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Analysis of the Variable Life Insurance Based on Log-Normal Distribution

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Abstract: Fixed rate, premiums and insurance coverage for policyholders and insurance companies in traditional life insurance have increased certain risks. For this reason, we consider studying variable life insurance. The biggest difference between the two insurance is that whether the actual death benefit of volatility is changeable. This paper studied the change of the premium when the premium changes in proportion to the death benefit and when it is fixed. And, it put forward a way to pay the death benefit, named "pay off increasing amount insurance". Finally, this paper simulated the mean and variance of the death benefit using Monte Carlo method, and also compared the advantage and disadvantages of each approach.

Keywords: Variable Life Insurance, The Actual Death Benefit, Change in Proportion, Fixed Premium, Pay off Increasing Amount Insurance, Monte Carlo Method

1. Introduction

Since the reform and opening up, Chinese insurance industry is rapidly becoming one of the fastest growing sectors of the national economy. With the gradual speeding up of the development of the insurance industry and the competition becoming increasingly fierce, the flaws of traditional life insurance are increasingly conspicuous. For policyholders, it is not conducive to change premiums based on their economic situation, and besides, they cannot gain benefits in economic growth. For life insurance companies, it will lead to increased risk if the scheduled interest rate is too high, or else it will be less attractive to clients. Unstable interest rates and fixed insurance coverage increase the risk of earnings, also lead to more people to terminate their traditional life insurance contracts.

In today's rapid economic development, traditional life insurance which is limited by its defects will gradually lose the market. In order to meet the demands of policyholders, and make clients benefited from economic growth while they obtain life insurance protection, Chinese life insurance companies should learn from foreign experience, develop variable life insurance product and research new pricing model of variable life insurance product.

The variable insurance was built in a paper by Duncan (1952). Since 1969, a flurry of activity on variable life insurance has appeared. It is the paper by Fraser, Miller, and Sternhell (1969), and its extensive discussion that be the basic reference. What is more, a less formal introduction and some numerical illustrations is provided by Miller (1971). For more discussions and additional information on variable annuities, see Bowers et al. (1997).

2. The Basic Introduction and Characteristics of Variable Life Insurance

The difference between variable life insurance and traditional life insurance is that the insured amount of policies is variable on the premise of the minimum amount, and this change depends on the benefits of separate accounts which policyholder choose to invest in. The investment-oriented policy is shown in figure 1.

In addition to the fixed minimum death benefit stipulated by the policy, the death benefit of variable life insurance also includes investment income from investment accounts, which is alterable. Insurance company opens a single account or multiple discrete sub-accounts which have different risks and different benefits. After deducting running expenses and the cost of death in premiums of policyholders, the insurance company will put the remaining costs into accounts, and establish funds with different investment direction and different risks for the insured. The policyholders could choose a fund or several funds, and insurance company will manage the funds by itself or commission professional company to manage them. The final benefits will return to the policyholders. The higher benefits of investment will lead to the higher cash value of the policy, and the insured amount would be higher too; on the contrary, lower investment income is, lower cash value of the policy and lower insured amount are.

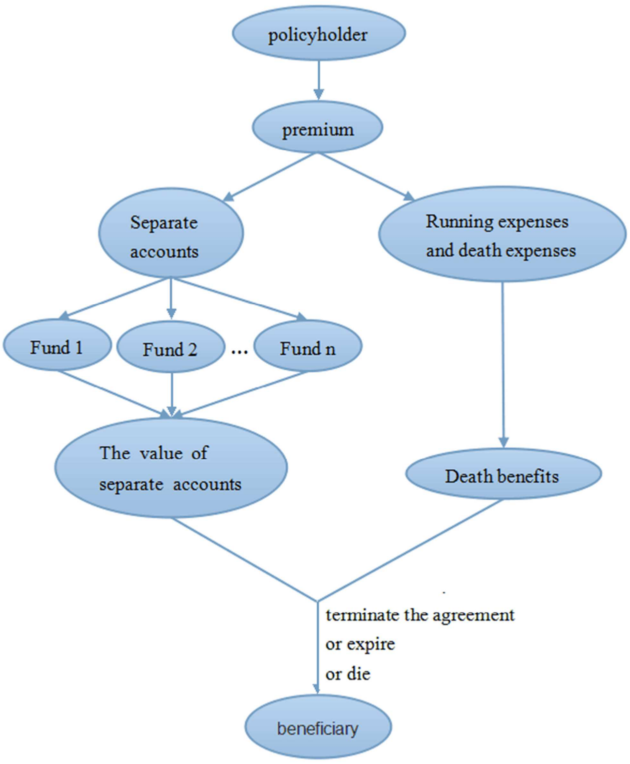

Figure 1. *The investment-oriented policy.*

Thus, the investment income of policyholders is exclusive to them, and they bear the responsibility of investment risk alone. For insurers, they need to bear only the risks that caused by mortality and expense, which is to say, the variable life insurance avoids the risk effectively which is caused by inflation and interest rate variation.

3. The Actuarial Study of Variable Life Insurance Death Benefit

The biggest difference between variable life insurance and traditional life insurance is the variability of death benefits. Therefore, the research on death benefit is the key to actuarial studies of variable life insurance. The actual death benefit is determined by the cumulative value of the policy, but not less than the scheduled minimum death benefit.

For example, consider discrete whole life insurance. Assume that the insured person age (x), the probability of survival for a year is p_x, the insured amount (the lowest death benefit) is N RMB, and the actuarial present value of the insured's permanent life insurance is A_x at age (x). $A^1_{x:\overline{n|}}$ indicates actuarial present value of permanent life insurance of n term when the insured ages (x). The liability reserve funds on the end of term k is ${}_kV(A_x)$, the net annual premium is $P(A_x)$, and the expected rate of return on investment of the period k is i_k, the actual rate of return on investment is i'_k.

There are two ways to determine the premium: fixed premium and premium changes in the same proportion with death benefit. We calculate the actual death benefit in both cases respectively:

3.1. Premiums Change in the Same Proportion with Death Benefits

Assume that b_k ($b_k \geq N$, $b_0 = N$) is the death benefit of term $k+1$. Therefore, the liability reserve funds in the beginning of term $k+1$ is $b_{k\,k}V(A_x)+b_kP(A_x)$.

Accumulating to the end of the term at the actual interest rate, the asset share becomes $(b_{k\,k}V(A_x)+b_kP(A_x)-b_kA^1_{x+k:\overline{1|}})(1+i'_{k+1})$, then

$$(b_{k\,k}V(A_x)+b_kP(A_x)-b_kA^1_{x+k:\overline{1|}})(1+i'_{k+1}) = p_{x+k}b_{k+1\,k+1}V(A_x) \quad (3.1)$$

According to the recurrence formula of liability reserve funds, we have

$$({}_kV(A_x)+P(A_x)-A^1_{x+k:\overline{1|}})(1+i_{k+1}) = p_{x+k\,k+1}V(A_x) \quad (3.2)$$

Divide (2.1) by (2.2), and we can get

$$b_{k+1} = \frac{1+i'_{k+1}}{1+i_{k+1}}b_k \quad (3.3)$$

When $i'_{k+1} = i_{k+1}$, we have $b_{k+1} = b_k$. That is to say, even if i'_{k+1}, the actual rate of return on i_{k+1}, investment of term $k+1$, fall back to the expected rate, the actual death benefit level will remain at last year's level without decreasing, which means the actual death benefit is relatively stable (see Duncan, Robert M, 1952).

3.2. The Premiums Are Fixed

Assume liability reserve funds change in the same proportion with death benefit, the premium is invariant, constant for $NP(A_x)$, and the liability reserve funds at the end of term k is $b_{k\,k}V(A_x)$, and the premium charged in the beginning of term $k+1$ is $NP(A_x)$, which accumulated to the end of the term at the rate of $b_k > N$, then we have

$$(b_{k\,k}V(A_x)+NP(A_x)-b_kA^1_{x+k:\overline{1|}})(1+i'_{k+1}) = p_{k+1}b_{k+1\,k+1}V(A_x) \quad (3.4)$$

Divide (2.4) by (2.5), we can get

$$b_{k+1} = \frac{({}_kV(A_x) + \frac{N}{b_k}P(A_x) - A^1_{x+k:\overline{1}|})(1+i'_{k+1})}{({}_kV(A_x) + P(A_x) - A^1_{x+k:\overline{1}|})(1+i_{k+1})} b_k \tag{3.5}$$

It is obvious that $b_k > N$. If $i'_{k+1} = i_{k+1}$, then we can get $b_{k+1} < b_k$. That means as long as i'_{k+1} fall back to the expected rate i_{k+1}, the actual death benefit level will drop below the level of the previous period (see Fraser, John C., Walter N.Miller, and Charles M. Sternhell, 1969, pp 175-243).

Therefore, the actual death benefit is affected by the actual investment return rate, and it is unstable, so we need to optimize the method of how to determine the actual death benefit when premium is fixed. Writers put forward a way named "pay off increasing amount insurance".

3.3. Pay off Increasing Amount Insurance

The bonus generated then (the asset share of the policy at the end of the term k mines liability reserves at the end of term k) is the premium of wholesale payment. Let b_k be the death benefit of term $k+1$, and the death benefit which is formed only by the bonus be $b_k - N$.

Therefore, the liability reserve in the beginning of term $k+1$ is

$$(b_k - N)A_{x+k} + N\,{}_kV(A_x) + NP(A_x).$$

The asset share accumulated to the end of the period at the rate of i'_{k+1} is

$$[(b_k - N)A_{x+k} + N\,{}_kV(A_x) + NP(A_x) - b_k A^1_{x+k:\overline{1}|}](1+i'_{k+1}),$$

and the death benefit of term $k+2$ is b_{k+1}. It follows that

$$[(b_k - N)A_{x+k} + N\,{}_kV(A_x) + NP(A_x) - b_k A^1_{x+k:\overline{1}|}](1+i'_{k+1}) = p_{x+k}[(b_{k+1} - N)A_{x+k+1} + N\,{}_{k+1}V(A_x)] \tag{3.6}$$

Yet

$$\begin{aligned} &[(b_k - N)A_{x+k} + N\,{}_kV(A_x) + NP(A_x) - b_k A^1_{x+k:\overline{1}|}](1+i_{k+1}) \\ &= [(b_k - N)(A^1_{x+k:\overline{1}|} + \frac{1}{1+i}p_{x+k}A_{x+k+1}) + N\,{}_kV(A_x) + NP(A_x) - b_k A^1_{x+k:\overline{1}|}](1+i_{k+1}) \\ &= [N\,{}_kV(A_x) + NP(A_x) - NA^1_{x+k:\overline{1}|}](1+i_{k+1}) + (b_k - N)p_{x+k}A_{x+k+1} \\ &= p_{x+k}N\,{}_kV(A_x) + (b_k - N)p_{x+k}A_{x+k+1} = p_{x+k}[(b_k - N)A_{x+k+1} + N\,{}_{k+1}V(A_x)] \end{aligned} \tag{3.7}$$

Dividing (2.6) by (2.7) gives

$$\frac{1+i'_{k+1}}{1+i_{k+1}} = \frac{(b_{k+1} - N)A_{x+k+1} + N\,{}_{k+1}V(A_x)}{(b_k - N)A_{x+k+1} + N\,{}_{k+1}V(A_x)} \tag{3.8}$$

According to the formula of liability reserves,

$${}_{k+1}V(A_x) = A_{x+k+1} - P(A_x)\ddot{a}_{x+k+1} = A_{x+k+1} - P(A_x)\frac{A_{x+k+1}}{P(A_{x+k+1})} = A_{x+k+1}(1 - \frac{P(A_x)}{P(A_{x+k+1})}) \tag{3.9}$$

Substituting (2.9) for (2.8) gives

$$\frac{1+i'_{k+1}}{1+i_{k+1}} = \frac{(b_{k+1} - 1)A_{x+k+1} + A_{x+k+1}(1 - \frac{P(A_x)}{P(A_{x+k+1})})}{(b_k - 1)A_{x+k+1} + A_{x+k+1}(1 - \frac{P(A_x)}{P(A_{x+k+1})})} = \frac{b_{k+1} - \frac{P(A_x)}{P(A_{x+k+1})}}{b_k - \frac{P(A_x)}{P(A_{x+k+1})}} \tag{3.10}$$

Hence,

$$b_{k+1} = [b_k - \frac{P(A_x)}{P(A_{x+k+1})}]\frac{1+i'_{k+1}}{1+i_{k+1}} + \frac{P(A_x)}{P(A_{x+k+1})} \tag{3.11}$$

$b_{k+1} = b_k$ when $i'_{k+1} = i_{k+1}$. That means rather than fall, the actual death benefit will remain at the level of last year even though the actual rate of return on investment drops to i_{k+1}. If and only if the actual rate of return on investment is lower than the assumed one, the actual death benefit will decrease, but it is never lower than the scheduled lowest death benefit (see Miller, Walter N, 1971).

4. The Comparison Between Three Methods of Actual Death Benefit

Now there are three approaches to calculate the actual death benefit. The first two are under the condition that the premium changes in proportion to the death benefit and it is fixed respectively. The last one is named "pay off increasing amount insurance".

Consider a variable whole life insurance of unit premium, the insured is 20 years old when sign the insurance contract. Use *China Life Insurance Mortality Table (2000-2003), CL1.*

Suppose that the ratio of two adjacent periods' rate is a log-normal random variable, that is

$$\ln\left(\frac{1+i'_{k+1}}{1+i'_k}\right) \sim N(\mu,\sigma^2) .$$

Set $i'_0 = 0.05, \mu = 0, \sigma = 0.003$, then the confidence interval is $\mu \pm 3\sigma = \left(-0.009, 0.009\right)$.

Yet $\ln\frac{1+0.04}{1+0.05} \approx -0.009$, $\ln\frac{1+0.06}{1+0.05} \approx 0.009$, according to the principle of 3σ, we obtain

$$P(\ln\frac{1+0.04}{1+0.05} \le \ln\frac{1+i'}{1+0.05} \le \ln\frac{1+0.06}{1+0.05}) \approx 99.73\% ,$$

i.e. $P(0.04 \le i' \le 0.06) \approx 99.73\%$,

which means i' almost always falls into interval (0.04,0.06). Therefore, the assumption is reasonable.

Simulate 10000 times the three methods, and the results is shown in Table 1.

Table 1. *The mean and variance of each approach after simulating 10000 times.*

	1st method	2ed method	3rd method
μ	1.4970	1.8163	1.5353
σ^2	0.0231	0.0395	0.0062

We can see from Table 1 that the first approach has smallest mean, and it is relatively stable; the second approach has biggest mean, but it is the most unstable; yet last approach, which is based on the former approach, is the most stable even if its mean is lower than the previous one.

Appendices

Appendix one: China Life Insurance Mortality Table (2000-2003), CL1

China Life Insurance Mortality Table (2000-2003), CL1						
age	**mortality**	**Survival number**	**death toll**	**Life Expectancy**	**Survival person-year**	
x	q_x	l_x	d_x	$\dot{e}_x$	L_x	T_x
0	0.000722	1,000,000	722	76.7	999,639	76,712,704
1	0.000603	999,278	603	75.8	998,977	75,713,065
2	0.000499	998,675	498	74.8	998,426	74,714,088
3	0.000416	998,177	415	73.9	997,969	73,715,662
4	0.000358	997,762	357	72.9	997,583	72,717,692
5	0.000323	997,405	322	71.9	997,244	71,720,109
6	0.000309	997,082	308	70.9	996,928	70,722,865
7	0.000308	996,774	307	70.0	996,621	69,725,937
8	0.000311	996,467	310	69.0	996,312	68,729,316
9	0.000312	996,157	311	68.0	996,002	67,733,004
10	0.000312	995,847	311	67.0	995,691	66,737,001
11	0.000312	995,536	311	66.0	995,381	65,741,310
12	0.000313	995,225	312	65.1	995,070	64,745,929
13	0.000320	994,914	318	64.1	994,755	63,750,860
14	0.000336	994,595	334	63.1	994,428	62,756,105
15	0.000364	994,261	362	62.1	994,080	61,761,677
16	0.000404	993,899	402	61.1	993,699	60,767,596
17	0.000455	993,498	452	60.2	993,272	59,773,898
18	0.000513	993,046	509	59.2	992,791	58,780,626
19	0.000572	992,536	568	58.2	992,253	57,787,835
20	0.000621	991,969	616	57.3	991,661	56,795,582
21	0.000661	991,353	655	56.3	991,025	55,803,922
22	0.000692	990,697	686	55.3	990,355	54,812,897
23	0.000716	990,012	709	54.4	989,657	53,822,542
24	0.000738	989,303	730	53.4	988,938	52,832,885

China Life Insurance Mortality Table (2000-2003), CL1						
age	**mortality**	**Survival number**	**death toll**	**Life Expectancy**	**Survival person-year**	
x	q_x	l_x	d_x	$\dot{e}_x$	L_x	T_x
25	0.000759	988,573	750	52.4	988,198	51,843,947
26	0.000779	987,823	770	51.5	987,438	50,855,749
27	0.000795	987,053	785	50.5	986,661	49,868,311
28	0.000815	986,268	804	49.6	985,866	48,881,651
29	0.000842	985,464	830	48.6	985,050	47,895,784
30	0.000881	984,635	867	47.6	984,201	46,910,735
31	0.000932	983,767	917	46.7	983,309	45,926,534
32	0.000994	982,850	977	45.7	982,362	44,943,225
33	0.001055	981,873	1,036	44.8	981,356	43,960,863
34	0.001121	980,838	1,100	43.8	980,288	42,979,507
35	0.001194	979,738	1,170	42.9	979,153	41,999,220
36	0.001275	978,568	1,248	41.9	977,944	41,020,066
37	0.001367	977,321	1,336	41.0	976,653	40,042,122
38	0.001472	975,985	1,437	40.0	975,266	39,065,469
39	0.001589	974,548	1,549	39.1	973,774	38,090,203
40	0.001715	972,999	1,669	38.1	972,165	37,116,430
41	0.001845	971,331	1,792	37.2	970,435	36,144,265
42	0.001978	969,539	1,918	36.3	968,580	35,173,830
43	0.002113	967,621	2,045	35.3	966,599	34,205,250
44	0.002255	965,576	2,177	34.4	964,488	33,238,652
45	0.002413	963,399	2,325	33.5	962,237	32,274,164
46	0.002595	961,074	2,494	32.6	959,827	31,311,928
47	0.002805	958,580	2,689	31.7	957,236	30,352,100
48	0.003042	955,891	2,908	30.8	954,437	29,394,865
49	0.003299	952,984	3,144	29.8	951,412	28,440,427
50	0.003570	949,840	3,391	28.9	948,144	27,489,016
51	0.003847	946,449	3,641	28.0	944,628	26,540,871
52	0.004132	942,808	3,896	27.1	940,860	25,596,243
53	0.004434	938,912	4,163	26.3	936,830	24,655,383
54	0.004778	934,749	4,466	25.4	932,516	23,718,553
55	0.005203	930,283	4,840	24.5	927,863	22,786,037
56	0.005744	925,442	5,316	23.6	922,785	21,858,174
57	0.006427	920,127	5,914	22.8	917,170	20,935,390
58	0.007260	914,213	6,637	21.9	910,894	20,018,220
59	0.008229	907,576	7,468	21.1	903,842	19,107,326
60	0.009313	900,107	8,383	20.2	895,916	18,203,484
61	0.010490	891,725	9,354	19.4	887,048	17,307,568
62	0.011747	882,371	10,365	18.6	877,188	16,420,520
63	0.013091	872,005	11,415	17.8	866,298	15,543,332
64	0.014542	860,590	12,515	17.1	854,333	14,677,035
65	0.016134	848,075	13,683	16.3	841,234	13,822,702
66	0.017905	834,392	14,940	15.6	826,922	12,981,468
67	0.019886	819,453	16,296	14.8	811,305	12,154,546
68	0.022103	803,157	17,752	14.1	794,281	11,343,241
69	0.024571	785,405	19,298	13.4	775,756	10,548,960
70	0.027309	766,107	20,922	12.8	755,646	9,773,205
71	0.030340	745,185	22,609	12.1	733,881	9,017,559
72	0.033684	722,576	24,339	11.5	710,406	8,283,678
73	0.037371	698,237	26,094	10.8	685,190	7,573,272
74	0.041430	672,143	27,847	10.2	658,220	6,888,082
75	0.045902	644,296	29,574	9.7	629,509	6,229,863
76	0.050829	614,722	31,246	9.1	599,099	5,600,354
77	0.056262	583,476	32,828	8.6	567,062	5,001,255
78	0.062257	550,648	34,282	8.1	533,508	4,434,193
79	0.068871	516,367	35,563	7.6	498,585	3,900,685
80	0.076187	480,804	36,631	7.1	462,488	3,402,100
81	0.084224	444,173	37,410	6.6	425,468	2,939,611
82	0.093071	406,763	37,858	6.2	387,834	2,514,143
83	0.102800	368,905	37,923	5.8	349,943	2,126,309
84	0.113489	330,982	37,563	5.4	312,200	1,776,366
85	0.125221	293,419	36,742	5.0	275,048	1,464,166
86	0.138080	256,677	35,442	4.6	238,956	1,189,118
87	0.152157	221,235	33,662	4.3	204,404	950,162

China Life Insurance Mortality Table (2000-2003), CL1						
age	**mortality**	**Survival number**	**death toll**	**Life Expectancy**	**Survival person-year**	
x	q_x	l_x	d_x	$\dot{e}_x$	L_x	T_x
88	0.167543	187,572	31,426	4.0	171,859	745,759
89	0.184333	156,146	28,783	3.7	141,754	573,899
90	0.202621	127,363	25,806	3.4	114,460	432,145
91	0.222500	101,557	22,596	3.1	90,258	317,685
92	0.244059	78,960	19,271	2.9	69,325	227,427
93	0.267383	59,689	15,960	2.6	51,709	158,102
94	0.292544	43,729	12,793	2.4	37,333	106,392
95	0.319604	30,937	9,887	2.2	25,993	69,059
96	0.348606	21,049	7,338	2.0	17,380	43,067
97	0.379572	13,711	5,204	1.9	11,109	25,686
98	0.412495	8,507	3,509	1.7	6,752	14,577
99	0.447334	4,998	2,236	1.6	3,880	7,825
100	0.484010	2,762	1,337	1.4	2,094	3,945
101	0.522397	1,425	745	1.3	1,053	1,851
102	0.562317	681	383	1.2	489	798
103	0.603539	298	180	1.0	208	309
104	0.645770	118	76	0.9	80	101
105	1.000000	42	42	0.5	21	21

Appendix two: The project used in the paper

```
function result=baoe(x,i,M,SI,n)
%b is the matrix of the death benefit, i is the expected rate, M and SI are actual rate and variance

%read the data
W=xlsread('D:\survival.xlsx');
p=W(:,3);
A=xlsread('D:\A.xls');

%simulating the n insureds
u=rand(n,1);
X=zeros(n,1);
for j=1:n
    k=0;
    m=p(x+k,1);
    while u(j,1)<m
        k=k+1;
        m=m*p(x+k,1);
    end
    X(j,1)=k;
end

%actual rate
Q=normrnd(M,SI,max(X),1);
Q=exp(Q);
I=zeros(max(X),1);

%I is the vector of actual rates
I(1,1)=0.04;
for j=1:max(X)-1
    I(j+1,1)=Q(j+1,1)*(I(j,1)+1)-1;
end

%generate the present value of n-year fixed life insurance
A1=zeros(max(X),1);
q=1-p;
for k=1:max(X)
    A1(k,1)=q(x+k-1,1)/(1+i);
end

%generate net premiums P
P=zeros(max(X),1);
for j=1:max(X)
    P(j,1)=i*A(j)/((1+i)* (1-A(j)));
end

%generate liability reserve V
V=zeros(max(X)-1,1);
for j=1:max(X)-1
    V(j,1)=A(j+1,1)*(1-(P(1,1)/P(j+1,1)))+0.002;
end
V=[0;V];

%b1  change in proportion
b1=ones(n,1);
for j=1:n
    b11=1;
    for k=1:X(j,1)-1
        b11=b11*(1+I(k,1))/(1+i);
    end
    b1(j,1)=b11;
end

%b2  fixed premium
b2=ones(n,1);
for j=1:n
    b21=1;
    for k=1:X(j,1)-1
b21=b21*(V(k,1)+(P(1,1)/b21-A1(k,1)))*(1+I(k,1))/((1+i)*V(k,1)+P(1,1)-A1(k,1));
    end
    b2(j,1)=b21;
```

```
end

%b3   pay off increasing amount insurance
b3=ones(n,1);
for j=1:n
    b31=1;
    for k=1:X(j,1)-1
b31=(b31-(P(1,1)/P(k+1,1)))*(1+I(k,1))/(1+i)+(P(1,1)/P(k+1,1));
    end
    b3(j,1)=b31;
end

b=[b1,b2,b3];
m=mean(b);
v=var(b);
result=[m;v];
```

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