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Performance Evaluation of Empirical Rain Rate Models for Computing Rain Attenuation

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Abstract: When computing rain attenuation with the International Telecommunication Union (ITU) model the rain rate is normally needed. However, only limited rain rate data are published by ITU for the various regions all over the globe. In order to obtain the missing rain rate data for ITU rain attenuation model, two empirical models used to generate the rain rate data for the fifteen ITU rain zones are presented in this paper. The performance of the two empirical models (referred to as Model 1 and Model 2) is also presented. Specifically, two ITU rain zones (N and P) are considered and Model 1 is used for estimating the rain rate in rain zone P, whereas, Model 2 is used for estimating the rain rate in rain zone N. The rain attenuation computed based on the ITU rain rate generated from the empirical models and the rain attenuation computed from the actual ITU rain rata data are compared in terms of Root Mean Square Error (RMSE), coefficient of determination (r^2) and prediction accuracy. According to the results, in respect of rain rate prediction for rain zone N, Model 2 has RMSE (3.491375), Coefficients of Determination (r^2= 0.99816) and Prediction Accuracy (92.49319835%). Also, for rain zone N, with respect to the rain attenuation, Model 2 had RMSE of 0.792219, the Coefficients of Determination (r^2) of 0.99855 and the Prediction Accuracy of 90.51054739%. Similarly, in respect of rain rate prediction for rain zone P, Model 1 has RMSE (4.924732), Coefficients of Determination (r^2= 0.99798) and Prediction Accuracy (92.29421163%). In addition, with respect to the rain attenuation for rain zone P, Model 1 has RMSE is 1.276727, the Coefficients of Determination (r^2) is 0.99812 and the Prediction Accuracy 90.31393492%. In all, the two prediction accuracies of the two empirical models are adjudged very good in predicting the rain rates. As such, the rain rates predicted by those models are suitable for computing the rain attenuations for the ITU rain zones.

Keywords: Rain Rate, Rain Attenuation, Empirical Models, ITU Rain Zone, RMSE, Coefficients of Determination, Prediction Accuracy

1. Introduction

According to experts, rain is the dominant propagation phenomena on wireless links with frequencies above 10 GHz [1]. The impairment of signal due to rain depends, among other things, on rain rate, size of raindrops and rain drop density [1-4]. The International Telecommunication Union–Radiocommunication Sector (ITU-R) model was adjudged the most widely accepted internationally for the prediction of the effect of rain on communication systems [5-8]. Generally, specific attenuation, γR(dB/km) is obtained from the rain rate R (mm/hr) using the power law relationship [4-7, 9-13]:

$$\gamma_R = kR^{\alpha} \qquad (1)$$

where α and k are coefficients that are dependent on polarization and frequency. Furthermore, the rain attenuation in dB is given as [13];

$$A_R = (\gamma_R)d \qquad (2)$$

where, d, is the effective path length; and γR, the specific attenuation, where, k and α, are frequency dependent constants.

According to *ITU*-R PN.837-1 recommendation, the world is split into 15 regions according to rain rates [14-15]. The rain rate data is normally given for a particular percentage of time exceeded. The data on the rain rate for different rain zones are published by ITU are for only seven different percentage of time exceeded. As such, for rain attenuation analysis requiring the rain rate for other percentage of time exceeded, models are developed to estimate the rain rates. The models are developed based on the available ITU rain rate data for different rain zones and percentage of time exceeded. The focus in this paper is to evaluate the effectiveness of using such model generated rain rate data in the computation of rain attenuation for the ITU rain zones. Specifically, two of such empirical models are considered in this paper and their application in the prediction of rain rate for two (2) ITU rain zones, namely, rain zone N and P are studied.

The earlier work in [17] used 15 different trendline model, one for each of the 15 ITU rain zones. The trendline models in [17] are more complex than the simplified two models presented in this paper. Also, the authors in [17] did not show the application of the rain rate predicted by such trendline models in the rain attenuation computation and how the computed rain attenuations differ from the rain attenuation computed from the actual ITU rain rate data.

Consequently, in this paper, first, the empirical rain rate models are used to estimate the rain rate for the rain zones and then the ITU rain attenuation model is used to compute the rain attenuation for the rain zones. Furthermore, the prediction efficiency of the models with respect to the rain rate and the rain attenuation are quantified in terms of Root Mean Square Error (RMSE), coefficient of determination (r^2) and the prediction accuracy.

2. Methodology

a The ITU Rain Fade Model

For frequencies under 40 GHz and path lengths shorter than 60 km specific attenuation originating from rainfall is defined by $\gamma_{R_{po}}$ in dB/km and modeled using the power-law equation as follows [9, 4]:

$$\gamma_{R_{po}} = max\{(k_v(R_{po})^{\alpha_v}), (k_h(R_{po})^{\alpha_h})\} \quad (3)$$

where R_{po} is the rain rate exceeded for po%. k_v, α_v, k_h and α_h are frequency dependent coefficients for vertical and horizontal polarization situations respectively [10, 12].

The rainfall rate, R_{po} has different values for each climate zone, also referred to as rain zone and po is the Percentage outage time (or Percentage unavailability time) of the link. [10]. Limited values of R_{po} for the fifteen (15) ITU rain zones are provided in [4, 11]. However, in this paper, two empirical models are used to generate the values of R_{po} for any given value of Po. The two empirical models are;

$$\text{MODEL 1; } Y = aX^b e^{cX} \quad (4)$$

$$\text{MODEL 2; } Y = \frac{1}{aX^b + c} \quad (5)$$

where Y is the rain rate in mm/h, X (in %) is the percentage of time the rain rate is exceeded, and the values of the model coefficient (a, b, c) and constant (Kopt) are given in Table 1 and Table 2 for all the fifteen (15) ITU rain zones.

Model 1 is used for computing the rain rate for rain zone A, C, D, E, G, J, P and Q. On the other hand, Model 2 is used for computing the rain rate in rain zones B, F, H, K, L, M and N.

Table 1. *Values For The Empirical Constants A, B and C and The Model Optimization Constant (Kopt) For Model 1 and The ITU Rain Zones Model 1 Applies To.*

MODEL 1				
	A	b	c	Kopt
A	1.261	-0.4148	-3.887	0.093228
C	2.387	-0.4148	-3.887	0.749839
D	3.976	-0.3416	-0.7553	0.161511
E	2.309	-0.4945	-1.877	0.234747
G	6.264	-0.3392	-0.8824	0.318893
J	13.71	-0.2027	-0.6551	0.762792
P	49.09	-0.2379	-2.091	3.741203
Q	53.91	-0.1668	-0.8625	1.118311

Table 2. *Values For The Empirical Constants A, B and C and The Model Optimization Constant (Kopt) For Model 2 and The ITU Rain Zones Model 2 Applies To.*

MODEL 2				
	A	b	C	Kopt
B	1.392	0.6512	0.01577	-0.25442
F	0.6567	0.6953	0.007375	0.169725
H	0.4108	0.601	0.005593	-0.48374
K	0.397	0.6739	0.006244	-0.76367
L	0.3157	0.7038	0.004226	-0.52586
M	0.1787	0.6364	0.006129	-0.74685
N	0.1083	0.6198	0.004093	-2.72616

If po is the Percentage outage time (or Percentage unavailability time) of the link and pa is the Percentage availability time of the link. Then

$$po = (100\% - pa) \quad (6)$$

Fundamentally, rain attenuation, $A_{R_{po}}$(dB) is the product of specific rain attenuation, $\gamma_{R_{po}}$ in dB/km and the propagation path length, d (km) between the transmitter and the receiver.

$$A_{R_{po}} = \left(\gamma_{R_{po}}\right) d \text{ (dB)} \quad (7)$$

b Goodness of Fit Measures

The Coefficients of Determination (r^2): If n is the number of data items and we denote y_i as the observed values of the dependent variable, $\bar{y}$ as its mean, and $\hat{y}_i$ as the fitted value, then the coefficient of determination is:

$$r^2 = \frac{\sum_{i=1}^{i=n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{i=n}(y_i - \bar{y})^2} \quad (8)$$

Prediction Accuracy: The prediction accuracy (PA in %) is calculated as follows:

$$PA = \left\{1 - \frac{1}{n}\left(\sum_{i=1}^{i=n}\left|\frac{|Y_{(measured)(i)} - Y_{(predicted)(i)}|}{PL_{(measured)(i)}}\right|\right)\right\} * 100\% \quad (9)$$

Root Mean Square Error (RMSE): The RMSE is calculated as follows:

$$\text{RMSE} = \sqrt[2]{\left\{\frac{1}{n}\left[\sum_{i=1}^{i=n}\left|Y_{(Atual)(i)} - Y_{(Predicted)(i)}\right|^2\right]\right\}} \quad (10)$$

where $Y_{(Atual)(i)}$ is the actual rain rate given by ITU and $Y_{(Predicted)(i)}$ is model predicted rain rate.

c The Algorithm For Comparative Analysis of The Rain Attenuation Computed Based On The Actual ITU Rain Rate Data and The Empirical Model Generated ITU Rain Rate Data

1) Input the operating frequency, (f) in GHz
2) Input The Path Length (d) in Km
3) Input the frequency constants for rain attenuation computation; (k_v, α_v, k_h and α_h)
4) Input rain zone (RZ)
5) Select model (MDL) based on Rain Zone
6) Input model coefficients (a, b, c) and the model constant (Kopt)
7) Let Po(1) = 1; Po(2) = 0.3; Po(3) = 0.01; Po(4) = 0.03; Po(5) = 0.00 1; Po(6) = 0.003; Po(7) = 0.0001;
8) For i = 1 to 7 Step 1
9) Input Actual Rain Rate Data (ARRT(i)) for rain zone RZ and percentage outage of Po(i)
10) Compute rain attenuation AAR(i) with ARRT(i)
11) Use Model MDL to compute Model Estimated Rain Rate Data (MRRT(i)) for rain zone (RZ) and percentage outage of Po(i)
12) Compute rain attenuation MAR(i) with MRRT(i) and the model for the given rain zone
13) Next i
14) Prepare table for Po(i), AAR(i) and MAR(i) where i = 1, 2, 3,…, 7
15) Compute Root Mean Square Error (RMSE)
16) Compute Coefficient of Determination (r^2)
17) Compute Prediction Accuracy
18) Plot Graph of AAR(i) and MAR(i) versus Po(i)
19) End

d Sub-headlines Italic

e Sub-headlines Italic

3. Results and Discussion

The effectiveness of the empirical models in estimating the ITU rain data and hence in the computation of rain attenuation is demonstrated through sample computations of rain attenuations for a 10 GHz microwave link with a path length of 5 Km. The microwave link is considered is two ITU rain zones which are found in Nigeria, namely, rain zone N and P. The data for the rain attenuation computation are given in Table 3.

Table 3. *The Input Data For The Rain Attenuation Computation For Rain Zones N and P.*

kh	Ah	Kv	Av	Frequency (f) in GHz	Path Length (d) in Km
0.01217	1.2571	0.01129	1.2156	10	5

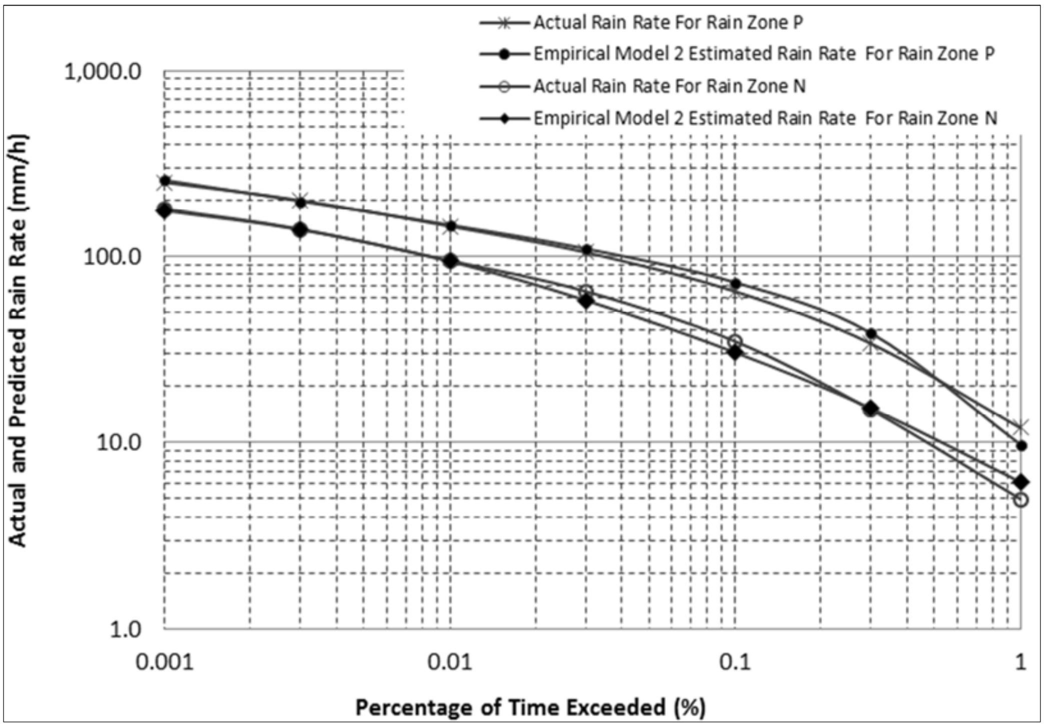

Figure 1. *Actual and Predicted Rain Rate (mm/h) Versus Percentage of Time Exceeded For ITU Rain Zones N and P.*

The actual ITU rain rate for the rain zones N and P are given in Table 4 and Figure 1 along with the rain rates estimated by the empirical models. Specifically, Model 1 is used for estimating the rain rate in rain zone P, whereas, Model 2 is used for estimating the rain rate in rain zone N. The graph of the actual and predicted rain rate is shown in Figure 1 while the goodness of fit of the models' prediction is given in Table 6. For rain zone N, Model 2 has RMSE (3.491375), Coefficients of Determination (r^2 = 0.99816) and Prediction Accuracy (92.49319835%). With coefficients of determination(r^2) being over 99% and also Prediction Accuracy over 92 %, it is obvious that the empirical model 2 can effectively estimate the rain rate for rain zone N.

Table 4. *Actual Rain Rate and The Model Predicted Rain Rate For Rain Zone P.*

Po(i)	Actual Rain Rate (mm/h) For Rain Zone P	Empirical Model 2 Estimated Rain Rate (mm/h) For Rain Zone P	Actual Rain Rate (mm/h) For Rain Zone N	Empirical Model 2 Estimated Rain Rate (mm/h) For Rain Zone N
1	12	9.806936	5	6.166002
0.3	34	38.65145	15	15.29986
0.1	65	72.61945	35	30.49596
0.03	105	109.9212	65	58.15793
0.01	145	147.5257	95	94.0347
0.003	200	198.0362	140	139.0651
0.001	250	257.1285	180	176.1306

Similarly, in Table 6, for rain zone P, Model 1 has RMSE (4.924732), Coefficients of Determination (r^2 = 0.99798) and Prediction Accuracy (92.29421163%). With coefficients of determination (r^2) being over 99% and also Prediction Accuracy over 92%, it is obvious that the empirical model 1 can effectively estimate the rain rate for rain zone P.

The rain attenuation computed from the actual ITU rain rate and the model predicted ITU rain rate for rain zone N and P are given in Table 4 and Figure 2. The graph of the actual and predicted rain attenuations are shown in Figure 2 while the goodness of fit of the models' prediction is given in Table 6. For rain zone N, Model 2, with respect to the rain attenuation, the RMSE is 0.792219, the Coefficients of Determination (r^2) is 0.99855 and the Prediction Accuracy 90.51054739%. With coefficients of determination (r^2) being over 99% and also Prediction Accuracy over 90 %, it is obvious that the empirical model 2 can effectively estimate the rain rate for computing the rain attenuation in rain zone N.

Table 5. *Rain Attenuation Computed From Actual Rain Rate and The Model Predicted Rain Rate For Rain Zone N and P.*

Po(i)	Rain Attenuation Based on Actual Rain Rate For Rain Zone P	Rain Attenuation Based on Empirical Model 1 Estimated Rain Rate For Rain Zone P	Rain Attenuation Based on Actual Rain Rate For Rain Zone N	Rain Attenuation Based on Empirical Model 2 Estimated Rain Rate For Rain Zone N
1	1.383245694	1.073290155	0.4602	0.5989
0.3	5.122508989	6.018478293	1.8312	1.8773
0.1	11.56841775	13.29812016	5.3126	4.4679
0.03	21.13964308	22.39257756	11.5684	10.0589
0.01	31.71876711	32.41485984	18.6405	18.4027
0.003	47.5209802	46.93514885	30.35	30.0954
0.001	62.9087372	65.17190556	41.6259	40.5042

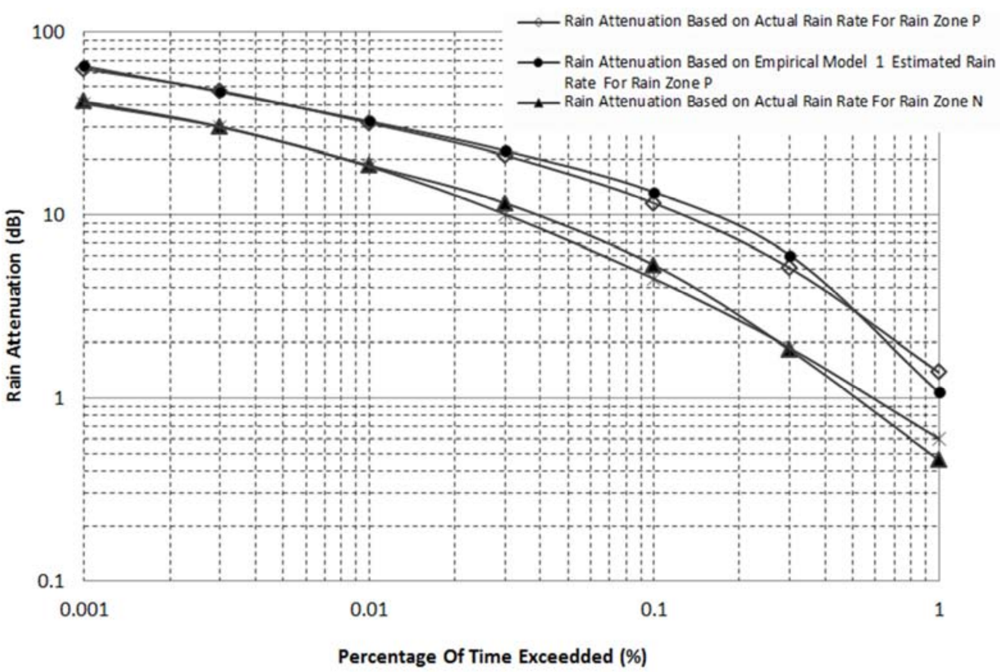

Figure 2. *Rain Attenuation computed from the Actual Rain Rate and the Model Predicted rain Rate For Rain Zone P and N versus Percentage of Time Exceeded.*

Table 6. *RMSE, Coefficients Of Determination (r^2) and Prediction Accuracy (%) Of The Models For The Two Rain Zones Considered.*

	RMSE	Coefficients of Determination (r-Squared):	Prediction Accuracy (%)
Rain Rate For Rain Zone N	3.491375	0.99816	92.49319835
Rain Attenuation For Rain Zone N	0.792219	0.99855	90.51054739
Rain Rate For Rain Zone P	4.924732	0.99798	92.29421163
Rain Attenuation For Rain Zone P	1.276727	0.99812	90.31393492

Similarly, in Table 6, for rain zone P, Model 1, with respect to the rain attenuation, the RMSE is 1.276727, the Coefficients of Determination (r^2) is 0.99812and the Prediction Accuracy 90.31393492%. With coefficients of determination (r^2) being over 99% and also Prediction Accuracy over 90 %, it is obvious that the empirical model 1 can effectively estimate the rain rate for computing the rain attenuation in rain zone P.

4. Conclusions

The prediction of rain rate for ITU rain zones using empirical models and the computation of rain attenuation based on the predicted rain rate is presented in this paper. The goodness of fit of the prediction models with respect to the actual or expected rain rates and rain attenuation are determined. The empirical models have very high prediction accuracies in all the cases and are hence recommended for used in the estimation of rain rate for the various ITU rain zones.

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