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On Clarification to the Previously Published Research Articles

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Abstract: In the recent literature, attempts have been made to propose new statistical distributions for modeling real phenomena of nature by adding one or more additional shape parameter (s) to the distribution of baseline random variable. The major contribution of these distributions are to obtain monotonic and non-monotonic shaped failure rates. This short article, offers a clarification to the research articles proposed by Al-Kadim and Boshi [4], El-Bassiouny et al. [9] and El-Desouky et al. [10]. A brief discussion on the properties of this general class is given. A future research motivation on this subject is also provided.

Keywords: Monotonic Failure Rate Function, Non-Monotonic Failure Rate Function, T-X Family, Exponential-Type Lifetime Distributions

1. Introduction

In the outlook of reliability modeling, some of the prominent interrelationships between the various functions such as CDF, PDF, HF, CHF and SF, for a continuous lifetime random variable, can be summarized as

$$h(z)=\frac{g(z)}{1-G(z)} \tag{1}$$

$$h(z)=\frac{g(z)}{S(z)} \tag{2}$$

$$H(z)=\int_0^z h(z)\,dz \tag{3}$$

$$S(z)=e^{-H(z)} \tag{4}$$

$$G(z)=1-e^{-H(z)} \tag{5}$$

In term of CHF the CDF of a random variable can be written in the form provided in (5). It is to be noted that all the cumulative hazard functions must fulfill the following two conditions:

(a) $H(z)$ is non-negative, differentiable and increasing function of z.

(b) $\lim_{z\to 0} H(z)\to 0$, and $\lim_{z\to\infty} H(z)\to\infty$.

In this short scenario, it will be showed that how (5) eases the introduction of exponential-type lifetime distributions. In the recent literature, many researchers have proposed new lifetime distributions based on the traditional exponential distribution. These distributions published in different journals are either not new, or description of the model suggested by Gurvich et al. [13]. A lifetime distribution proposed by Gurvich et al. [13] has the CDF given by

$$G(z)=1-e^{-\lambda F(z)} \tag{6}$$

A number of research articles have been proposed by using the approach provided in (6). In the recent literature, attention has been focused to introduce new family of distributions by introducing new additional parameter (s). The introduction of additional parameter (s) into model offers a more flexible distribution. The most well-known family of distributions are: beta-G by Eugene et al. [11], Log-Gamma-G Type-2 of Amini et al. [7], Gamma-G Type-1 proposed by Zografos and Balakrishnan [20], Gamma-G Type-2 due to

Risti´c and Balakrishnan [16] Gamma-G Type-3 of Torabi and Montazeri [17], Exponentiated T-X by Alzaghal et al. [6], Logistic-G studied by Torabi and Montazeri [18] and Weibull–G family of Bourguinion et al. [8]. Recently, Alzaatreh et al. [6] introduced T-X family of distributions defined by

$$G(z)=\int_0^{V[F(z)]} f(y)\,dy \qquad (7)$$

where $f(y)$ stands for the PDF of a random variable say Y, where $Y\in[s,\ t]$ for $-\infty\le s<t<\infty$. Let $V[F(z)]$ be any function of CDF of Z such that $V[F(z)]$ satisfies the settings given below.

(c) $V[F(z)]\in[s,t]$

(d) $V[F(z)]$ is monotonically increasing and differentiable.

(e) $V[F(z)]\to s$ as $Z\to-\infty$ and $V[F(z)]\to t$ as $Z\to\infty$.

Table 1 shows the $V[F(z)]$ functions for some parametric models of the T-X family.

***Table 1.** Different $V[F(z)]$ functions for special models of the T-X family.*

$V[F(z)]$	Range of T	Members of T-X family
$F(z)$	$[0,1]$	Beta-G (Eugene et al., 2002), Mc-G (Alexander et al., 2012)
$-\log(1-F(z))$	$(0,\infty)$	Gamma-G Type-1 (Zografos and Balakrishnan, 2009)
$-\log(F(z))$	$(0,\infty)$	Log-Gamma-G Type-2 (Amini et al., 2012)
$\frac{F(z)}{1-F(z)}$	$(0,\infty)$	Gamma-G Type-3 (Torabi and Montazeri, 2012)
$-\log(1-F^a(z))$	$(0,\infty)$	Exponentiated T-X (Alzaghal et al., 2013)
$\log\left(\frac{F(z)}{1-F(z)}\right)$	$(-\infty,\infty)$	Logistic-G (Torabi and Montazeri, 2014)
$\frac{1}{1-F(z)}$	$(0,\infty)$	Exponential Pareto Distribution (Al-Kadim and Boshi, 2013) (Under Clarification)
$\frac{1}{1-F(z)}$	$(0,\infty)$	Exponential Lomax Distribution (El-Bassiouny et al., 2015) (Under Clarification)
$\frac{1}{1-F(z)}$	$(0,\infty)$	Exponential Flexible Weibull Extension Distribution (El-Desouky et al., 2016) (Under Clarification)

Zografos and Balakrishnan [19], proposed Gamma-G Type-1 by changing the upper limit of integral in (7) with $-\log(1-F(z))$, so, the expression in (7), can be re-written as

$$G(z)=\int_0^{-\log(1-F(z))} f(y)\,dy \qquad (8)$$

Similarly, Amini et al. [7] studied Log-Gamma-G Type-1 family of distributions by replacing the upper limit of integral in (8) with $-\log(1-F(z))$. Recently, using the expression in (8), Ahmad and Iqbal [1] proposed the GFWEx distribution. Al-Kadim and Boshi [4], introduced a new life time model entitled EP distribution by replacing the upper limit of integral in (7), with $\left(\frac{1}{1-F(z)}\right)$, and $f(y)$ with the density function of the exponential distribution. So the expression provided in (7), becomes.

$$G(z)=\int_0^{\frac{1}{1-F(z)}} \lambda e^{-\lambda z} dz. \qquad (9)$$

On solving, one might get the following expression

$$G(z)=1-e^{-\lambda\left(\frac{1}{1-F(z)}\right)} \qquad (10)$$

Where, $F(z)$ is a monotonically increasing function of z. Using the expression given in (9), El-Bassiouny et al. [9], proposed a new lifetime distribution titled as EL distribution. Also, using the same technique El-Desouky et al. [10], introduced another lifetime model named as The EFWEx distribution. By comparing (9) to (7) or (8), it is noted that the class characterized by (9) is a familiar general result in literature. Also, comparing (9) to (5), it is instantly noted that $\lambda\left(\frac{1}{1-F(z)}\right)=H(z)$. But, the quantity used in (9) or (10), is $\left(\frac{1}{1-F(z)}\right)$, does not meet the conditions, stated in (a)-(e). Therefore, it is observed that the function suggested in (9), is recognized by the common audience of journals. So, the claim of work provided in (9), is therefore, incorrect in the view of above discussion. For further detail about such class of distributions, one may call to Pham and Lai [15]. The key aims of the newly proposed distributions are to develop statistical models capable of modeling real phenomena with monotonic and non-monotonic shaped failure rates.

2. Characteristics of Hazard Rates

A lifetime model can be classified based on its shape of $h(z)$, for the sake of simplicity, the HF of the lifetime classes are often mentioned by the abbreviations as IFR, DFR, BTFR, UMFR and MUMFR etc. By Differentiating $h(z)$ with respect to z, one may can get different shapes of the HF's as follows. The HF must have,

(f) Monotonically increasing (non-decreasing) shape over time, if the first derivative of $h(z)$ with respect to z yields a positive value $\forall z$.

If the HF of a distribution increases over time then it is a very good model for describing the lifecycle of a machine's component. For example, the failure rate of a machine's component in its fifteenth year of working may be many times greater than its failure rate during the very first month of service. A distribution with increasing HF is said to be a lighter tailed distribution.

(g) Monotonically decreasing (non-increasing) shape over time, if the first derivative of $h(z)$ with respect to z yields a negative value $\forall z$.

If the HF of a distribution decreases over time, then it is considered a very good candidate model for modeling skewed data. A decreasing HF can be obtained by improving the system. For example, if a machine's component is defective, then by removing the defects, the lifetime of machine may be increased, resulting in a decreasing failure rate. A distribution with decreasing HF is said to be a heavier tailed distribution.

(h) Constant shape over time, if the value of the first derivative of $h(z)$ with respect to z is zero $\forall z$.

If the HF of a distribution is constant (neither increasing nor decreasing) then the distribution is said to be memory less, such as exponential distribution (in the continuous class of distributions) and geometric distribution (in the discrete class of distributions). There are attractive and wide ranges of applications of constant hazard function in reliability engineering. It reveals that the past information is entirely ignored and only the current information is utilized to take decision about the future event. For example, if a machine performed up to a specific time units say t, then the probability of performing up to another time units say t_0 is entirely independent of the past information that how long the machine has been performing. Alternatively, we can say that the hazard function will be constant, if the survival time is distributed exponentially. A distribution with a constant HF is said to have medium tails.

(i) Bathtub shaped HF, if the first derivative of $h(z)$ with respect to z yields a negative value $\left(i.e.\ h^{/}(z)<0\right)$ for $z \in (0, z_0)$ and positive value for $z > z_0$, that is $h^{/}(z)>0$, where the value z_0 has a unique as well as positive solution of $h^{/}(z_0)=0$.

The bathtub shaped HF initially decreases, followed by a less or more constant pattern (known as useful life period), then increases (known as wearing out period). A bathtub HF is very useful for describing the behavior of human mortality, where initially, during infant period (0-6 months after birth) the hazard rate is very high, then slowly decreases and followed up by less or more constant period (between 25- 45 years of age), and then increases (in older ages 70 years and above).

(j) Unimodal (upside-down bathtub) shaped HF, if the first derivative of $h(z)$ yields a positive value s for $z \in (0, z_0)$ and negative value for $z > z_0$, that is $h^{/}(z)<0$ and the value z_0 has a unique as well as positive solution of $h^{/}(z_0)=0$.

A distribution is said to have unimodal (also called upside-down bathtub) HF, if it's $h(z)$ has a unique mode having two phases. Initially, the $h(z)$ increases and then decreases. It is very useful for determining the time period having maximum risk.

(k) Modified unimodal shaped, if the HF initially, has unimodal shape and then increasing.

A distribution is said to have modified unimodal (also called modified upside down bathtub) failure rate function, if it's $h(z)$ gradually increases in the initial phase, then declines and finally again increases. The death rate of cancer patients is observed to have unimodal or modified unimodal shapes, where initially the failure rate is very high, after surgical removal the failure declines and finally again increases. The figure 1 & 2, displays monotonic and non-monotonic HF.

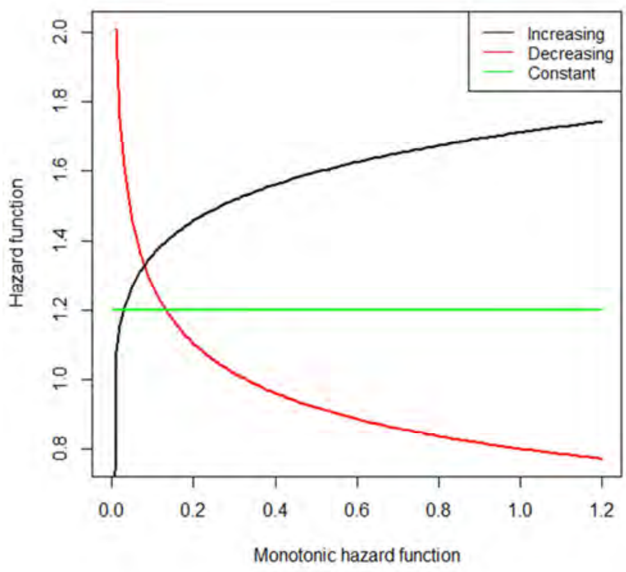

Figure 1. *Different Monotonic HFs.*

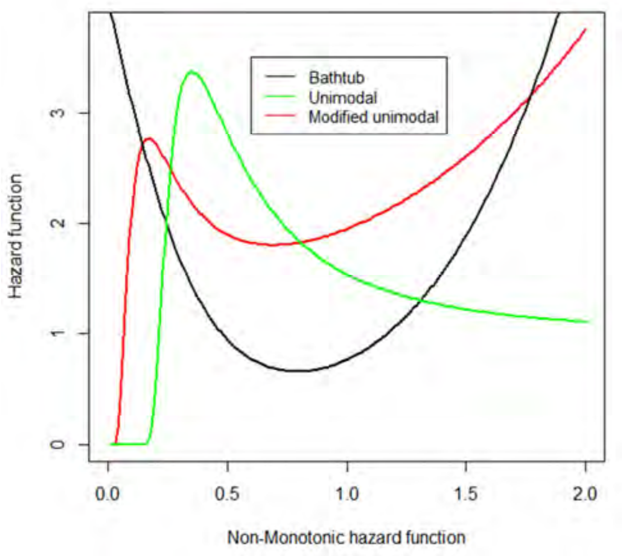

Figure 2. *Different Non-monotonic HFs.*

For further detail about aging classes one may refer to Almalki [5]. In spite of its appreciation, and wide range of applications in reliability theory, the traditional exponential distribution is unable to model data with increasing, decreasing, unimodal, modified unimodal or bathtub shaped HF. Therefore, to obtain different shapes of the HF, many different extensions of exponential distribution such as EE family by Gupta and Kundu [12], the GEE distribution due to Ristić and Balakrishnan [16], and TEE distribution proposed by Merovci [14], etc. have been proposed in the literature.

3. Research Motivations

This article offers straight forward conditions and encourages the researchers, how to propose new lifetime distributions. Taking the advantage of the expression provided in (5) or (7), one can construct a generalized version of exponential-type distributions or any other lifetime models by choosing a simply differentiable function. For example, Ahmad and Hussain [2], proposed a new lifetime model, named as NEx-W Distribution by replacing the quantity $H(z)$, in (5), with $e^{\beta z^{\alpha} - \frac{\sigma}{z^2}}$. Also, Ahmad and Hussain [3] studied another new lifetime distribution, named as VFW distribution by replacing the quantity $H(z)$, in (5), with $e^{\beta z^{\alpha} - \frac{1}{z}}$.

4. Conclusion

This short research article, confirmed a clarification to the previously published research papers, as the generator used in these papers is inappropriate in the view of recent literature. Furthermore, in order to propose new lifetime models, straight forward conditions are provided for a generator to use. A complete description of the failure rate functions are also discussed. Moreover, a simple and straight way is suggested to generate new functions.

It is hoped that this paper will be helpful for authors in future research work.

Notations

Z	Lifetime random variable
$g(z)$	Probability density function (PDF)
$G(z)$	Cumulative distribution function (CDF)
$h(z)$	Hazard function (HF)
$H(z)$	Cumulative hazard function (CHF)
$S(z)$	Survival function (SF)

Abbreviations

IFR	Increasing failure rate
DFR	Decreasing failure rate
BTFR	Bathtub failure rate
UMFR	Unimodal failure rate.
MUMFR	Modified unimodal failure rate.
GFWEx	Generalized flexible Weibull extension
EL	Exponential Lomax
EP	Exponential Pareto
EFWEx	Exponential flexible Weibull extension
GEE	Gamma exponentiated exponential
EE	Exponentiated exponential
TEE	Transmuted exponentiated exponential
NEx-W	New extended Weibull
VFW	Very flexible Weibull

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Biography

Zubair Ahmad S/O Wali Muhammad, research scholar at Quaid-i-Azam University, obtained his Master degree in Statistics in 2014 at University of Malakand and received his M. Phil. Degree in Statistics in 2017 at Quaid-i-Azam University, Islamabad, Pakistan. His research topic in M. Phil. was “On Different Modifications of Weibull Distribution” under the guidance & supervision of Dr. Zawar Hussain, Quaid-i-Azam University, Islamabad, Pakistan.