>
<td>1179.348</td>
</tr>
</tbody>
</table>

8.2.2. Analysis of the Crack Length in the SIF $K_{\text{II}}$ Calculation

Table 6. Analysis of the crack length in the SIF calculation. With $a=i*10$ (cm) ($i=1:8$)

From Table 6 and Fig. 3 show that the error of EFG method are small. When the crack length $a = 40$ cm, the maximum error calculation is 0.7030%. When the crack length $a = 60$ cm, the minimum error calculation is 0.2799%.
8.3. Simulation Results of Displacement Fields and Stress Field

Figure 14. The node displacement fields after 2 steps

Fig. 14 presents the displacement field of edge crack finite plates. Where we can see that the node displacement fields after 2 steps is reflected under the effect of loads.

Figure 15. x, y direction stress field of center crack under tension

Fig. 15 and Fig. 16 present the stress contour plot of $\sigma_{xx}$ and $\sigma_{yy}$ direction of center and edge crack under tension. It also shows that the stress concentration at the crack, the singularity of the crack tip stress and the stress field at the crack location outside is smooth.

All the problems have been simulated by EFG codes (algorithms) writing in MATLAB (R2010b).

8.4. Simulation of Crack Propagation

Simulate the growth of crack in quasi-static is evaluated by equivalent stress intensity factor, and if its exceeds the fracture toughness of the material then the crack will be extended for some finite length ($da$) in a particular direction that is found by a suitable crack growth criterion. The step size ($da$) is a used to determine parameters, and da should be chosen enough minor to get an accurate crack growth path. The stress intensity factors are recalculated for the new crack geometry, and the next crack is extended according to a new direction.

In this paper, the following equation is calculated for each step of the cracking step [24]:

$$da_n = \frac{K_e^{(n)}}{K_e^{(0)}} da_0$$

Where $da_0$ is initial cracking step, $K_e^{(0)}$ is initial equivalent stress intensity factor, $da_n$ is step n for cracking step, $K_e^{(n)}$ is step n the equivalent stress intensity factor.

The obtained results of model crack propagation analysis by using a relatively coarse discretization of $40 \times 40$ nodes for centre crack and $20\times40$ nodes for edge crack. The crack growth increment is selected 0.1a ($a=40$) for this study, and the crack growth are simulated for 10 steps and 15 step. The result of crack path is shown as Fig. 17 and Fig. 18.

Figure 17. Crack growth path of centre crack

Figure 18. Crack growth path of single edge crack
9. Conclusions and Discussion

The Meshless computation has some outstanding advantages compared to the traditional finite element method (FEM) in treatment of arbitrary evolving discontinuity. Because of the independence of elements, the adaptive refinement can be easily achieved. Therefore, the crack-propagation analysis can be done easily and dramatically used to allow crack growth without remeshing. By introducing enriched functions, the extensions are added in the approximation of traditional Meshless method, the computation accuracy was improved. Through the computational results of stress intensity factors for edge and centre crack are compared to the results of element finite method and exact method, proving the Meshless method is very convenient for crack problem. In both cases the crack growth that is edge and centre crack are simulated for predicting fatigue crack propagation path. This method is used to allow crack growth without remeshing.

Through the analyses of numerical examples demonstrate, we can see that enriched EFG method which can solve the fracture problems is effective, and has practical merits for modeling crack growth problem. It is very promising in engineering application.

References


