International Journal of Management and Fuzzy Systems
2015; 1(2): 15-20
Published online July July, 2015 (http://www.sciencepublishinggroup.com/j/ijmfs)
doi: 10.11648/j.ijmfs.20150102.11

A Novel Strategy for Comparative Points in Facility Layout Problem with Fuzzy Logic

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To cite this article:
Erfan Ghasem Khani, Mostafa Ali Beigi. A Novel Strategy for Comparative Points in Facility Layout Problem with Fuzzy Logic. *International Journal of Management and Fuzzy Systems*. Vol. 1, No. 2, 2015, pp. 15-20. doi: 10.11648/j.ijmfs.20150102.11

Abstract: Distance measure is one of the most important component in facility layout problems. Many distance approaches have been proposed so far. However, there is no method that can always give a satisfactory solution to every situation. In this paper, first we review on some distance methods, then we present a new strategy for comparative points in facility layout with fuzzy logic, which it is very useable, specifically when it is hard (or impossible) to use other methods to solve uncertain points. Finally, some numerical examples illustrate the presented method as well as comparing it with other various ones.

Keywords: Multi Attribute Decision Making (MADM), Facility Layout (FL), Distance Measure, Fuzzy Logic, Uncertain Points, MOER Method, Decision Making (DM)

1. Introduction

Nowadays the concept of Facility Layout (FL) is acquiring more and more attention in the representation of intelligent automation. In many cases it is necessary to known in what manner which method selecting for distance measure, how various data differ or agree with each other, and what the measure of their comparison is. A good placement of facilities is dependent on data in problem and selecting method of measure distance.

Facility Layout (FL) Problem is one of the classical problems in which the planning for the placement of different types of facilities such as machines, employee workstations, utilities, customer service areas, restrooms, material storage areas, lunchrooms, drinking fountains, offices, and internal walls is discussed [Francis, White and McGinnis, 1992].

Many distance method for facility plant have been proposed, such as Euclidean distance, Squared Euclidean distance, Chebyshev distance, rectilinear distance. Sometimes we receive lots of information from factories and most of them are approximate. FL sometimes occurs in a fuzzy environment where the available information is imprecise/uncertain which may confuse the designer in the FL problem. There are many misconceptions about fuzzy logic. To begin with, fuzzy logic is not fuzzy. Basically, fuzzy logic is a precise logic of imprecision and approximate reasoning [Zade, 1975- Zade, 1979].

More specifically, fuzzy logic may be viewed as an attempt at formalization/mechanization of two remarkable human capabilities. First, the capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, conflicting information, partiality of truth and partiality of possibility – in short, in an environment of imperfect information. And second, the capability to perform a wide variety of physical and mental tasks without any measurements and any computations [Zade, 2008].

Applications of fuzzy numbers for indicating uncertain and vague information in facility layout, Multiple-criteria decision analysis, linguistic controllers, data mining, and etc. Fuzzy distance can be widely usage in attribute importance. Many fuzzy distance indices have been proposed since 1965. Some of the methods used crisp number to calculate the distance between two trapezoidal fuzzy numbers [Bloch, 1999- Saha, Wehrli, Gomberg, 2002- Pedrycz, 2007]. Human intuition says that the distance between two uncertain numbers should as a collection of points with different degrees of belongingness, then the distance between two fuzzy numbers is noting but the collection of pairwise distance between the elements of the respective fuzzy numbers [Voxman, 1998- Jahantigh and Hajighasemi, 2012]. Therefore, in this study, we pay to other methods for fuzzy distance, which used fuzzy distance to calculate the distance between two fuzzy numbers and introduce a fuzzy distance for normal fuzzy numbers.

In this paper, first we review on some distance methods, then we present a new strategy for comparative points in facility layout with fuzzy logic, which it is very useable, specifically when it is hard (or impossible) to use other methods to solve uncertain points. Finally, some numerical examples illustrate the presented method as well as comparing it with other various ones. New method named MOER distance (MOER abbreviation of authors name).

The rest of the paper is organized as follows:

Section 2 contains the basic definitions and notations that will be used in the remaining parts of the paper. In Section 3, we review some distance methods. Section 4 includes a new Approach to determine fuzzy distance measure for distance between several points. Some numerical examples demonstrate the advantages of the reviewed methods and compared results in section 5. The paper is concluded in Section 6.

2. Basic Concepts and Notations

Definition 1 (Distance): [Deza and Deza, Encyclopedia of Distances]

A distance space (X,d) is a set X (carrier) equipped with a distance d .

A function $d: X \times X \to R$ is called a distance (or dissimilarity) on X if, for all $x, y \in X$, it holds:

$Dist(x,y) \geq 0$ (nonnegativity)

- $Dist(x,y) = Dist(y,x)$ (symmetry)
- $Dist(x,x) = 0$ (reflexivity)

In Topology, a distance with $Dist(x,x) = 0$ implying $x = y$ is call a symmetric.

For any distance d , the function $Dist_1$ defined for $x \neq y$ by $Dist_1(x,y) = d(x,y) + c,$ where $c = \max_{x,y,z \in X}(d(x,y) - d(x,z), d(y,z)),$ and $Dist(x,x) = 0,$ is a metric. Also, $Dist_2(x,y) = d(x,y)^c$ is a metric for sufficiently small $c \geq 0$.

Definition 2 (Metric): [Deza and Deza, Encyclopedia of Distances]

Let X be a set. A function $d: X \times X \to R$ is called a metric on X if, for all $x, y, z \in X$, it holds:

- $Dist(x,y) \geq 0$ (nonnegativity);
- $Dist(x,y) = 0$ if and only if $x = y$ (identity of indiscernibles);
- $Dist(x,y) = Dist(y,x)$ (symmetry);
- $Dist(x,x) \leq Dist(x,z) + Dist(z,y)$ (triangle inequality)

In general, a generalized fuzzy number $\tilde{A}$ is described as any fuzzy subset of real line R , whose membership $\mu_{\tilde{A}}(x)$ can be defined as [Dubios and Prade, 1978]:

$$\mu_{\tilde{A}}(x) = \begin{cases} L_A(x) & a \leq x \leq b \\ \omega & b \leq x \leq c \\ R_A(x) & c \leq x \leq b \\ 0 & otherwise, \end{cases} \tag{1}$$

Where $0 \leq \omega \leq 1$ is a constant, and $L_{\tilde{A}} : [a,b] \to [0,\omega]$ and $R_{\tilde{A}} : [c,d] \to [0,\omega]$ are two strictly monotonical and continuous mapping from R to closed interval $[0,\omega]$. If $\omega = 1$, then $\tilde{A}$ is a normal fuzzy number; otherwise, it is a trapezoidal fuzzy number and is usually denoted by $\tilde{A} = (a,b,c,d,\omega)$ or $\tilde{A} = (a,b,c,d)$ if $\omega = 1$.

In particular, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $\tilde{A} = (a,b,d,\omega)$ or $\tilde{A} = (a,b,d)$ if $\omega = 1$. Therefore, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since $L_{\tilde{A}}(x)$ and $R_{\tilde{A}}(x)$ are both strictly monotonical and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. Let $L^{-1}_{\tilde{A}} : [0,\omega] \to [a,b]$ and $R^{-1}_{\tilde{A}} : [0,\omega] \to [c,d]$ be the inverse functions of $L_{\tilde{A}}(x)$ and $R_{\tilde{A}}(x)$, respectively. Then $L^{-1}_{\tilde{A}}(r)$ and $R^{-1}_{\tilde{A}}(r)$ should be integrable on the close interval $[0,\omega]$. In other words, both $\int_0^{\omega} L_{\tilde{A}}^{-1}(r)dr$ and $\int_0^{\omega} R_{\tilde{A}}^{-1}(r)dr$ should exist. In the case of trapezoidal fuzzy number, the inverse functions $L^{-1}_{\tilde{A}}(r)$ and $R^{-1}_{\tilde{A}}(r)$ can be analytically expressed as

$$L_{\tilde{A}}^{-1}(r) = a + (b-a)r / \omega, \; 0 \leq \omega \leq 1 \tag{2}$$

$$R_{\tilde{A}}^{-1}(r) = d - (d-c)r / \omega \;\; 0 \leq \omega \leq 1 \tag{3}$$

The set of all elements that have a nonzero degree of membership in $\tilde{A}$, it is called the support of $\tilde{A}$, i.e.

$$\text{Supp}\,(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}}(x) \succ 0\} \tag{4}$$

The set of elements having the largest degree of membership in $\tilde{A}$, it is called the core of $\tilde{A}$, i.e.

$$\text{Core}\,(\tilde{A}) = \left\{ x \in X \mid \mu_{\tilde{A}}(x) = \sup_{x \in X} L_{\tilde{A}}(\tilde{A}) \right\} \tag{5}$$

In the following, we will always assume that $\tilde{A}$ is continuous and bounded support Supp $(\tilde{A})$. The strong support of A should be $\text{Supp}\,(\tilde{A}) = [a,d]$.

Definition 3: A function $s:[0,1] \to [0,1]$ is a reducing function if is s increasing and $s(0) = 0$ and $s(1) = 1$. We say that s is a regular function if $\int_0^1 s(r)dr = \frac{1}{2}$.

Definition 4: If $\tilde{A}$ is a fuzzy number with r-cut representation, $\left(L_{\tilde{A}}^{-1}(r), R_{\tilde{A}}^{-1}(r)\right)$ and s is a reducing function, then the value of $\tilde{A}$ (with respect to s); it is defined by

$$Val(\tilde{A}) = \int_0^1 s(r)[L_{\tilde{A}}^{-1}(r) + R_{\tilde{A}}^{-1}(r)]dr \quad (6)$$

Definition 5: If $\tilde{A}$ is a fuzzy number with r-cut representation $\left(L_{\tilde{A}}^{-1}(r), R_{\tilde{A}}^{-1}(r)\right)$, and s is a reducing function then the ambiguity of $\tilde{A}$ (with respect to s) is defined by

$$Amb(\tilde{A}) = \int_0^1 s(r)[R_{\tilde{A}}^{-1}(r) - L_{\tilde{A}}^{-1}(r)]dr \quad (7)$$

Definition 6: Let $\tilde{A}$ is a fuzzy number. The absolute value of the fuzzy number $\tilde{A}$ is denoted by $|\tilde{A}|$ and defined as follows [13]:

$$\left|\tilde{A}(x)\right| = \begin{cases} 0 & x \prec 0 \\ \tilde{A}(x) \vee \tilde{A}(-x) & x \geq 0 \end{cases}, \quad (8)$$

And for all $r \in [0,1]$,

$$\left[|\tilde{A}|\right]_r = \begin{cases} [\tilde{A}]_r & L_{\tilde{A}}^{-1}(r) \geq 0 \\ \left[0, |L_{\tilde{A}}^{-1}(r)| \vee R_{\tilde{A}}^{-1}(r)\right] & L_{\tilde{A}}^{-1}(r) \leq 0 \leq R_{\tilde{A}}^{-1}(r) \\ \left[-R_{\tilde{A}}^{-1}(r), -L_{\tilde{A}}^{-1}(r)\right] & L_{\tilde{A}}^{-1}(r) \leq R_{\tilde{A}}^{-1}(r) \leq 0 \end{cases}$$

where $[\tilde{A}]_r = \left[L_{\tilde{A}}^{-1}(r), R_{\tilde{A}}^{-1}(r)\right]$ is the r cut representation of $\tilde{A}$ and $[|\tilde{A}|]_r$ is the $r-$cut representation of $|\tilde{A}|$, respectively.

Fuzzy Distance Measure [Ali Beigi, Hajjari and Ghasem Khani, 2015]

In this subsection, we describe "M. Ali Beigi, T. Hajjari, E. Ghasem Khani" fuzzy distance measure [Ali Beigi et al., 2015], which use in this paper.

Let $\tilde{A} = (x_1, x_2, x_3)$ and $\tilde{B} = (y_1, y_2, y_3)$ are two triangular fuzzy numbers and distance between $\tilde{A}$ and $\tilde{B}$ is denoted by $Dist(\tilde{A}, \tilde{B})$ where $Dist(\tilde{A}, \tilde{B}) = (d_1, d_2, d_3)$. We calculate d_1, d_2 and d_3 as follows:

If $x_1 = y_1$ then $d_1 = 0$, otherwise

$$d_1 = \begin{cases} |x_1 - y_3| & x_2 \geq y_2, \\ |y_1 - x_3| & x_2 \prec y_2 \end{cases} \quad (9)$$

$$d_2 = \max\left\{|x_2 - y_2|, d_1\right\} \quad (10)$$

and

$$d_3 = \begin{cases} 0 & x_3 = y_3, \\ \max\{y_3 - x_1, x_3 - y_1\} & x_3 \neq y_3 \end{cases} \quad (11)$$

Remark 1: The proposed method is a metric.

- $Dist(\tilde{A}, \tilde{B}) \geq 0$
- $Dist(\tilde{A}, \tilde{B}) = Dist(\tilde{B}, \tilde{A})$
- $Dist(\tilde{A}, \tilde{C}) \leq Dist(\tilde{A}, \tilde{B}) + Dist(\tilde{B}, \tilde{C})$.

The proposed method can be developed in n-demotion.

Let $\tilde{A}$ and $\tilde{B}$ are two points in n-Dimensional space with triangular fuzzy number values in each dimensions.

The points $\tilde{A}$ and $\tilde{B}$ can be shown as:

$$A = \left(\left(x_1^n, x_2^n, x_3^n\right), n = 1, \ldots, k\right)$$

$$B = \left(\left(y_1^n, y_2^n, y_3^n\right), n = 1, \ldots, k\right)$$

$Dist^n(\tilde{A}, \tilde{B}) = (d_1^n, d_2^n, d_3^n)$ Is the distance of the nth component of $\tilde{A}$ from the nth component of $\tilde{B}$ and d_1^n, d_2^n and d_3^n are related to from the left point, the centre and the right point of this distance respectively. The distance between each component can be calculated by the same method.

Then we define the total fuzzy distance between $\tilde{A}$ and $\tilde{B}$ as following:

$$Dist(\tilde{A}, \tilde{B}) = Dist^1(\tilde{A}, \tilde{B}) + Dist^2(\tilde{A}, \tilde{B}) + \ldots + Dist^k(\tilde{A}, \tilde{B}) = \left[d_1^1 + d_1^2 + \ldots + d_1^k, d_2^1 + d_2^2 + \ldots + d_2^k, d_3^1 + d_3^2 + \ldots + d_{3k}\right] \quad (12)$$

3. Some Existing Distance Methods

In this section, we briefly review some existing distance measure.

Different authors have constructed different distance measure. Some of them are discussed here.

3.1. Rectilinear Distance

The rectilinear distance [Floudas and Pardalos, 2009] $d_R(X, P_i)$ between two points X and P_i is defined as following (see Fig. 1):

$$d_R(X, P_i) = |x - a_i| + |y - b_i| \quad (13)$$

Where $X = (x, y)$ and $P_i = (a_i, b_i)$.

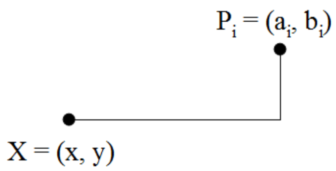

Fig. 1. *Rectilinear Distance* $d_R(X, P_i)$

3.2. Euclidean Distance or Straight Line

In order to determine the Euclidean distance between [Floudas and Pardalos, 2009] $X=(x,y)$ and $P_i=(a_i,b_i)$ is given by (see Fig. 2):

$$d_E(X,P_i)=\sqrt{(x-a_i)^2+(y-b_i)^2} \qquad (14)$$

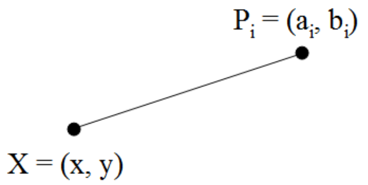

Fig. 2. *Euclidean distance* $d_E(X,P_i)$

3.3. Squared Euclidean Distance

The Square Euclidean distance between $X=(x,y)$ and $P_i=(a_i,b_i)$ is calculated as:

$$d_{SE}(X,P_i)=(x-a_i)^2+(y-b_i)^2 \qquad (15)$$

3.4. Manhattan Distance [Grabusts, 2011]

Manhattan distance or city block distance represents distance between points in a city road grid. It computes the absolute differences between coordinates of a pair of objects:

$$d_{man}(X,P_i)=|x-a_i|+|y-b_i| \qquad (16)$$

3.5. Chebyshev Distance [Grabusts, 2011]

Chebyshev distance is also called Maximum value distance. It computes the absolute magnitude of the differences between coordinates of a pair of objects(seeFig.3):

$$d_{ch}(X,P_i)=\max\{|x-a_i|,|y-b_i|\} \qquad (17)$$

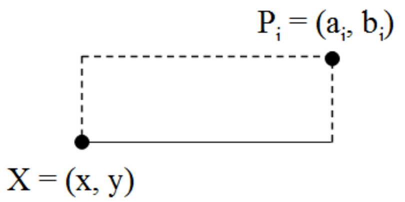

Fig. 3. *Chebyshev distance* $d_{ch}(X,P_i)$.

4. New Method to Determine Fuzzy Distance Measure for Distance Between Several Points

In this section, we present a new method for distance measure between several points.

This method include three parts.

Part One:

One of the most important steps for using fuzzy applications is converting crisp data to fuzzy. In order to convert (x_i,y_i) to triangular fuzzy number shown as following (see Fig. 4):

$$x_i=(x_i-1,x_i,x_i+1)$$
$$y_i=(y_i-1,y_i,y_i+1)$$

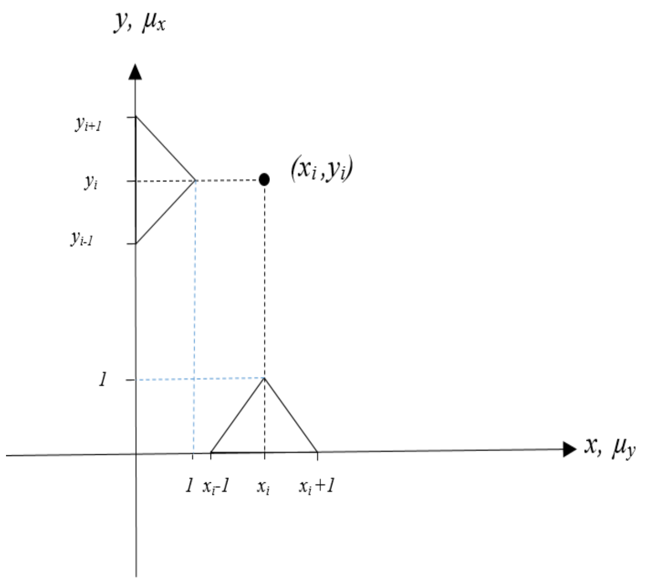

Fig. 4. *Convert* (x_i,y_i) *to triangular fuzzy number.*

Part Two:

Applying fuzzy distance measure [Ali Beigi et al., 2015] (definition 5) to converted known points, in other word, we calculate the fuzzy distance measure between point that converted in part one.

For using this method, first we calculate distance between x_i and x_j, then calculate distance between y_i and y_j. Finally, we should calculate distance between result from x and y, which it is total distance that named MOER distance (MOER abbreviation of authors name, in other hand the final answer formulized as following:

$$Dist(x)=Dist(x_i,x_j)$$

$$Dist(y)=Dist(y_i,y_j)$$

$$Dist_{total}=Dist_{MOER}=Dist(x)+Dist(y) \qquad (18)$$

One of the important advantage of MOER distance is far simple and give a better result when points are uncertain and vague.

Part Three: In this part, we will present defuzzify of our method. In order to comparative proposed method with crisp method, we should be defuzzify MOER distance.

Defuuzify of proposed method shown as following:

$$Defuzzify_{MOERDist}=d_2$$

Which d_2 is the mean of $Dist_{total}=(d_1,d_2,d_3)$.

Note: In many cases, d_2 is similar to rectilinear distance result.

Extension Proposed Method

If the points were approximately and we have the maximum distance between certain point and approximate point (MCP) we define Fuzzy number as following:

$$\tilde{x}_i = (x_i - MCP, x_i, x_i + MCP)$$

$$\tilde{y}_i = (y_i - MCP, y_i, y_i + MCP)$$

5. Numerical Examples

This section uses several numerical examples to compare the distance results of proposed method with some other existing distance methods.

Example 5.1. Consider coordinate of two point $X_1 = (1,2)$ and $X_2 = (3,4)$. We compare the new distance with five methods as following:

The rectilinear distance, the Euclidean distance, The Square Euclidean distance, Manhattan distance, Chebyshev distance.

Comparison between the results of the proposed similarity measure and other methods are shown in Table 1.

According to our proposed method, we have:

$$Dist_{\tilde{x}} = (0,2,4), \quad Dist_{\tilde{y}} = (0,2,4)$$
$$\Rightarrow \quad Dist_{MOER} = Dist_{\tilde{x}} + Dist_{\tilde{y}} = (0,4,8)$$

One of the interesting advantages of presented method that is mean of $Dist_{MOER}$ is similar rectilinear distance result, in most cases.

Example 5.2. Let $X_1 = (3,7)$ and $X_2 = (-2,-5)$ are two point in coordinate.

By rectilinear distance $Dist_R = 17$. From the proposed Method, the result is

$$Dist_{MOER} = Dist_{\tilde{x}} + Dist_{\tilde{y}} = (13,17,21)$$

We can see the distance results by rectilinear approach is same with our method.

Example 5.3. Suppose $X_1 = (1,2)$ and $X_2 = (5,6)$.

The results obtained by the proposed method and some others methods are calculate as following:

$$Dist_{\mathrm{R}} = 8$$
$$Dist_{\mathrm{E}} = 4$$
$$Dist_{\mathrm{SE}} = 16$$
$$Dist_{\mathrm{CH}} = 4$$
$$Dist_{MOER} = (4,8,12)$$

Example 5.4. Considered we have two approximate points. The problem is distance between "approximately two" and "approximately four".

From the previous methods we cannot get a result, in other word, when we have approximate point, we cannot calculate distance by other methods, which is not a satisfactory result for managers, but our method can solve this problem and calculate above distance.

By proposed method, we have:

$$Approximately 2 = \tilde{A} = (1,2,3)$$

$$Approximately 4 = \tilde{B} = (3,4,5)$$

$$\Rightarrow Dist_{MOER} = (0,2,4)$$

Example 5.5. Consider two approximate point $X_1 = (3,7)$ and $X_2 = (4,5)$, MCP =2 By presented method, we have:

$$\tilde{x}_1 = (1,3,5) \quad \tilde{x}_2 = (2,4,6)$$
$$\tilde{y}_1 = (5,7,9) \quad \tilde{y}_2 = (3,5,7)$$

$$Dist_{\tilde{x}} = (3,3,5), \quad Dist_{\tilde{y}} = (2,2,6)$$
$$\Rightarrow \quad Dist_{MOER} = Dist_{\tilde{x}} + Dist_{\tilde{y}} = (5,5,11)$$

And this is one of the other advantage of proposed method.

Table 1. *Comparison of the proposed method with some other methods (Example 5.1.).*

Points	X_1	X_2	Result
Rectilinear	$(1,2)$	$(3,4)$	4
Euclidean	$(1,2)$	$(3,4)$	$\sqrt{8}$
Square Euclidean	$(1,2)$	$(3,4)$	8
Manhattan	$(1,2)$	$(3,4)$	4
Chebyshev	$(1,2)$	$(3,4)$	2
Proposed Method	$\tilde{x}_1 = (0,1,2)$ $\tilde{y}_1 = (1,2,3)$	$\tilde{x}_2 = (2,3,4)$ $\tilde{y}_2 = (3,4,5)$	$Dist_{MOER} = (0,4,8)$

6. Conclusion

Distance measure play an important role in facility layout. Many methods have been proposed for distance between points. Each of these techniques has been shown to produce nonintuitive results in certain cases.

In this paper, first we reviewed on some distance methods, then we presented a new strategy for comparative points in facility layout with fuzzy logic, which it is very useable, specifically when it is hard (or impossible) to use other methods to solve uncertain points. Finally, some numerical examples illustrated the presented method as well as comparing it with other various ones. New method which we called MOER distance.

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