Study of the effects of roll motion on transverse stability of a small boat

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Abstract: The aim of our paper is to study the effects of roll motion on a transverse stability of a small boat. Roll frequency will be calculated and analyzed for reducing motion sickness based on stability criteria and optimization technique. The practical solution of ship roll motion will be also investigated in our paper.

Keywords: Ship Roll Motion, Transverse Stability, Optimization

1. Introduction

To comfort the exclusive client in yacht industry with highly competition, almost all prominent yacht builders and designers will cope with stability, ship motion and space arrangement optimization. In the 18th international ship and offshore structures congress (ISSSC 2012), ship motion is one of important and necessary topics because oscillation of yacht will substantially effect on the onboard comfort of yacht owners and crew [1]. The designers have to contemplate the demand of yacht owner with the sea conditions and also consider the effects of yacht motions on safe operational capability of yacht [2]. In yacht design, the roll motion evaluation is essential to the ship safety [3]. In 1757, effect of the metacenter height GM on roll motion was analyzed through Bernoulli [4]. Our research will focus on using stability criteria and optimization technique for a solution of roll motion.

2. Roll Motion and Stability Criteria

2.1. Ship Motion

Ship motions are described as an object with respect to six degrees of freedom in translation and rotation as figure 1. It is included includes surge, sway and heave and the rotation movement includes roll (heel), pitch (trim) and yaw [2]. In the actual operation condition, ship motions have a complicated relationship. However, ship motions can be split into two categories. The first category consists of pitch, heave and roll influenced by sea waves and the second category consists of surge, sway and yaw produced by propeller force, rudder, current and wind [5].

\[ f(t) - c \ddot{z} - kz = m\ddot{z}, \]  

(1)

Figure 1. Six degree of ship motions [6]
Where

- \( f(t) \): Exciting force
- \( \dot{z} \): Vertical velocity
- \( \ddot{z} \): Vertical acceleration
- \( c \): Damping coefficient
- \( k \): spring constant
- \( m \): mass of the suspended weight

After rearranging the above equation, the vibration equation is

\[
f(t) = m\ddot{z} + c\dot{z} + k\ddot{z}
\]  

(2)

There are three general cases in ship motions including the undamped free vibration, damped free vibration and forced vibration with harmonic excitation (Johnson, 1982). In the first case, the condition of the undamped free vibration provides

\[
0 = m\ddot{z} + k\ddot{z} = 0
\]  

(3)

Where: \( \omega_n^2 = \frac{k}{m} \), \( \omega_n \) is undamped natural frequency, so the undamped natural period is as follow:

\[
T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m}{k}}
\]  

(4)

Then

\[
\ddot{z} + \omega_n^2\ddot{z} = 0
\]  

(5)

Solution of the above equation:

\[
z(t) = A\cos\omega_n t + B\sin\omega_n t
\]  

(6)

### 2.2. Tuning Factor

**Figure 2.**Rolling with tuning factor [3]

Tuning factor \( A \) is the ratio of the excitation frequency to the undamped natural frequency \( \omega_n \) [2]. As the figure 2, when the tuning factor equals one, the yacht may capsize due to the frequency resonance and the lowest curve shows that roll motion will be dampened quickly. The yachtsman has to change the course immediately to reduce or increase the excitation frequency [3]. Hence the yacht will not cope with the frequency resonance. In design stage, the yacht can be designed to stay away from the frequency resonance by optimizing the mass moment of inertia around a longitudinal [3].

#### 2.3. Heave Motion

Heave motion is closely related to the transverse stability because the acceleration impacts on the apparent mass of the ship [2]. To describe the heave motion independently, Johnson (1982) suggests that the spring constant of ship \( k \) is in connection with the Tons-per-inch immersion (TPI) and the added mass of water \( m \) accelerated by heave motion is calculated as a percentage of the mass of ship \( x_e \):

\[
k = \text{TPI. 12 in/ft}
\]  

(7)

\[
m = \frac{\Delta}{g} (1 + x_e)
\]  

(8)

Hence, the natural period of heave motion:

\[
T_n z = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{\Delta(1+x_e)}{12gTPI}}
\]  

(9)

#### 2.4. Undamped Roll Motion and Undamped Pitch Motion

Undamped roll motion and undamped pitch motion are determined by the below equation (Johnson, 1982):

\[
I(1 + x_e)\ddot{\phi} + \Delta G\bar{M}\phi = 0
\]  

(10)

Where:
- \( I \): moment of inertia about the appropriate axis, \( \text{ft}^4 \);
- \( x_e \): the added mass coefficient for particular motion;
- \( \ddot{\phi} \): the angular acceleration of roll motion, it will be replaced by \( \theta \) in pitch motion, rad;  
- \( \Delta G\bar{M}\phi \): the righting moment (with small heel angle \( \phi \) < 10° in radians) and the righting arm \( \bar{G}Z = \bar{G}M \phi \) with the metacenter height \( GM \).

For roll motion, \( I_x = mk_x^2 \) with \( k_x \) is the radius of gyration.

For pitch motion, \( I_y = mk_y^2 \) with \( k_y \) is the radius of gyration.

As the result, the equation describes the undamped roll motion [2] as:

\[
\ddot{\phi} + \frac{\Delta G\bar{M}}{g(1+x_e)k_x^2}\phi = \ddot{\phi} + \omega_{n\phi}^2\phi = 0
\]  

(11)

The natural frequency \( \omega_{n\phi} \) and the undamped natural period of the undamped roll motion as:

\[
\omega_{n\phi}^2 = \frac{g\bar{G}M}{(1+x_e)k_x^2}
\]  

(12)

\[
T_{n\phi} = 2\pi k_x \sqrt{\frac{1+x_e}{g\bar{G}M}}
\]  

(13)
3. Roll Motion

3.1. Roll Frequency

In roll motion, the above equation of the undamped natural period can be used to estimate the radius of gyration $k_x$, and the added mass coefficient $x_A$ because the damping of rolling is not strong [2]. Based on the equation of $T_{ng}$, we can realize that the distance between center of gravity and metacenter GM (the metacenter height) will deeply influence the roll motion. A ship with small GM will be sailed more comfortable than the other. Unfortunately, the radius of gyration $k_x$ is hardly defined, so the empirical equation will be applied in real condition with small heel angle (below 10°) [2], as follow:

$$T = \frac{C \cdot B}{\sqrt{GM}}$$  \hspace{1cm} (14)

Where:
- C - is an empirical constant, for large ship C = 0.38 ÷ 0.55;
- B - is maximum beam, ft;
- T - is the period of roll motion, s.

The equation (14) shows that the period of roll motion $T$ does not depend on the amplitude of roll motion $\phi$.

![Figure 3. Response amplitude operators (RAO)](image)

In the ship motions, the non-dimensional diagram of $\nu / \sqrt{\lambda}$ based on the ratio of wave length and ship length, the ratio of motion amplitude and wave amplitude is commonly applied, its ordinates are known as Response Amplitude Operators (RAO), see fig. 3 [6,7]. In RAO for rolling, the vertical scale will be placed by the ratio of roll motion amplitude and wave amplitude [3].

3.2. Roll Motion Caused by Wave Motion (Roll Motion in Irregular Sea)

In a sea, the yacht will encounter wave sea. Hence, the period of roll motion of ship in wave (is called as forced rolling, forced oscillation) will combine the natural roll period of the ship (is called as free oscillation illustrated above) and the period of the wave. If the ship’s motion period equals to the wave’s period, the ship will be unstable and suffer the huge roll motion [2].

3.3. Motion Sickness Incidence (MSI)

In the early 1970s, motion sickness incidence (motion sickness index, MSI) is a term used in the research of effects ship motion upon the human body supported by the US Navy [8]. Since 1974, the motion sickness incidence (MSI) is suggested by O’Hanlon and McCauley to forecast the percentage of persons who will get motion sickness, it is expressed as the below expression [9]:

$$\text{MSI} = K_m \cdot \text{MSDV}_z$$  \hspace{1cm} (15)

Where
- MSDV$_z$ is the vertical motion sickness dose value, m/s$^{1.5}$
- $K_m$ is a coefficient depends upon the exposed population. In survey of mixed group with un-adapted male and female, $K_m = 1/3$ will be applied [9]. Depend on ISO 2631/3-1985, the criterion of MSI is 10% with respect to the exposure period of 2 hours [10].

Ship motion will produce the low frequency oscillation of human body in the vertical z-axis (normally less than 0.5Hz) at particular sea conditions in connection with ship speed, passengers will experience motion sickness [11,12]. The higher frequency vibrations is also determined between 1 to 80 Hz by American Bureau of Shipping’s (ABS) guide for passenger comfort on ships (2014) and guide for comfort yacht (2014), it will cause the passengers to discomfort due to effects of vibrations and forces upon the human body [11,12]. The vertical Motion Sickness Dose Value MSDV$_z$ used for analyzing motion sickness is applied by ABS guides as follow [11,12]:

$$\text{MSDV}_z = \sqrt{\int_0^T a_w^2(t)dt}$$  \hspace{1cm} (16)

Where
- MSDV$_z$ is the vertical motion sickness dose value, m/s$^{1.5}$
- $a_w(t)$ is the z-axis acceleration, it is a function of time, m/s$^2$;
- $T$ is the duration of the motion, s.

In the shorter exposure periods, MSDV$_z$ will be calculated based on the weighted root-mean-square (RMS) acceleration value $a_w[\text{m/s}]$ and the exposure period $T_0[\text{s}]$ as follow [11]:

$$\text{MSDV}_z = a_w^{1/2}$$  \hspace{1cm} (17)

The weighted RMS acceleration value $a_w[\text{m/s}]$ can be determined based on the function of time of the weighted acceleration $a_w(t) [\text{m/s}^2]$ and the duration of the measurement $T[\text{s}]$ as follow [11]:

$$a_w = \sqrt{\frac{1}{T} \int_0^T a_w^2(t)dt}$$  \hspace{1cm} (18)

To comply with ABS regulation of motion sickness restriction, the ship must satisfy the essential criteria to obtain the notation COMF+ from ABS.

In frequency range from 0.1 to 0.5 Hz, maximum of MSDV$_z$ is 30 m/s$^{1.5}$, in the frequency range from 1 to 80 Hz, maximum of the weight RMS acceleration value $a_w$ is 2 mm/s
3.4. The Recommendation of Optimization Technique for Ship Roll Motion Solution

According to Singiresu (1996), optimization is “the process of finding the conditions that give the maximum or minimum value of a function”[13]. There are three categories of optimization methods including:
1. Mathematical programming techniques;
2. Stochastic process techniques;

The mathematical programming techniques are convenient to determine the minimum of a function of several variables with prescribed constraints. In mathematical programming techniques, the objective function \( f(X) \) will be determined the minimize values based on an n-dimensional vector \( X \) (the design vector), the constrained optimization problem [13].

The statement of a constrained optimization problem in mathematical programming techniques as follow [13]:

Find design vector \( X = \{ x_1, \ldots, x_n \} \) which minimizes the objective function \( f(X) \); and \( x_i \) (with \( i = 1, 2, \ldots, n \)) are called decision variables.

Constrained optimization problem:

\[
\begin{align*}
\text{The inequality constraints } &: g_j(X) \leq 0, \text{ with } j = 1, 2, \ldots, m \\
\text{The equality constraints } &: h_j(X) = 0, \text{ with } j = 1, 2, \ldots, p
\end{align*}
\] (18)

If there are not any constraints, the above statement will be unconstrained optimization problem [13].

In our research, we offer the process for roll motion analysis as figure 4.

![Figure 4. The analysis process of roll motion](image)

We recommend the optimization algorithm to optimize the mass moment of inertia around the longitudinal axis of yacht for controlling the roll motion. Input data of ship model will be optimized and the wave model combines essential factors including wave frequency and wave length will be controlled before importing to Maxsurf1. The results of roll motion analysis will cover aspects of motion sickness, tuning factor, transverse stability and RAO. In the first analysis, motion sickness will be carried on by criteria of MSI index. In the second analysis, RAO and tuning factor will be used in roll motion analysis. In the final analysis, transverse stability will be estimated by the metacenter height \( GM \) and the right arm \( GZ \). The criteria of transverse stability and motion sickness relating to roll motion will also be analyzed.

4. Conclusion and Future Works

There is a lot of optimization techniques, the best suitable optimization methods should be discovered before applying for roll motion analysis. Wave data and criteria of transverse stability and rolling frequency should also be determined based on the appropriate conditions.

Up to now, there are a lot of ways to cope with the roll motion and almost all solution using motion-damping devices such as bilge keels, controllable fins, anti-rolling tanks and gyro-stabilizer based on gyroscope [2]. Suggested by Eliasson (2000) [3], to reduce the rolling frequency of yacht, there are three categories including using damping solutions created by friction between yacht and water, damping created by wave generation on water surface, and damping created by vortices generation from the keel, rudder, sharp bilges and sails.

After optimization of roll motion, it is very convenient that the designers refer the appropriate solution for dampening roll motion.

References


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**Biography**

**Nguyen Anh TUAN** (1987, Saigon), B.Eng on Naval Architecture and Marine Engineering (HCMUT, 2010). He is a director of TyVy Company Limited which focuses on yacht services. Mr Tuan has possessed the III-class Captain and the Examiner Certificate (Vietnam Inland Waterways Administration, 2012). Now, he is a Master’s researcher at the NAME, HCMUT (2014-2015). He has experience in yacht design, CAD/CAM/CAE/PLM, Vietnam Inland Waterways Law, and Statistical Analysis.

**Tat-Hien LE** (1981, Saigon). Naval Architect (2004, HCMUT) and then M.Eng (2006) and PhD (2009) in Naval architecture, both from Pukyong University, Busan, Korea. His researches were applications of real coded Genetic Algorithm for ship hull surface fitting. Since 2012 Dr. Le is Head of the Naval Architecture and Marine System Engineering (NAME) department of the HCMUT, VNU-HCM. He is interested in ship design optimizing and modeling for ship stability analysis and so on.

