The Life Predicting Calculations in Whole Process Realized by Calculable Materials Constants from short Crack to Long Crack Growth Process

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Abstract: To use the theoretical approach, to adopt the simple stress-, or strain-parameter method, by means of the conventional material constants, to establish some new calculation models in whole crack growth process for elastic-plastic steels, which are the equations of the driving forces and the life predictions from micro to macro crack; To provide yet several crack growth-rate-linking-equations and life calculating expressions in whole process, for which are under different loading conditions in high cycle and low cycle fatigue. For key parameters A_1 , A_2 and B_2 have proposed some new concept, and to define their physical and geometrical meanings. For the transition crack size from crack forming stage to crack growth stage, provide concrete calculation processes and methods. Thereby to realized the lifetime predicting calculations in whole process based on traditional calculable material parameters. The purpose is to try to make the modern fatigue-fracture discipline depended on tests become gradually calculable subjects as the traditional material mechanics. So that will be having practical significances for saving testing manpower and funds, for promoting applying and development relevant disciplines.

Keywords: Elastic-Plastic Materials, Fatigue Fracture, Crack Propagation Modeling, Low Cycle Fatigue, High Cycle Fatigue, Lifetime Prediction

1. Introduction

As everyone knows for the traditional material mechanics, that is a calculable subject, and has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the life problems for some structures when it is pre-existing flaws and under repeated loading. In that it has no to contain such calculable parameters as the crack variable a or as the damage variable D in their calculating models. On the other hand, inside the fracture mechanics and the damage mechanics, due to there are these variables, so they can just calculate above problems. But nowadays latter these disciplines are all subjects mainly depended on fatigue, damage and fracture tests.

Author thinks, in the mechanics and the engineering fields, in which are also to exist such scientific principles of similar to genetic and clone technology in life science. Author has done

some of works used the theoretical approach as above the similar principles [1-8]. For example, for some strength calculation models from micro to macro are provided by reference [1], for some damage growth rate calculation models from micro to macro damage growth are proposed by references [2-8] which are models in each stage even in whole process, under different loading conditions. Two years ago, in order to do the lifetime calculations in whole process on fatigue-damage-fracture for an engineering structure, author was by means of Google Scholar to search the lifetime prediction models, as had been no found for this kind of calculation equations. After then, author continues to research this item, and bases on was provided and now is complemented on the comprehensive figure 1 of material behaviors [3]; still applies above genetic principles, to study and analyze data in references, thereby to provide some new calculable models for

2. The Life Prediction Calculations for Elastic-Plastic Materials Containing Pre-Flaws

About life curves of short crack growth as the first stage are just described with curves 1 ($\sigma < \sigma_s, \sigma_m = 0$), 2 ($\sigma > \sigma_s$) and 3 ($\sigma < \sigma_s, \sigma_m \neq 0$) in reversed direction coordinate system inside attach figure.1, where depicting for the

coordinate system constituting and each curve meaning had explained in references [1-3].

(1) Under work stress $\sigma < \sigma_s$ (high cycle fatigue) condition

Under work stress $\sigma < \sigma_s$ condition, it is with a_1 as variable to adopt the simple stress parameter σ as its calculating one, and with the short crack-stress-factor range ΔH to express the life prediction equation, that corresponding reversed curves 1 and 3 can be calculated by following equations

$$N_1 = \int_{a_{01}}^{a_r} \frac{da_1}{A_1 \times (\Delta H_1)^{m_1}}, (\text{cycle}) \quad (1-1)$$

or

$$N_1 = \int_{a_{01}}^{a_r} \frac{da_1}{A_1 \times (\Delta \sigma)^{m_1}}, (\text{cycle}) \quad (1-2)$$

Where

$$H_1 = \sigma \cdot a_1^{1/m_1}, (MPa \cdot mm^{1/m_1}) \quad (2)$$

$$\Delta H_1 = \Delta \sigma \cdot a_1^{1/m_1} (MPa \cdot mm^{1/m_1}) \quad (3)$$

Here the H in eqn (2) is defined as short crack stress intensity factor, it is driving force of short crack growth under monotonic loading, and the crack stress intensity factor range ΔH_1 in eqn (3) it is driving force under fatigue loading. The A_1 is defined as the comprehensive and calculable material

Where

$$v_{eff} = \ln(a_{1f} / a_0) / N_{1fc} - N_{01} = [\ln(a_{1f} / a_0) - \ln a_1 / a_{01}] / N_{1f} - N_{01}, (mm/cycle) \quad (6)$$

or

$$v_{eff} = [a_{1f} \ln(1/1-\psi)] / N_{1f} - N_{01}, (mm/cycle) \quad (7)$$

The v_{eff} in eqns (6-7) is defined as an effective damage history correction factor in first stage, its physical meaning is the effective crack growth rate of whole failure to cause specimen material in a cycle, its unit is $mm/cycle$. ψ is a reduction of area. a_0 is pre-micro-crack value that is no effect to fatigue damage under prior cycle loading [10]. a_{01} is an initial micro-crack value, a_f is a critical fracture size before failure. N_{01} is initial life in first stage, $N_{01} = 0$; N_{1f} is failure life, $N_{1f} = 1$. By the way, here is also to adopt those material constants $\sigma'_f, b'_f, \epsilon'_f, c'_f$ as “genes” in the fatigue damage subject.

So, for the eqn (1), its final expansion equation corresponded reversed to curves 1 ($A_1 A$) is as below form:

$$N_1 = \frac{\ln a_{tr} - \ln a_1}{2(\sigma'_f)^{-m_1} \times (v_{eff})^{-1} (\Delta \sigma)^{m_1}}, (Cycle), (\sigma < \sigma_s, \sigma_m = 0) \quad (8)$$

And its final expansion equation corresponded reversed to

constant. Author researches and thinks, its physical meaning of the A_1 is a concept of power, that just is a maximal increment value to give out energy in one cycle before to cause material failure. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 1 ($\sigma_m = 0$) or 3 ($\sigma_m \neq 0$) on the y-axis, also is an intercept between $O_1 - O_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m_1 of the formula (4-5). The comprehensive material constant A_1 in formulas (4-5) is a calculable one, it has function relation with other material constants m_1 and σ'_f , etc. the σ'_f is a fatigue strength coefficient.

$$A_1 = 2(2\sigma'_f)^{-m_1} \times (v_{eff})^{-1}, (MPa^{m_1} mm/cycle), (\sigma_m = 0) \quad (4)$$

$$A_1 = 2[2\sigma'_f(1 - \sigma_m / \sigma'_f)]^{-m_1} \times (v_{eff})^{-1}, (MPa^{m_1} mm/cycle), (\sigma_m \neq 0) \quad (5)$$

Here should yet explain the A_1 in eqn (4) is corresponding reversed curves 1, its mean stress $\sigma_m = 0$; The A_1 in eqn (5) is corresponding curves 3, its $\sigma_m \neq 0$. And the correctional method for its mean stress $\sigma_m \neq 0$ can be corrected by reference [9].

curves 3 ($D_1 D$) should be:

$$N_{oi} = \frac{\ln a_{oi} - \ln a_1}{2[2\sigma'_f(1 - \sigma_m / \sigma'_f)]^{-m_1} \times (v_{eff})^{-1} \times (\Delta \sigma)^{m_1}}, (\sigma_m \neq 0) \quad (9)$$

Where a_{tr} is a transitional crack size from short crack to long crack growth process, $a_{tr} \approx a_{mac}$, a_{mac} is a long crack size corresponding forming macro crack. a_{oi} is a medial crack size between initial crack size and transitional crack size corresponding medial life N_{oi} .

(2) Under work stress $\sigma > \sigma_s$ (low cycle fatigue) condition.

Under $\sigma > \sigma_s$ condition, here it is still with a_1 as variable, but it should adopt the simple strain ϵ_p as its calculating parameter, and it should adopt the short crack-strain-factor range ΔI to express its life equation. That is corresponded to reversed direction curve $C_1 C$ in Fig1, is as following form

$$N_1 = \int_{a_1}^{a_r} \frac{da_1}{B_1 \times (\Delta I)^{m_1}}, (Cycle), (\sigma > \sigma_s) \quad (10)$$

Here

$$\Delta I_1 = (\Delta \varepsilon_p)^{m_1'} \cdot a_1 \quad (11)$$

$$N_1 = \int_{a_1}^{a_r} \frac{da_1}{B_1 \times (\Delta \varepsilon_p)^{m_1'} a_1} (Cycle), (\sigma > \sigma_s) \quad (12)$$

Where the B_1 is also calculable comprehensive material constants.

$$B_1 = 2[2\varepsilon_f']^{-m_1'} \times (v_{eff})^{-1}, (\%)^{m_1'} (mm / cycle) \quad (13)$$

$$A_1 = 2[2\sigma_f'(1 - \sigma_m / \sigma_f)]^{-m_1} (v_{eff})^{-1}, (MPa)^{m_1} mm / cycle, (\sigma_m \neq 0) \quad (16)$$

Or

$$A_1 = 2K'^{-m_1} [2\varepsilon_f'(1 - \sigma_m / \sigma_f)]^{1/c'} \times (v_{eff})^{-1}, (MPa)^{m_1} mm / cycle (\sigma_m \neq 0) \quad (17)$$

Where K' is a cyclic strength coefficient. m_1' is defined to be fatigue ductility exponent, $m_1' = -1 / c_1'$, $m_1 = -1 / c_1' \times n'$, c_1' just is a fatigue ductility exponent under low cycle fatigue, $n' = b_1' / c_1'$, n' is a strain hardening exponent. So that, its final expansion equation for (10) is as below form,

$$N_{oi} = \frac{\ln a_{oi} - \ln a_1}{2[2\sigma_f'(1 - \sigma_m / \sigma_f)]^{-m_1} (v_{eff})^{-1} \times (\Delta \sigma / 2)^{m_1}} (\sigma > \sigma_s, \sigma_m \neq 0) \quad (19)$$

If to take formula (17) to replace A_1 into eqn. (14), its final

If via the crack stress factor amplitude $\Delta H_1 / 2$ in eqn (10) to express it, due to plastic strain occur cyclic hysteresis loop effect, it should be

$$N_1 = \int_{a_1}^{a_r} \frac{da_1}{A_1 \times (\Delta \sigma / 2)^{m_1} \times a_1} (Cycle), (\sigma > \sigma_s) \quad (14)$$

Where the A_1 is also a calculable comprehensive material constant:

$$A_1 = 2(2\sigma_f')^{-m_1} (v_{eff})^{-1}, (MPa)^{m_1} mm / cycle (\sigma_m = 0) \quad (15)$$

$$N_1 = \frac{\ln a_{tr} - \ln a_1}{2(2\varepsilon_f')^{-m_1'} \times (v_{eff})^{-1} (\Delta \varepsilon_p)^{m_1'}}, (Cycle), (\sigma > \sigma_s) \quad (18)$$

Its final expansion equation for (14) is as following form,

$$N_{oi} = \frac{\ln a_{oi} - \ln a_1}{2K'^{-m_1} [2\varepsilon_f'(1 - \sigma_m / \sigma_f)]^{1/c'} \times (v_{eff})^{-1} \times (\Delta \sigma / 2)^{m_1}} (\sigma > \sigma_s, \sigma_m \neq 0) \quad (20)$$

Here influence of mean stress in eqn (19-20) can be ignored.

Where

$$K_2 = \sigma \sqrt{\pi a_2} \quad (22)$$

$$\Delta K_2 = \Delta \sigma \sqrt{\pi a_2} \quad (23)$$

2.2. The Calculations for Fatigue-Damage in Long Crack Growth Process (or Called the Second Stage)

In Fig.1, the residual life curves of long crack growth in the second stage are just described with curves 1' ($\sigma < \sigma_s, \sigma_m = 0$), 2' ($\sigma > \sigma_s$) and 3' ($\sigma < \sigma_s, \sigma_m \neq 0$) at reversed direction coordinate system.

(1) Under work stress $\sigma < \sigma_s$ condition

Here it is divided two methods: K_2 -factor method and σ -method:

1) K_2 -factor method

For life prediction equation corresponded to reversed curves $A_2 A_1$ and $D_2 D_1$ should be as following

$$N_2 = \int_{a_r}^{a_{eff}} \frac{da_2}{A_2 \times [y_2(a/b) \Delta K_2]^{m_2}} (cycle) \quad (21)$$

As it is known the $K_2 = K_1$ -factor is just the macro crack stress intensity factor, and the ΔK_2 is the macro-crack stress intensity factor range. The $y_2(a/b)$ is correction factor [11] related for long crack form and structure size. The A_2 in eqn.(21) is defined as comprehensive material constants of macro-crack, for $\sigma_m = 0$, it is corresponding curve $A_1 A_2$, and it is also calculable one as following

$$A_2 = 2(2K_{2eff})^{-m_2} \times v_{pv}, (MPa \sqrt{m})^{m_2} \times mm / cycle, (\sigma_m = 0) \quad (24)$$

Or

$$A_2 = \frac{2}{2-m_2} (a_{2eff}^{1-\frac{m_2}{2}} - a_{02}^{1-\frac{m_2}{2}}) \cdot (MPa\sqrt{m})^{m_2} \times mm / cycle, (\sigma_m = 0) \quad (25)$$

For $\sigma_m \neq 0$, the A_2 is corresponded to curve D_1D_2 , it should be as following form

$$A_2 = 2 \left[2K_{2eff} (1 - K_{2m} / K_{2fc}) \right]^{-m_2} \times v_{pv}, (\sigma_m \neq 0) \quad (26)$$

Where K_{2m} is mean crack stress intensity factor, K_{2eff} is an effective crack stress intensity factor, K_{2fc} is a critical crack stress intensity factor, which they are parameters under cyclic loading. It should be point that the physical meaning for the A_2 is also a concept of power, that just is a maximal increment value to give out energy in one cycle before failure. Its geometrical meaning is also a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 1' ($\sigma_m = 0$) or 3' $\sigma_m \neq 0$ on the y-axis, also is an intercept between $O_3 - O_4$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m_2 of the formula (24~26). Here,

$$v_{pv} = \frac{(a_{2pv} - a_{02})}{N_{2eff} - N_{02}} \approx 3 \times 10^{-5} \sim 3 \times 10^{-4} = v^* (mm / Cycle) \quad (27)$$

Author research and think, the parameter v_{pv} is defined to be the virtual rate, it is an equivalent rate caused in precrack, it can take similar dimension with the " v^* " ($m / cycle$) by reference [12]. But its unit is different, here unit of the v_{pv} is " $mm / cycle$ ". The crack size a_{2pv} is a virtual crack size as equivalent to a precrack size, a_{02} is an initial as equivalent to the initial micro-crack size. N_{02} is an initial life, $N_{02} = 0$. N_{pv} is a virtual life, $N_{2eff} = 1$. In references [13-14], all refer to effective stress intensity factor, here to propose to take

$$N_{2eff} = \frac{\frac{2}{2-m_2} (a_{2eff}^{1-\frac{m_2}{2}} - a_{02}^{1-\frac{m_2}{2}})}{2[2K_{2eff} (1 - K_m / K_{2fc})]^{-m_2} \times v_{pv} \times [(Ya / b)^{m_2} \Delta \sigma^{m_2} \pi^{\frac{m_2}{2}}]}, (\sigma_m \neq 0) \quad (35)$$

Its medial life N_{2oj} in second stage is

$$N_{2oj} = \frac{\frac{2}{2-m_2} (a_{oj}^{1-\frac{m_2}{2}} - a_{tr}^{1-\frac{m_2}{2}})}{2[2K_{2eff} (1 - K_{2m} / K_{2fc})]^{-m_1} \times v_{pv} [Y_2(a / b) \Delta \sigma \sqrt{\pi}]^{m_2}}, (cycle) (\sigma_m \neq 0) \quad (36)$$

Where a_{tr} is a transitional crack size between two stages from short crack a_{mic} to long crack a_{mac} growth process, $a_{tr} \approx a_{mac}$, the a_{oj} is a medial size. $a_{o2} < a_{oj} < a_{2eff}$.

2) σ -method

Due to word stress is still $\sigma / \sigma_s \ll 1$ ($\sigma \leq 0.5\sigma_s$), in the long crack growth process, its residual life equation of

equivalent values as follow

$$K_{2eff} \approx \sqrt{K_{th} K_{1c}}, \quad (28)$$

$$K_{1c} = \sigma_f \sqrt{\pi a_c} \quad (29)$$

Here the K_{th} is a threshold crack stress intensity factor value.
or

$$K_{2eff} \approx (0.25 - 0.4) K_{2fc} \quad (30)$$

$$K_{2fc} = \sigma'_f \sqrt{\pi a_{2fc}}, (MPa\sqrt{m}) \quad (31)$$

$$K_{2eff} = \sigma'_f \sqrt{\pi a_{2eff}}, (MPa\sqrt{m}) \quad (32)$$

$$K_{2m} = (K_{2max} + K_{2min}) / 2 \quad (33)$$

It should be point that σ_f is a true fracture stress under monotonous load, and the σ'_f is the fatigue strength coefficient under fatigue load.

So the effective life expanded equation corresponding reversed direction curve A_2A_1 is following forming.

$$N_{2eff} = \frac{\frac{2}{2-m_2} (a_{2eff}^{1-\frac{m_2}{2}} - a_{02}^{1-\frac{m_2}{2}})}{2[2K_{2eff}]^{-m_2} \times v_{pv} \times [(Ya / b)^{m_2} \Delta \sigma^{m_2} \pi^{\frac{m_2}{2}}]}, (\sigma_m = 0) \quad (34)$$

And the effective life expanded equation corresponding reversed direction curve D_2D_1 should be

corresponding reversed direction curve A_2A_1 and D_2D_1 in fig.1 is as following form

$$N_1 = \int_{a_{tr}}^{a_{2eff}} \frac{da_1}{B_2 \times (\Delta \delta_1)^{m_2}}, (Cycle) \quad (37)$$

Where

$$\delta_i = \pi a_2 \sigma_s \times (\sigma / \sigma_s)^2 / E, (mm) \quad (38)$$

$$\Delta \delta_i = \frac{\beta \Delta \sigma^2 \pi a_2}{4 \sigma_s E}, (mm), (\sigma_m = 0) \quad (39)$$

The δ_i is a crack tip open displacement, it is driving force of short crack growth under monotonic loading; and the $\Delta \delta_i$ is a crack tip open displacement range, it is driving force under fatigue loading. For the coefficient β in eqn (39), it equal

$$B_2 = 2 \left(\frac{\beta (\sigma_f'^2 \times a_{2eff} \times \pi / \sigma_s^2) \sigma_s}{E} \right)^{-m_2} \times v_{pv}, (mm^{m_2} \times mm / cycle), (\sigma = 0) \quad (40)$$

And for $\sigma \neq 0$ that is

$$B_2 = 2 \left(\frac{2 \beta (\sigma_f'^2 \times a_{2eff} \times \pi / \sigma_s^2) \sigma_s}{E} \left[1 - \frac{a_{02} (\sigma_{\max}^2 + \sigma_{\min}^2)}{2 \times a_{2eff} \sigma_s^2} \right] \right)^{-m_2} v_{pv}, (mm^{m_2} \times mm / cycle) (\sigma \neq 0) \quad (41)$$

Where $\sigma_{\max}$ and $\sigma_{\min}$ are maximum and minimum work stress. The a_{2eff} can be calculable effective crack size.

So its final expansion form corresponded reversed direction

$$N_{2eff} = \frac{(4E \cdot \sigma_s)^{m_2} \times \frac{1}{1-m_2} (a_{2eff}^{1-m_2} - a_{tr}^{1-m_2})}{2 \left(\frac{2 \beta (\sigma_{2eff}^2 \times a_{2eff} \times \pi / \sigma_s^2) \sigma_s}{E} \right)^{-m_2} v_{pv} [\gamma_2 (a/b) \beta \times \Delta \sigma^2 \pi]^{m_2}}, (\sigma_m = 0), (cycle) \quad (42)$$

And the life equation corresponded to reversed direction curve $D_2 D_1$ is following

$$N_{2eff} = \frac{(2E \cdot \sigma_s)^{m_2} \times \frac{1}{1-m_2} (a_{2eff}^{1-m_2} - a_{tr}^{1-m_2})}{2 \left(\frac{2 \beta (\sigma_{2eff}^2 \times a_{2eff} \times \pi / \sigma_s^2) \sigma_s}{E} \left[1 - \frac{a_{02} (\sigma_{\max}^2 + \sigma_{\min}^2)}{2 \times a_{2eff} \times \sigma_s^2} \right] \right)^{-m_2} v'_{pv} [\gamma_2 (a/b) \beta \Delta \sigma^2 \pi]^{m_2}}, (cycle) (\sigma_m \neq 0) \quad (43)$$

(2) Under work tress $\sigma > \sigma_s$ condition

Under $\sigma > \sigma_s$ condition, its effective life models corresponded to reversed curve $C_2 C_1$ in figure 1 is as below form,

$$N_{2eff} = \int_{a_r}^{a_{2eff}} \frac{da_2}{B_2 \times (\Delta \delta_i / 2)^{\lambda_2}}, (Cycle), (\sigma > \sigma_s) \quad (44)$$

Where B_2 is also a calculable comprehensive material constant,

$$B_2 = 2 \left[(\pi \sigma_s (\sigma_f' / \sigma_s + 1) a_{2eff} / E) \right]^{-\lambda_2} \times v_{pv}, (mm^{\lambda_2} \times mm / cycle), (\sigma_m = 0) \quad (45)$$

$$B_2 = 2 \left[(\pi \sigma_s (\sigma_f' / \sigma_s + 1) (1 - \sigma_m / \sigma_f') a_{2eff} / E) \right]^{-\lambda_2} \times v_{pv}, (mm^{\lambda_2} \times mm / cycle), (\sigma_m \neq 0) \quad (46)$$

The λ_2 is defined to be ductility exponent in long crack growth process, $\lambda_2 = -1 / c_2'$, c_2' is a fatigue ductility exponent in second stage.

So that, the final expansion equations is derived from above mentioned eqn. (44) as follow

For $\sigma_m = 0$,

$$N_{2eff} = \frac{\frac{1}{1-\lambda_2}(a_{2eff}^{1-\lambda_2} - a_{02}^{1-\lambda_2})}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)a_{2eff}/E)\right]^{\lambda_2} \times v_{pv} \left[\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s + 1)}{E}\right]^{\lambda_2}}, (cycle), \quad (47)$$

For $\sigma_m \neq 0$, it should be

$$N_{2eff} = \frac{\frac{1}{1-\lambda_2}(a_{2eff}^{1-\lambda_2} - a_{02}^{1-\lambda_2})}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1-\sigma_m/\sigma'_f)a_{2eff}/E)\right]^{\lambda_2} \times v_{pv} \left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)}{E}\right]^{\lambda_2}}, (cycle) \quad (48)$$

In the eqn (48), influence to mean stress usually can ignore.

Where, the a_{2eff} is an effective crack size, it can calculate from effective crack tip opening displacement δ_{2eff}

$$a_{2eff} = \frac{E \times \delta_{2eff}}{\pi\sigma_s(\sigma'_f/\sigma_s + 1)}, (mm) \quad (49)$$

And

$$\delta_{2eff} = (0.25 \sim 0.4)\delta_c, (mm) \quad (50)$$

Where the δ_c is critical crack tip displacement. So the a_{2eff} in (47-48) can be calculated by δ_c -value by means of equations (49-50).

2.3. The Life Prediction Calculations for Crack Propagation in Whole Process

$$\begin{aligned} \frac{da_1}{dN} &= \left\{ 2[2\sigma'_f]^{-m_1} \times (a_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} a \right\}_{a_{01} \rightarrow a_{tr}} \leq \frac{da_{tr}}{dN} = \\ &= \leq \frac{da_2}{dN_2} = \left\{ 2 \left(\frac{2\beta(\sigma_{2eff}^2 \times a_{eff} \pi / \sigma_s^2) \sigma_s}{E} \right)^{-m_2} v_{pv} \times \left(\frac{y_2(a/b)\beta\Delta\sigma^2 \pi a}{4\sigma_s E} \right)^{m_2} \right\}_{a_{tr} \rightarrow a_{eff}}, (\sigma_m = 0), (mm/cycle) \end{aligned} \quad (52)$$

For $\sigma < \sigma_s, \sigma_m \neq 0$, its expanded the crack rate-linking-equation for eqn (51) corresponded to positive curve DD_1D_2 is as following form

$$\begin{aligned} \frac{da_1}{dN} &= \left\{ 2[2\sigma'_f(1-\sigma_m/\sigma'_f)]^{-m_1} \times (a_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} a \right\}_{a_{01} \rightarrow a_{tr}} \leq \frac{da_{tr}}{dN} = \\ &= \leq \frac{da_2}{dN_2} = \left\{ 2 \left(\frac{2\beta(\sigma_{2eff}^2 \times a_{eff} \times \pi / \sigma_s^2) \sigma_s}{E} \left[1 - \frac{a_{02}(\sigma_{max}^2 + \sigma_{min}^2)}{2a_{meff}\sigma_s^2} \right] \right)^{-m_2} v_{pv} \times \left(\frac{y_2(a/b)\beta\Delta\sigma^2 \pi a}{2\sigma_s E} \right)^{m_2} \right\}_{a_{tr} \rightarrow a_{eff}}, \\ &mm/cycle, (\sigma_m \neq 0) \end{aligned} \quad (53)$$

And the life equations in whole process corresponding to reversed direction curves A_2A_1A and D_2D_1D should be as below

$$\Sigma N = N_1 + N_2 = \int_{a_{01}}^{a_{tr}} \frac{da}{A_1 \times (\Delta\sigma)^{m_1} \times a} + \int_{a_{tr}}^{a_{2eff}} \frac{da}{A_2 (\Delta\delta_t)^{m_2}}, \quad (54)$$

Its expanded equation corresponding to reversed direction curves A_2A_1A is as following form

$$\begin{aligned} \Sigma N = N_1 + N_2 = & \int_{a_{01}}^{a_{tr}} \frac{da}{2[2\sigma'_f]^{-m_1} \times (a_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} \times a} \\ & + \int_{a_{tr}}^{a_{2eff}} \frac{da}{2 \left(\frac{2\beta(\sigma_{eff}^2 \times \pi \times a_{eff} / \sigma_s^2) \sigma_s}{E} \right)^{-m_2} v_{pv} [2\gamma_2 \beta \Delta \sigma^2 \pi a / 4E \sigma_s]^{m_2}}, (cycle), (\sigma_m = 0) \end{aligned} \quad (55)$$

But for $\sigma_m \neq 0$, its expanded equation corresponding to reversed direction curves $D_2 D_1 D$ should be

$$\begin{aligned} \Sigma N = N_1 + N_2 = & \int_{a_{01}}^{a_{tr}} \frac{da}{2[2\sigma'_f (1 - \sigma_m / \sigma'_f)]^{-m_1} \times (a_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} \times a} \\ & + \int_{a_{tr}}^{a_{2eff}} \frac{da}{2 \left(\frac{2\beta(\sigma_{eff}^2 \times \pi \times a_{eff} / \sigma_s^2) \sigma_s}{E} \left[1 - \frac{a_{02}(\sigma_{max}^2 + \sigma_{min}^2)}{2a_{meff} \sigma_s^2} \right] \right)^{-m_2} v_{pv} [2\gamma_2 \beta \Delta \sigma^2 \pi a / 2E \sigma_s]^{m_2}}, (cycle) (\sigma_m \neq 0) \end{aligned} \quad (56)$$

(2) Under work stress $\sigma > \sigma_s$ for eqn (51) corresponding to positive curve $CC_1 C_2$ is as
Under work stress $\sigma > \sigma_s$, its expanded rate link equation following form

$$\begin{aligned} \frac{da_1}{dN} = & \left\{ 2K'^{-m_1} [2\epsilon'_f]^{1/c'} \times (v_f \times a_{tr})^{-1} \times (\Delta\sigma / 2)^{m_1} \times a \right\}_{a_{01} \rightarrow a_{tr}} \leq \frac{da_{tr}}{dN} = \\ \frac{da_2}{dN_2} = & \left\{ 2 \left[(\pi \sigma_s (\sigma'_f / \sigma_s + 1) a_{eff} / E) \right]^{\lambda_2} \times v_{pv} \left[\frac{0.5\pi \sigma_s \gamma_2 (\Delta\sigma / 2 \sigma_s + 1) a}{E} \right]^{\lambda_2} \right\}_{a_{tr} \rightarrow a_{2eff}}, mm/cycle, (\sigma \neq 0) \end{aligned} \quad (57)$$

The life equations in whole process corresponding to reversed direction curve $C_2 C_1 C$ should be as following

$$\Sigma N = N_1 + N_2 = \int_{a_{01}}^{a_{tr}} \frac{da}{B_1 \times (\Delta\sigma / 2)^{m_1} \times a} + \int_{a_{tr}}^{a_{2eff}} \frac{da}{B_2 (\Delta\sigma / 2)^{\lambda_2}}, \quad (58)$$

And the life prediction expanded expression in whole process corresponded reversed curve $C_2 C_1 C$, it should be

$$\begin{aligned} \Sigma N = & \int_{a_{01}}^{a_{tr}} \frac{da}{2K'^{-m_1} [2\epsilon'_f]^{1/c'} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma / 2)^{m_1} \times a} \\ & + \int_{a_{tr}}^{a_{2eff}} \frac{da}{2 \left[(\pi \sigma_s (\sigma'_f / \sigma_s + 1) a_{2eff} / E) \right]^{\lambda_2} \times v_{pv} \left[\frac{0.5\pi \sigma_s \gamma_2 (\Delta\sigma / 2 \sigma_s + 1) a}{E} \right]^{\lambda_2}}, (cycle) \end{aligned} \quad (59)$$

It should point that the calculations for rate and life in whole process should be according to different stress level, to select appropriate calculable equation. Here it have to explain that its meaning of the crack rate-linking-equation (51-53, 57) is to make a calculable linking formula between the first stage crack rate and the second stage one, which it should be calculated by the short crack growth rate equation before the transition point size a_{tr} ; it should be calculated by the long crack growth rate equation after the transition point a_{tr} , note that it is not added together by the crack growth rates for two stages. But the life calculations for two stages can be added together. About calculation method, it can calculate by means of computer doing computing by different crack size [16].

3. Calculating Example

3.1. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b = 573MPa$, yield limit $\sigma_s = 361MPa$, fatigue limit $\sigma_{-1} = 267.2MPa$, reduction of area is $\psi = 0.51$, modulus of elasticity $E = 200000MPa$; Cyclic strength coefficient $K' = 1165MPa$, strain-hardening exponent $n' = 0.187$; Fatigue strength coefficient $\sigma'_f = 947.1MPa$, fatigue strength exponent $b'_1 = -0.111$, $m_1 = 9.009$; Fatigue ductility coefficient $\epsilon'_f = 0.464$, fatigue ductility exponent $c'_1 = -0.5395$,

$m'_1 = 1.8536$. Threshold value $\Delta K_{th} = 8.6 \text{ MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c} = K_{1c} = 92.7 \text{ MPa}\sqrt{\text{m}}$. Working stress $\sigma_{\max} = 450 \text{ MPa}$, $\sigma_{\min} = 0$ in pressure vessel. And suppose that for long crack shape has been simplified via treatment

become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1, i.e. $y_2(a/b) = 1$. Other computing data are all included in table 1.

Table 1. Computing data.

$K_{1c}, \text{MPa}\sqrt{\text{m}}$	$K_{eff}, \text{MPa}\sqrt{\text{m}}$	$K_{th}, \text{MPa}\sqrt{\text{m}}$	v_{pv}	m_2	δ_c, mm	λ_2	$y_2(a/b)$	a_{th}, mm
92.7	28.23	8.6	2×10^{-4}	3.91	0.18	2.9	1.0	0.07

3.2. Required Calculating Data

Try to calculate respectively as following different data and depicting their curves:

- (1) To calculate the transitional point crack size a_{tr} between two stages;
- (2) To calculate the crack growth rate at transitional point (at crack size a_{tr})
- (3) To calculate the life N_1 in first stage from micro crack $a_1 = 0.02 \text{ mm}$ growth to transitional point a_{tr}
- (4) To calculate the life N_2 in second stage N_2 from transitional point a_{tr} to long crack size $a_{2eff} = 5 \text{ mm}$;
- (5) Calculating the whole service lifetime $\sum N$.
- (6) Depicting their life curves in whole process.

3.3. Calculating Processes and Methods

The concrete calculation methods and processes are as follows

- (1) Calculations for relevant parameters
 - 1) Stress range and mean stress calculations:
Stress range: $\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 450 - 0 = 450 \text{ (MPa)}$
Mean stress:
 $\sigma_m = (\sigma_{\max} + \sigma_{\min}) / 2 = (450 - 0) / 2 = 225 \text{ MPa}$
 - 2) According to formulas (7), calculation for correction

coefficient v_{eff} in first stage.

For the effective crack sizes a_{eff} in first stage and the second stage, it both can be calculated respectively, and can take smaller one of both, here to take same value with the second stage, $a_{1eff} = a_{2eff} = 2 \text{ mm}$. For example, according to formulas (49), Calculating effective size a_{eff}

$$a_{eff} = \frac{E \times \delta_{eff}}{\pi \sigma_s (\sigma_f / \sigma_s + 1)} = \frac{200000 \times 0.25 \times 0.18}{\pi 361 (947.1 / 361 + 1)} = 2.1 \text{ (mm)},$$

Take $a_{eff} = 2.0 \text{ mm}$

$$v_{eff} = a_{eff} \ln[1 / (1 - \psi)] = 2 \times \ln[1 / (1 - 0.51)] = 1.43 \text{ (mm/cycle)}$$

3) By eqn (27), to select virtual rate v_{pv} in second stage, here take:

$$v_{pv} = \frac{a_{2eff} - a_{02}}{N_{2f} - N_{02}} \approx 2.0 \times 10^{-4} \text{ (mm / Cycle)}, \quad N_{2f} = 1, \quad N_{02} = 0$$

(2) To calculate the crack size a_{tr} of transitional point between two stages

1) To select calculating equation of short crack growth rate

At first, calculation for comprehensive material constant B_1 by eqn (17)

$$A_1 = 2K^{-m_1} [2\epsilon'_f (1 - \sigma_m / \sigma'_f)]^{1/c'} \times (a_{ef} \times v_f)^{-1} = 2 \times 1165^{-9.01} \times [2 \times 0.464 (1 - 225 / 947.1)]^{1/-0.5395} (2 \times 0.713)^{-1} \\ = 6.28 \times 10^{-28} \text{ (MPa)}^{m_1} \text{ mm / cycle}$$

Here select the crack growth rate linking equation (57), and for its growth rate to simplify as follow form,

$$da_1 / dN_1 = A_1 \times (\Delta\sigma / 2)^{m_1} \times a_1 = 3.193 \times 10^{-28} \times (450 / 2)^{9.01} \times a_1 = 6.28 \times 10^{-28} \times 1.56 \times 10^{21} \times a_1 = 9.8 \times 10^{-7} \times a_1$$

2) By rate linking equation (57), calculating for crack growth rate in long crack growth process, Calculation for comprehensive material constant B_2 by eqn (46)

$$B_2 = 2 \left[(\pi \sigma_s (\sigma'_f / \sigma_s + 1) (1 - \sigma_m / \sigma'_f) a_{eff} / E) \right]^{-\lambda_2} \times v_{pv} = 2 \left[2 (3.1416 \times 361 (947.1 / 361 + 1) (1 - 225 / 947.1) \times 2 / 200000) \right]^{-2.9} \\ \times 2 \times 10^{-4} = 9.1988 \text{ (mm)}^{-\lambda_2} \times \text{mm / Cycle}$$

For the long crack rate in second stage, to take brief calculations as follow form,

$$da_2 / dN_2 = B_2 \left[\frac{0.5\pi\sigma_s y_2 (\Delta\sigma / 2\sigma_s + 1)a_2}{E} \right]^{\lambda_2} = 9.1988 \times \left[\frac{0.5\pi 361(450 / (2 \times 361) + 1)a_2}{E} \right]^{2.9} = 9.1988 \times 1.6698 \times 10^{-7} a_2^{2.9}$$

$$= 1.5384 \times 10^{-6} a_2^{2.9}$$

3) Calculation for crack size a_{tr} at transitional point

According to the equations (51) and (57), for calculation the crack size a_{tr} at transitional point, it can make equal between both rate expansion equations, and to simplify it as following

$$6.28 \times 10^{-28} \times 1.56 \times 10^{21} \times a_{tr} = 9.1988 \times 1.6698 \times 10^{-7} \times a_{tr}^{2.9}$$

$$da_1 / dN_1 = da_{tr} / dN_{tr} = 9.8 \times 10^{-7} a_1 = 9.8 \times 10^{-7} \times 0.789 = 7.74 \times 10^{-7} (mm / cycle)$$

$$da_2 / dN_2 = da_{tr} / dN_{tr} = 1.5384 \times 10^{-6} a_{tr}^{2.9} = 1.5384 \times 10^{-6} \times (0.79)^{2.9}$$

$$= 7.74 \times 10^{-7} (mm / cycle)$$

Here it can be seen, the growth rate at the transition point is same.

(4) Life prediction calculations in whole process

1) To select eqn (20), to calculate the life N_1 from

$$N_1 = \frac{\ln a_{tr} - \ln a_{01}}{2K' \cdot m_1 [2\epsilon'_f (1 - \sigma'_m / \sigma'_f)]^{1/c'} \times (a_{eff} \times v_f)^{-1} \times (\Delta\sigma / 2)^{m_1} \times a}$$

$$= \frac{\ln 0.789 - \ln 0.02}{2 \times 1165^{-9.01} \times [2 \times 0.464(1 - 225 / 947.1)]^{1/-0.5395} (2 \times 0.713)^{-1} \times (450 / 2)^{9.01}}$$

$$= \frac{3.675}{6.28 \times 10^{-28} \times 1.56 \times 10^{21}} = \frac{3.675}{9.8 \times 10^{-7}} = 3751260 (Cycle)$$

So predicting life in first stage $N_1 = 3751260 (Cycle)$

And for above formulas, we can derive simplified life equation corresponded to different crack size as follow form

$$N_1 = \frac{1}{9.8 \times 10^{-7} a_1}$$

$$N_2 = \frac{\frac{1}{1 - \lambda_2} (a_{2eff}^{1 - \lambda_2} - a_{tr}^{1 - \lambda_2})}{2 \left[(\pi\sigma_s (\sigma'_f / \sigma_s + 1) (1 - \sigma'_m / \sigma'_f) a_{eff} / E) \right]^{\lambda_2} \times v_{pv} \left[\frac{0.5\pi\sigma_s y_2 (\Delta\sigma / 2\sigma_s + 1)}{E} \right]^{\lambda_2}}$$

$$= \frac{\frac{1}{1 - 2.9} (5^{1 - 2.9} - 0.789^{1 - 2.9})}{2 \left[(3.1416 \times 361 (947.1 / 361 + 1) (1 - 225 / 947.1) \times 2 / 200000) \right]^{2.9} \times v_{pv}}$$

$$\times \frac{1}{\left[\frac{0.5\pi 361 (450 / (2 \times 361) + 1)}{E} \right]^{2.9}} = \frac{0.8}{9.1988 \times 1.6698 \times 10^{-7}} = \frac{0.8}{1.5384 \times 10^{-6}} = 520625 (Cycle)$$

From above formulas, we can also derive simplified life equation corresponding different crack size as follow form

$$N_2 = \frac{1}{1.5384 \times 10^{-6} a_2^{2.9}}$$

Therefore, predicting lifetime in whole process is

$$\sum N = N_1 + N_2 = 3751260 + 520625 = 4271885 (Cycle)$$

$$a_{tr} = (0.638)^{\frac{1}{1.9}} = (0.638)^{0.5263} = 0.789 (mm)$$

Result, to obtain crack size $a_{tr} = 0.789 (mm)$ at the transitional point.

(3) To calculate the crack growth rate at transitional point

micro-crack $a_{01} = 0.02 mm$ to transitional point $a_{tr} = 0.789 mm$ is as follow,

2) To select eqn (48), to calculate the life N_2 from transitional point $a_{tr} = 0.789 mm$ to $a_{2eff} = 5 mm$ is as follow,

The life data corresponded to different crack length is all included in table 2, 3 and 4. Author finds these data are calculated by the simple stress- or strain-parameter method are basically close as compared with another result data that are calculated by the two-parameter-multiplication-method in damage mechanics, where it has been published recently in reference [3]. Especially, those data in second stage are closer, and the calculating models and method is simpler than the two-parameter-method.

Table 2. Crack growth life data in whole process.

Data point of number	1	2	3	4	5
Crack size (mm)	0.02	0.04	0.1	0.2	0.4
Data of the first stage	51020408	25510204	10204082	5102041	2551020
Data of the second stage	Invalid section				

Table 3. Crack growth life data in whole process.

Data point of number	5	6	7 transition point	8
Crack size (mm)	0.5	0.6	0.789	1.0
Data of the first stage	2040816	1700680	1293293	1020408
Data of the second stage	4851966	2859513	1292431	650026

Table 4. crack growth life data in whole process.

Data point of number	9	10	11	12	13
Crack size (mm)	1.5	2.0	3.0	4	5
Data of the first stage	680272	510204	Invalid section		
Data of the second stage	200570	87085	26871	11667	6108

(5) To depict the life curves in the whole process

By means of the data in tables 2-4 mentioned above have depicted the life curves for two stages and whole course are respectively in figure 2 and 3.

Figure 2. life curve in whole course (in decimal coordinate system).

- (A) 1-1--- data curve in first stage obtained by single-parameter calculating method;
 (B) 1-2---data curve in second stage obtained by single-parameter calculating method;
 (C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789mm).

Figure 3. life curve in whole course (in logarithmic coordinate system).

- (A) 1-1--- data curve in first stage obtained by single-parameter calculating method;
 (B) 1-2---data curve in second stage obtained by single-parameter calculating method;
 (C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789mm).

4. Discussions and Conclusions

- (1) About comparison of calculating results data: The

lifetime data calculated by the simple stress- or strain-parameter-method as compared with the two-parameter-multiplication-method in damage mechanics [3], which is basically close, the calculating models and method is also simpler than it.

(2) About new understanding for some material constants: True material constants must show the inherent characters of materials, such as the σ_s and E, δ, ψ etc in the material mechanics; for instance the σ_f and σ'_f ; ε_f , and ε'_f ; b_1 and b'_1 ; c_1 and c'_1 and so on in the fatigue damage mechanics; K_{Ic} , δ_c , m_2 , λ_2 and so on in the fracture mechanics, above which could all be checked and obtained from general handbooks. But some key materials constants A_1 , A_2 and B_2 in the fracture mechanics which are essentially functional formulas with other parameters the σ'_f and ε'_f ; b'_1 , and c'_1 etc. to have functional relations, for which can all be calculated by means of the relational expressions e.g. eqns (4-5), (13,15-16), (24-26), (40-41), (45-46), etc. that has to combine experiments and to verify. Therefore for this kind of material constants can be defined as the calculable comprehensive materials constants.

(3) About cognitions to the physical and geometrical meanings for key parameters A_1 and A_2 , B_2 : The parameters A_1 in the first stage and A_2 , B_2 in the second stage, they are all a concept of power, just are a maximal increment value paying energy in one cycle before to cause failure. Their geometrical meanings are a maximal micro-trapezium area approximating to beeline.

(4) About the methods for crack propagation rate and lifetime calculations in whole process: Calculation for crack transition size a_{tr} it can be calculated from the crack growth-rate-linking-equation (51-53) and (57); calculation for the short crack rate it should be calculated by the short crack rate-equations (da_1/dN_1) before at transition point a_{tr} ; for the long crack rate it should be calculated via long crack-rate-equations (da_2/dN_2) after transition point a_{tr} . But for the lifetime calculations in whole process can be added together by life cycle number of two stages.

(5) Based on the traditional material mechanics is a calculable subject; in consideration of the conventional materials constants there are "the hereditary characters"; In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions of the (1)~(4); then for the fatigue and the fracture disciplines, if make them become calculable subjects, that will be to exist possibility.

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subjects. Due to they hard research, make to discover the fatigue damage and crack behavioral law for materials, to form the modern fatigue, damage-, and fracture-mechanics; due to they work like a horse, make to develop the fatigue, damage-, and fracture mechanics subjects, gain huge benefits for accident analysis, safety design and operation for which are mechanical equipments in engineering fields. Particularly should explain that author cannot have so many of discovery and provide above the calculable mathematical models and the figure 1, if have no their research results.

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