Lagrangian Dynamical Systems with Three Para-complex Structures

Ibrahim Yousif Ibrahim Abad Alrhman

Departmentof Mathematics, Faculty of Education, West Kordufan University, Alnhoud City, Sudan

Email address:
iyibrahimi@gmail.com

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Abstract: This paper aims to present Lagrangian Dynamical systems formalism for mechanical systems using Three Para-Complex Structures, which represent an interesting multidisciplinary field of research. As a result of this study, partial differential equations will be obtained for movement of objects in space and solutions of these equations. In this study, some geometrical, relativistic, mechanical, and physical results related to Three Para- Complex Structures mechanical systems broad applications in mathematical physics, geometrical optics, classical mechanics, analytical mechanics, mechanical systems, thermodynamics, geometric quantization and applied mathematics such as control theory.

Keywords: Differential Geometry, Para-complex Structure, Lagrangian Dynamics

1. Introduction

The geometric study of dynamical systems is an important chapter of contemporary mathematics due to its applications in Mechanics, Theoretical Physics. If M is a differentiable manifold that corresponds to the configuration space, a dynamical system can be locally given by a system of ordinary differential equations of the form $\dot{x}^i = f^i(t; x)$, which are called equations of evolution. Globally, a dynamical system is givenby a vector field X on the manifold $M \times R$ whose integral curves, $c(t)$ aregiven by the equations of evolution, $X \circ c(t) = \dot{c}(t)$. The theory of dynamicalsystems dealwith the integration of such systems.

Mehmet and Murat Sari obtained On Para –Euler Lagrange and Para Hamiltonian Equations and constrained para complex Mechanical Equations [1].

Tekkoyun submitted paracomplex analogue of the Euler-Lagrange equations was obtained in the framework of para-Kahlerian manifold and the geometric results on a paracomplex mechanical systems were found [2].

Kasap and Tekkoyun obtained Lagrangian and Hamiltonian formalism for mechanical systems using para/ pseudo-Kahler manifolds, representing an interesting multidisciplinary. field of research. Also, the geometrical, relativistical, mechanical and physical results related to para/ pseudo-Kahler mechanical systems were given, too [3].

Oguzhan and Kasap submitted Mechanical Equations with Two Almost Complex Structures on Symplectic Geometry, using two complex structures, examined mechanical systems on Symplectic geometry [4].

In this paper, we study dynamical systems with Three Almost para Complex Structures. After Introduction in Section 1, we consider Historical Background paper basic. Section 2 deals with the study paracomplex Structures. Section 3 is devoted to study Lagrangian Dynamics.

2. Preliminaries

In this preliminary chapter, we recall basic definitions, results and formulas which we shall use in the subsequent chapters of the paper. Most of material included in this chapter occurs in standard literatures namely.

Definition 2.1. [6]
An almost product structure J on a tangent bundle $T\mathcal{M}$ of m-real dimensional configurationmanifold $\mathcal{M}$ is a (1, 1) tensor field J on $T\mathcal{M}$ such that $I^2 = I$. Here, the pair $(T\mathcal{M}, J)$ is calledan almost product manifold.

Definition 2.2. [8]
An almost Para -complex structure on $\mathcal{M}$ amanifold is a differentiable map $I: T\mathcal{M} \to T\mathcal{M}$ on the tangent bundle $T\mathcal{M}$ of $\mathcal{M}$ such that I preserves each fiber, A manifold with affixed almost para -complex structure is called an almost

para -complex manifold.

Theorem 2.3. [10]

Suppose that $\{x_1, x_2, x_3, x_4, x_5, x_6\}$, be a real coordinate system on $(\mathcal{M}, J)$. Then we denote by.

$$\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}\right\}$$

$$\{dx_1, dx_2, dx_3, dx_4, dx_5, dx_6\}$$

$$J\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2} = j\left(\frac{\partial}{\partial x_1}\right), J\left(\frac{\partial}{\partial x_2}\right) = \frac{\partial}{\partial x_1} = j\left(\frac{\partial}{\partial x_2}\right)$$

$$J\left(\frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_4} = j\left(\frac{\partial}{\partial x_3}\right), J\left(\frac{\partial}{\partial x_4}\right) = \frac{\partial}{\partial x_3} = j\left(\frac{\partial}{\partial x_4}\right)$$

$$J\left(\frac{\partial}{\partial x_5}\right) = \frac{\partial}{\partial x_6} = j\left(\frac{\partial}{\partial x_5}\right), J\left(\frac{\partial}{\partial x_6}\right) = \frac{\partial}{\partial x_5} = j\left(\frac{\partial}{\partial x_6}\right) \quad (1)$$

Definition 2.4.

Let $Z_1 = x_1 + ix_2, Z_2 = x_3 + ix_4, Z_3 = x_5 + ix_6, i^2 = -1$, be Para complex manifold. In local coordinates system on a neighborhood V of $T\mathcal{M}$. We define the vector fields by.

$$I\left(\frac{\partial}{\partial Z_1}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right), I\left(\frac{\partial}{\partial \bar{Z}_1}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)$$

$$I\left(\frac{\partial}{\partial Z_2}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x_3} - i\frac{\partial}{\partial x_4}\right), I\left(\frac{\partial}{\partial \bar{Z}_2}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x_3} + i\frac{\partial}{\partial x_4}\right)$$

$$I\left(\frac{\partial}{\partial Z_3}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x_5} - i\frac{\partial}{\partial x_6}\right), I\left(\frac{\partial}{\partial \bar{Z}_3}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x_5} + i\frac{\partial}{\partial x_6}\right)$$

And the dual convector fields

$$I(dZ_1) = \frac{1}{2}(dx_1 - idx_2), I(d\bar{Z}_1) = \frac{1}{2}(dx_1 - idx_2)$$

$$I(dZ_2) = \frac{1}{2}(dx_3 - idx_4), I(d\bar{Z}_2) = \frac{1}{2}(dx_3 - idx_4)$$

$$I(dZ_3) = \frac{1}{2}(dx_5 - idx_6), I(d\bar{Z}_1) = \frac{1}{2}(dx_5 - idx_6)$$

Theorem 2.5.

Suppose that $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a real coordinate system on $(\mathcal{M}, J)$. Then we denote by.

$$\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}\right\}$$

$$\{dx_1, dx_2, dx_3, dx_4, dx_5, dx_6\}$$

$$I\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2}, I\left(\frac{\partial}{\partial x_2}\right) = \frac{\partial}{\partial x_1}$$

$$I\left(\frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_4}, I\left(\frac{\partial}{\partial x_4}\right) = \frac{\partial}{\partial x_3}$$

$$I\left(\frac{\partial}{\partial x_5}\right) = \frac{\partial}{\partial x_6}, I\left(\frac{\partial}{\partial x_6}\right) = \frac{\partial}{\partial x_5} \quad (2)$$

If I is defined as a Para complex manifold $\mathcal{M}$ then $I^2 = I \circ I = 1$

Proof.

$$I^2\left(\frac{\partial}{\partial x_1}\right) = I\left(I\left(\frac{\partial}{\partial x_1}\right)\right) = I\left(\frac{\partial}{\partial x_2}\right) = \frac{\partial}{\partial x_1}$$

$$I^2\left(\frac{\partial}{\partial x_2}\right) = I\left(I\left(\frac{\partial}{\partial x_2}\right)\right) = I\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2}$$

$$I^2\left(\frac{\partial}{\partial x_3}\right) = I\left(I\left(\frac{\partial}{\partial x_3}\right)\right) = I\left(\frac{\partial}{\partial x_4}\right) = \frac{\partial}{\partial x_3}$$

$$I^2\left(\frac{\partial}{\partial x_4}\right) = I\left(I\left(\frac{\partial}{\partial x_4}\right)\right) = I\left(\frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_4}$$

$$I^2\left(\frac{\partial}{\partial x_5}\right) = I\left(I\left(\frac{\partial}{\partial x_5}\right)\right) = I\left(\frac{\partial}{\partial x_6}\right) = \frac{\partial}{\partial x_5}$$

$$I^2\left(\frac{\partial}{\partial x_6}\right) = I\left(I\left(\frac{\partial}{\partial x_6}\right)\right) = I\left(\frac{\partial}{\partial x_5}\right) = \frac{\partial}{\partial x_6}$$

3. Lagrangian Dynamical Systems

In this section we introduce the concept of Lagrangian Dynamical Systems. We start by the following definition.

Definition 3.1. [5]

A Lagrangian function vector field X on $\mathcal{M}$ is a smooth function L : $T\mathcal{M} \rightarrow$ R such that.

$$i_X\phi_L = dE_L \quad (3)$$

Let ξ be the vector field by.

$$\xi = X_1 \frac{\partial}{\partial x_1} + X_2 \frac{\partial}{\partial x_2} + X_3 \frac{\partial}{\partial x_3} + X_4 \frac{\partial}{\partial x_4} + X_5 \frac{\partial}{\partial x_5} + X_6 \frac{\partial}{\partial x_6} \quad (4)$$

And.

$$X_1 = \dot{x}_1, X_2 = \dot{x}_2, X_3 = \dot{x}_3, X_4 = \dot{x}_4, X_5 = \dot{x}_5, X_6 = \dot{x}_6$$

$$J(\xi) = J\left(X_1 \frac{\partial}{\partial x_1} + X_2 \frac{\partial}{\partial x_2} + X_3 \frac{\partial}{\partial x_3} + X_4 \frac{\partial}{\partial x_4} + X_5 \frac{\partial}{\partial x_5} + X_6 \frac{\partial}{\partial x_6}\right) \quad (5)$$

$$J(\xi) = X_1 J\left(\frac{\partial}{\partial x_1}\right) + X_2 J\left(\frac{\partial}{\partial x_2}\right) + X_3 J\left(\frac{\partial}{\partial x_3}\right) + X_4 J\left(\frac{\partial}{\partial x_4}\right) + X_5 J\left(\frac{\partial}{\partial x_5}\right) + X_6 J\left(\frac{\partial}{\partial x_6}\right)$$

$$U = J(\xi) = -X_1 j \frac{\partial}{\partial x_1} + X_2 j \frac{\partial}{\partial x_2} - X_3 j \frac{\partial}{\partial x_3} + X_4 j \frac{\partial}{\partial x_4} - X_5 j \frac{\partial}{\partial x_5} + X_6 j \frac{\partial}{\partial x_6}$$

Let that Liouville Vector field on complex manifold $(\mathcal{M}, U)$.

Definition 3.2. [11]
Kinetic energy given $T: T\mathcal{M} \rightarrow \mathcal{M}$.

$$T = \frac{1}{2} m_i (\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{x}_4^2 + \dot{x}_5^2 + \dot{x}_6^2)$$

Potential energy $P: T\mathcal{M} \rightarrow \mathcal{M}$.

$$P = m_i g h$$

Definition 3.3. [14]
The Lagrangian function (energy function)

$$L = T - P$$

$$E_L^J = U_{G_1}(L) - L$$

Definition 3.4. [2]
Is vertical derivation (differentiation) d_J is defined

$$d_{G_J} = \left[i_{G_J}, d \right] = i_{G_J} d - d i_J$$

$\phi_L = d d_1 L$ such that.

$$d_J = \frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6 \tag{6}$$

Defined by operator $d_J: A(\mathcal{M}) \rightarrow \Lambda^1 \mathcal{M}$

$$d_J L = \left(\frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6 \right) L$$

$$d_J L = -\boldsymbol{j} \frac{\partial L}{\partial x_2} dx_1 + \boldsymbol{j} \frac{\partial L}{\partial x_1} dx_2 - \boldsymbol{j} \frac{\partial L}{\partial x_4} dx_3 + \boldsymbol{j} \frac{\partial L}{\partial x_3} dx_4 - \boldsymbol{j} \frac{\partial L}{\partial x_6} dx_5 + \boldsymbol{j} \frac{\partial L}{\partial x_5} dx_6 \tag{7}$$

That.

$$\phi_L = -d(d_{G_1}) = -d \left(-\boldsymbol{j} \frac{\partial L}{\partial x_2} dx_1 + \boldsymbol{j} \frac{\partial L}{\partial x_1} dx_2 - \boldsymbol{j} \frac{\partial L}{\partial x_4} dx_3 + \boldsymbol{j} \frac{\partial L}{\partial x_3} dx_4 - \boldsymbol{j} \frac{\partial L}{\partial x_6} dx_5 + \boldsymbol{j} \frac{\partial L}{\partial x_5} dx_6 \right)$$

$$\phi_L = \boldsymbol{j} \frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 \wedge dx_1 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 \wedge dx_2 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 \wedge dx_3 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 \wedge dx_4 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 \wedge dx_5$$

$$- \boldsymbol{j} \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 \wedge dx_6 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 \wedge dx_1 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 \wedge dx_2 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 \wedge dx_3 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2$$

$$\wedge dx_4 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 \wedge dx_5 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 \wedge dx_6 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 \wedge dx_1 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 \wedge dx_2$$

$$+ \boldsymbol{j} \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 \wedge dx_3 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 \wedge dx_4 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 \wedge dx_5 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 \wedge dx_6 + \boldsymbol{j} \boldsymbol{v} \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4$$

$$\wedge dx_1 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 \wedge dx_2 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 \wedge dx_3 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 \wedge dx_4 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 \wedge dx_5$$

$$- \boldsymbol{j} \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 \wedge dx_6 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 \wedge dx_1 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 \wedge dx_2 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 \wedge dx_3 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5$$

$$\wedge dx_4 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 \wedge dx_5 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 \wedge dx_6 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 \wedge dx_1 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 \wedge dx_2$$

$$+ \boldsymbol{j} \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 \wedge dx_3 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 \wedge dx_4 + \boldsymbol{j} \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 \wedge dx_5 - \boldsymbol{j} \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 \wedge dx_6$$

Calculate $\phi_L(\xi)$.

$$i_X \phi_L = \phi_L(\xi) = \left(j \frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 \wedge dx_1 - j \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 \wedge dx_2 + j \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 \wedge dx_3 - j \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 \wedge dx_4 + j \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 \wedge dx_5 - \right.$$

$$j \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 \wedge dx_6 + j \frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 \wedge dx_1 - j \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 \wedge dx_2 + j \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 \wedge dx_3 - j \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2 \wedge dx_4 + j \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 \wedge$$

$$dx_5 - j \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 \wedge dx_6 + j \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 \wedge dx_1 - j \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 \wedge dx_2 + j \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 \wedge dx_3 - j \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 \wedge dx_4 +$$

$$j \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 \wedge dx_5 - j \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 \wedge dx_6 + j \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 \wedge dx_1 - j \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 \wedge dx_2 + j \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 \wedge dx_3 - j \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 \wedge$$

$$dx_4 + j \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 \wedge dx_5 - j \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 \wedge dx_6 + j \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 \wedge dx_1 - j \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 \wedge dx_2 + j \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 \wedge dx_3 + \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 \wedge$$

$$dx_4 - \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 \wedge dx_5 + \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 \wedge dx_6 - \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 \wedge dx_1 + \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 \wedge dx_2 + j \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 \wedge dx_3 - j \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 \wedge$$

$$\left. dx_4 + j \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 \wedge dx_5 - j \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 \wedge dx_6 \right) \left(X^1 \frac{\partial}{\partial x_1} + X^2 \frac{\partial}{\partial x_2} + X^3 \frac{\partial}{\partial x_3} + X^4 \frac{\partial}{\partial x_4} + X^5 \frac{\partial}{\partial x_5} + X^6 \frac{\partial}{\partial x_6} \right) \quad (7)$$

From the energy equation we get.

$$E_L = V(L) - L = X^1 \frac{\partial L}{\partial x_2} - X^2 \frac{\partial L}{\partial x_1} + X^3 \frac{\partial L}{\partial x_4} - X^4 \frac{\partial L}{\partial x_3} + X^5 \frac{\partial L}{\partial x_6} - X^6 \frac{\partial L}{\partial x_5} - L \qquad (8)$$

In the equation of the energy equation we obtain.

$$dE_L = \left(-j \frac{\partial}{\partial x_2} dx_1 + j \frac{\partial}{\partial x_1} dx_2 - j \frac{\partial}{\partial x_4} dx_3 + j \frac{\partial}{\partial x_3} dx_4 - j \frac{\partial}{\partial x_6} dx_5 + j \frac{\partial}{\partial x_5} dx_6 \right) \left(X^1 \frac{\partial L}{\partial x_2} - X^2 \frac{\partial L}{\partial x_1} + X^3 \frac{\partial L}{\partial x_4} - X^4 \frac{\partial L}{\partial x_3} \right.$$

$$\left. + X^5 \frac{\partial L}{\partial x_6} - X^6 \frac{\partial L}{\partial x_5} - L \right)$$

$$dE_L = -jX^1 \frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 - jX^1 \frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 - jX^1 \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 - jX^1 \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 - jX^1 \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 - X^1 \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6$$

$$+jX^2 \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 + jX^2 \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 + jX^2 \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 + jX^2 \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 + jX^2 \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 + jX^2 \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6$$

$$-jX^3 \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 - jX^3 \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 - jX^3 \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 - jX^3 \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 - jX^3 \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 - jX^3 \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6$$

$$+jX^4 \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 + jX^4 \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2 + jX^4 \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 + jX^4 \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 + jX^4 \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 + jX^4 \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6$$

$$-jX^5 \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 - jX^5 \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 - jX^5 \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 - jX^5 \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 - jX^5 \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 - jX^5 \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6$$

$$+jX^6 \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 + jX^6 \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 + jX^6 \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 + jX^6 \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 + jX^6 \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 + jX^6 \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6$$

$$-\frac{\partial L}{\partial x_1} dx_1 - \frac{\partial L}{\partial x_2} dx_2 - \frac{\partial L}{\partial x_3} dx_3 - \frac{\partial L}{\partial x_4} dx_4 - \frac{\partial L}{\partial x_5} dx_5 - \frac{\partial L}{\partial x_6} dx_6 \qquad (9)$$

Equation of Equation (8) with Equation (9) we obtain.

$$i_X \phi_L = dE_L$$

$$j \left(X^1 \frac{\partial}{\partial x_1} + X^1 \frac{\partial}{\partial x_2} + X^1 \frac{\partial}{\partial x_3} + X^1 \frac{\partial}{\partial x_4} + X^1 \frac{\partial}{\partial x_5} dx_5 + X^1 \frac{\partial}{\partial x_6} \right) \left(\frac{\partial L}{\partial x_2} \right) dx_1 + \frac{\partial L}{\partial x_1} dx_1$$

$$-j \left(X^1 \frac{\partial}{\partial x_1} + X^1 \frac{\partial}{\partial x_2} + X^1 \frac{\partial}{\partial x_3} + X^1 \frac{\partial}{\partial x_4} + X^1 \frac{\partial}{\partial x_5} dx_5 + X^1 \frac{\partial}{\partial x_6} \right) \left(\frac{\partial L}{\partial x_1} \right) dx_2 + \frac{\partial L}{\partial x_2} dx_2$$

$$+j \left(X^1 \frac{\partial}{\partial x_1} + X^1 \frac{\partial}{\partial x_2} + X^1 \frac{\partial}{\partial x_3} + X^1 \frac{\partial}{\partial x_4} + X^1 \frac{\partial}{\partial x_5} dx_5 + X^1 \frac{\partial}{\partial x_6} \right) \left(\frac{\partial L}{\partial x_3} \right) dx_3 + \frac{\partial L}{\partial x_3} dx_3$$

$$-j \left(X^1 \frac{\partial}{\partial x_1} + X^1 \frac{\partial}{\partial x_2} + X^1 \frac{\partial}{\partial x_3} + X^1 \frac{\partial}{\partial x_4} + X^1 \frac{\partial}{\partial x_5} dx_5 + X^1 \frac{\partial}{\partial x_6} \right) \left(\frac{\partial L}{\partial x_4} \right) dx_4 + \frac{\partial L}{\partial x_4} dx_4$$

$$j\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_5}\right)dx_5+\frac{\partial L}{\partial x_5}dx_5$$

$$-j\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_6}\right)dx_6+\frac{\partial L}{\partial x_6}dx_6=0 \tag{10}$$

Be an integral curve. in local coordinates it is obtained that.
Suppose that a curve.

$$\alpha: I \subset R \rightarrow T^*\mathcal{M} = R^{2n}$$

is an integral curve of the Lagrangian vector field X_H, i. e.,

$$X_L\big(\alpha(t)\big) = \frac{d\alpha(t)}{dt}, t \in I$$

In the local coordinates, if it is considered to be

$$\alpha(t) = \big(x_1(t), x_2(t), x_2(t), x_4(t), x_5(t), x_6(t)\big)$$

we obtain

$$\frac{d\alpha(t)}{dt}=\frac{dx_1}{dt}\frac{\partial}{\partial x_1}+\frac{dx_2}{dt}\frac{\partial}{\partial x_2}+\frac{dx_3}{dt}\frac{\partial}{\partial x_3}+\frac{dx_4}{dt}\frac{\partial}{\partial x_4}+\frac{dx_5}{dt}\frac{\partial}{\partial x_5}+\frac{dx_6}{dt}\frac{\partial}{\partial x_6}X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}=\frac{\partial}{\partial t} \tag{11}$$

Taking the equation (10) = the equation (11).

$$j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_2}\right)dx_1+\frac{\partial L}{\partial x_1}dx_1=0 \rightarrow j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_2}\right)+\frac{\partial L}{\partial x_1}=0$$

$$-j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_1}\right)dx_2+\frac{\partial L}{\partial x_2}dx_2=0 \rightarrow j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_1}\right)-\frac{\partial L}{\partial x_2}=0$$

$$j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_3}\right)dx_3+\frac{\partial L}{\partial x_3}dx_3=0 \rightarrow j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_3}\right)+\frac{\partial L}{\partial x_3}=0$$

$$-j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_4}\right)dx_4+\frac{\partial L}{\partial x_4}dx_4=0 \rightarrow j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_4}\right)-\frac{\partial L}{\partial x_4}=0$$

$$j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_5}\right)dx_5+\frac{\partial L}{\partial x_5}dx_5=0 \rightarrow j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_5}\right)+\frac{\partial L}{\partial x_5}=0$$

$$-j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_6}\right)dx_6+\frac{\partial L}{\partial x_6}dx_6=0 \rightarrow j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_6}\right)-\frac{\partial L}{\partial x_6}=0$$

And.

$$j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_2}\right)+\frac{\partial L}{\partial x_1}=0, j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_1}\right)-\frac{\partial L}{\partial x_2}=0, j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_3}\right)+\frac{\partial L}{\partial x_3}=0$$

$$j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_4}\right)-\frac{\partial L}{\partial x_4}=0, j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_5}\right)+\frac{\partial L}{\partial x_5}=0, j\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_6}\right)-\frac{\partial L}{\partial x_6}=0 \tag{12}$$

Hence the triple $(\mathcal{M}, \phi_L, \xi)$ is called apara complex Lagrangian mechanical system which are deduced by means of an almost real structure j and using of basis $\left\{\frac{\partial}{\partial x_i}: i = 1,2,3,4,5,6\right\}$ on the distributions $\mathcal{M}$.

4. Conclusions

From the study, we obtain that Lagrangian formalisms in generalized Classical Mechanics and field theory can be intrinsically characterized on $(T\mathcal{M}, \phi_L, \xi)$ being a model of Three Para-Complex Structures. So, the paths of semisprayξ on $J^{(2,0)}\mathcal{M}$ are the solutions of the Euler-Lagrange equations given by (12) on the mechanical system $(T\mathcal{M}, \phi_L, \xi)$.

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