On the Equilibrium Without Loss in the Discrete Time Models of Economic Dynamics

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Abstract: The model of economic dynamics with a fixed budget is considered. The conditions are derived under which the model with a fixed budget has an equilibrium state with the equilibrium prices. The necessary and sufficient conditions for the existence of equilibrium prices are found.

Keywords: Production Function, Equilibrium State, Growth Rate

1. Introduction

Let's consider a model defined at the moment t by the productive mapping $a(x)$ [1-3, 10]:

$$a(x) = \left\{ \tilde{x} = (\tilde{x}^1, \dots, \tilde{x}^n) \in (R_+^n)^n \mid 0 \leq \sum_{i=1}^n \tilde{x}^i \leq \sum_{i=1}^n B^k \cdot x^{k^*} + (F^1(x^1), \dots, F^n(x^n)), x^{k^*} = (x^{k1}, \dots, x^{kn}), k = \overline{1, n} \right\},$$

where $x = (x^1, \dots, x^n) \in (R_+^n)^n$, B^k is a diagonal matrix, the main diagonal of which has a form

$$(v^{k1}, \dots, v^{kn}), v^{ki} \in [0, 1] \quad (k, i = \overline{1, n}),$$

$$F^j(x) = \min_{i=\overline{1, n}} \frac{x^i}{c^{ij}}, c^{ij} > 0 \quad (i, j = \overline{1, n}).$$

The productive mapping a^k of the branch k is in the form

$$a^k(x) = \langle 0, B^k x + (0, \dots, 0, F^k(x), 0, \dots, 0) \rangle \quad (x \in R_+^n).$$

Let's define the conditions under which the model with fixed budgets

$$M = \{y, U(\ell), \Omega\}$$

has an equilibrium state $(P, x^1, \dots, x^n, y)$ with equilibrium prices $P = (P^1, \dots, P^n)$. Here as usual $U(\ell) = (U^1(\ell, \cdot), \dots, U^n(\ell, \cdot))$, in its turn $U^i(\ell, x) = [\ell, B^k x] + \ell^i \cdot F^k(x)$; $\Omega = (\lambda^1, \dots, \lambda^n)$ is a budget vector, $\ell = (\ell^1, \dots, \ell^n)$ is a prices vector.

Let $I = \{1, 2, \dots, n\}$.

The following sets are introduced in [4]

$$I_1(x) = \{i \in I \mid x^i = 0\}.$$

$$I_2(x) = \{i \in I \mid x^i > 0\},$$

$$R^k(x) = \left\{ i \in I \mid \frac{x^i}{c^{ik}} = \min_{j \in I} \frac{x^j}{c^{jk}} \right\}, (k \in I),$$

$$Q^k(x) = I \setminus R^k(x), k \in I.$$

The problem of k -th consumer is as follows:

$$U^k(\ell, x) \rightarrow \max, x \in V = \{x \geq 0, [P, x] = 1\}. \quad (1)$$

Let $\bar{x}^{k^*}$ be a maximum point in (1) ($k \in I$).

2. Main Part

Definition. The equilibrium $(P, \bar{x}^1, \dots, \bar{x}^n, \Omega, y)$ is called equilibrium without loss if for all $k \in I$ is valid

$$R^k(\bar{x}^{k^*}) = I.$$

Definition. The prices $P = (P^1, \dots, P^n)$ defined in the equilibrium without loss is called an equilibrium prices without loss.

Note that due to the consequence of the lemma 1 [4] we have $I_1(\bar{x}^k) = \emptyset$.

First, consider the consumer problem without loss, and then the equilibrium without loss.

Note that the solution of the k – th problem without loss has the following property

$$\bar{x}^k = \gamma^k \cdot c^k \quad (k \in I), \quad (2)$$

where $\gamma^k \geq 0$ is some constant that is equal to the value of the k – th production function in the point $\bar{x}^k$.

Indeed from the conditions $R^k(\bar{x}^k) = I, I_1(\bar{x}^k) = \emptyset$ we get that

$$\frac{\bar{x}^{k1}}{c^{1k}} = \dots = \frac{\bar{x}^{kn}}{c^{nk}} = \gamma^k, \bar{x}^{ki} > 0 \quad (k, i \in I).$$

Consider the problem of kth consumer without loss. To analyze the problem (1) we apply the necessary and sufficient conditions for the extremum, wherein in the point $\bar{x}$ the maximum is reached if and only if when

$$(U^k)'(\bar{x}, g) \leq 0 \text{ for all } g \in G_{\bar{x}}(V),$$

where

$$G_{\bar{x}}(V) = \{g \in R^n \mid [P, g] = 0, g^i \geq 0 \forall i \in I_1(\bar{x})\}.$$

As is known [2], $(U^k)'(\bar{x}, g) = q^k(g)$, where

$$q^k(g) = [\ell_v^k, g] + \ell^k \cdot \min_{i \in R^k(\bar{x})} \frac{g^i}{c^{ik}} \quad (k \in I).$$

Then in our case (without loss) we get that the necessary and sufficient optimality conditions for $\bar{x}$ of the branch k take the forms

$$q^k(g) = [\ell_v^k, g] + \ell^k \cdot \min_{i \in I} \frac{g^i}{c^{ik}} \leq 0 \quad \forall g \in \Omega = \{g \in R^n \mid [P, g] = 0\}. \quad (3)$$

Lemma 1. The number μ^k ($k \in I$), defined in the lemma 4 [4] in the case of without loss ($R^k(\bar{x}) = I$) coincides with the maximal growth rate of the total wealth of the k – th branch and is equal to

$$\mu^k = \frac{\ell^k + [\ell_v^k, c^k]}{[P, c^k]} \quad (k \in I), \quad (4)$$

where $\ell_v^k = (\ell^1 \cdot v^{k1}, \dots, \ell^n \cdot v^{kn})$.

Proof. In the case, when $R^k(x) = I$, as follows from lemma 4 [4] the number μ^k is in the form of (4).

From the other hand let $\bar{x}$ be a maximum point in the problem (1) of the kth consumer that is indeed similar to the following relations

$$\max_{x \geq 0} \frac{U^k(\ell, x)}{[P, x]} = \frac{U^k(\ell, \bar{x})}{[P, \bar{x}]} \quad (k \in I),$$

where the functions $U^k(\ell, x)$ are defined in the introduction.

From the definition of μ^k and (2) we get that

$$\max_{x \geq 0} \mu^k(x) = \mu^k,$$

where μ^k is of the form (4).

Lemma is proved.

Theorem 1. Let strongly positive vector $P = (P^1, \dots, P^n)$ be given, index $k \in I$ and the number μ^k is defined by (4). The vector $\bar{x}$ is a solution of the problem (1), satisfying the relation

$$R^k(\bar{x}) = I.$$

Then and only then

$$\ell^j \cdot v^{kj} \leq \mu^k \cdot P^j \quad \forall j \in I, \quad (5)$$

$$P \in \frac{1}{\mu^k} \cdot (\ell_v^k + \partial \tilde{q}^k), \quad (6)$$

where $\ell_v^k, \partial \tilde{q}^k$ are defined in the lemma 3 [4] for the case $R^k(\bar{x}) = I$.

The proof follows from the consequence of 1 [4], theorem 1 [4] and lemma 1

Remark. By given μ^k ($k \in I$), the equality (3) may be considered as a system of n linear equation with respect to variables- coordinates of equilibrium vector of prices P without loss

$$[P, c^k] = \frac{1}{\mu^k} \cdot [\ell^k + [\ell_v^k, c^k]] \quad (k \in I),$$

where $c^k = (c^{1k}, \dots, c^{nk})$, $\ell_v^k = (\ell^1 \cdot v^{k1}, \dots, \ell^n \cdot v^{kn})$ and in contrary by the given prices P from the equality (3) are defined uniquely the maximal growth rate μ^k of the total wealth of the k – th branch ($k \in I$).

Let's consider the following problem. Let $v^{ki} \geq 0, c^{ik} > 0$ ($i, k \in I$) be given. By which $\ell^i > 0, \mu^k > 0$ ($i, k \in I$) there exists the vector $P = (P^1, \dots, P^n)$, that for some $\lambda^1, \dots, \lambda^n, y$ is an equilibrium prices without loss in the model M , defined by the set $\{\ell, \lambda^1, \dots, \lambda^n, y\}$.

Note that it follows from the lemma 3 [4] that in the case of lossless ($R^k(\bar{x}) = I$) subdifferential $\partial \tilde{q}^k$ takes the form [5, 6]

$$\partial \tilde{q}^k = \left\{ f = \ell^k \cdot (f^1, \dots, f^n) \mid \exists \alpha^i \geq 0: \sum_{i \in I} \alpha^i = 1, f^i = \frac{\ell^k \cdot \alpha^i}{c^{ik}} \quad \forall i \in I \right\}. \quad (7)$$

It can be shown that for each set $\mu^1 > 0, \dots, \mu^n > 0$ there are weights $\lambda^1, \dots, \lambda^n$ such that the growth rate in the model defined by the weights $\lambda^1, \dots, \lambda^n$, coincides up to those cells in which $\lambda^i = 0$ and the growth rate does not depend on the choice of the equilibrium price P .

Lemma 2. Let the numbers $\ell^i > 0, \mu^k > 0, v^{ji} \geq 0, c^{ij} > 0$ ($i, j, k \in I$) be given and $(P, x^1, \dots, x^n)$ is an equilibrium without loss in the model with utility functions U^j , defined in the introduction section by the budgets $\lambda^i = [P, x^j]$ and distributed by the vector $y = \sum_{i=1}^n x^i$ ($j \in I$). The relation (6) is fulfilled for $\forall k \in I$ then and only then when for any

$v^{ji} \geq 0$ and u^j ($i, j \in I$), satisfying the relations

$$\sum_{j \in I} \mu^j \cdot (v^{ji} + u^j \cdot c^{ij}) = 0 \quad \forall i \in I, \quad (8)$$

the inequality below is valid

$$\sum_{j \in I} (\sum_{i \in I} v^{ji} \cdot \ell^i \cdot v^{ji} + u^j \cdot (\ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij})) \leq 0. \quad (9)$$

Proof. It follows from (7) that

$$\partial \tilde{q}^k = \left\{ (\beta^1, \dots, \beta^n) \mid \sum_{i \in I} c^{ik} \cdot \beta^i = \ell^k, \beta^i \geq 0 \right\} \quad (k \in I).$$

Denote the set of the form $\frac{1}{\mu^k} (\ell_v^k + \partial \tilde{q}^k)$ by Φ^k , that considering lemma 3 [4] (in the case $R^k(\bar{x}) = I$) indeed has a form

$$\begin{aligned} \Phi^k &= \left\{ \frac{1}{\mu^k} \cdot (\gamma^1, \dots, \gamma^n) \mid \gamma^i = \ell^i \cdot v^{ki} + \beta^i, \sum_{i \in I} c^{ik} \cdot \beta^i \right. \\ &\quad \left. = \ell^k, \beta^i \geq 0 \right\} = \\ &= \left\{ \frac{1}{\mu^k} \cdot (\gamma^1, \dots, \gamma^n) \mid \sum_{i \in I} c^{ik} \cdot (\gamma^i - \ell^i \cdot v^{ki}) = \ell^k, \gamma^i \right. \\ &\quad \left. \geq \ell^i \cdot v^{ki} \right\} = \\ &= \left\{ \frac{1}{\mu^k} \cdot \gamma \mid \sum_{i \in I} c^{ik} \cdot \gamma^i = \ell^k + \sum_{i \in I} \ell^i \cdot v^{ki} \cdot c^{ik}, \gamma^i \right. \\ &\quad \left. \geq \ell^i \cdot v^{ki} \right\} \quad (k \in I). \end{aligned}$$

Since for all $k \in I$ due the conditions of the lemma3 [4] (6) is fulfilled we have

$$P \in \bigcap_{k \in I} \Phi^k, \text{ where } P = (P^1, \dots, P^n).$$

Thus

$$\begin{aligned} \mu^j \cdot P^i &\geq \ell^i \cdot v^{ji} \quad (i, j \in I), \\ \mu^j \cdot \sum_{i \in I} P^i \cdot c^{ij} &= \ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij} \quad (j \in I). \end{aligned}$$

This system may be written as a system of inequalities

$$\begin{cases} \mu^j \cdot P^i \geq \ell^i \cdot v^{ji} \quad (i, j \in I), \\ \mu^j \cdot \sum_{i \in I} P^i \cdot c^{ij} \geq \ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij} \quad (j \in I), \\ -\mu^j \cdot \sum_{i \in I} P^i \cdot c^{ij} \geq -(\ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij}) \quad (j \in I). \end{cases} \quad (10)$$

Let's introduce the denotations

$$\begin{cases} f^{ji} = e^i \cdot \mu^j, \\ f^j = \mu^j \cdot (c^{1j}, c^{2j}, \dots, c^{nj}), \\ f^{n+j} = -f^j, \\ \beta^{ji} = \ell^i \cdot v^{ji} \quad (i, j \in I), \\ \beta^j = \ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij}, \\ \beta^{n+j} = -\beta^j, \end{cases} \quad (11)$$

where e^i is i -th ort in the space R_+^n .

Rewriting the system (10) in new denotation considering (11) we obtain

$$\begin{cases} [f^{ji}, P] \geq \beta^{ji} \quad (i, j \in I), \\ [f^j, P] \geq \beta^j \quad (j \in I), \\ [f^{n+j}, P] \geq \beta^{n+j} \quad (j \in I). \end{cases} \quad (12)$$

Then, by the theorem of [7] for the compatibility of the system (12) is necessary and sufficient that for any $v^{ji} \geq 0, v^j \geq 0, v^{n+j} \geq 0$ ($i, j \in I$) from the equality

$$\sum_{i, j \in I} v^{ji} \cdot f^{ji} + \sum_{j \in I} v^j \cdot f^j + \sum_{j \in I} v^{n+j} \cdot f^{n+j} = 0$$

follow the inequality

$$\sum_{i, j \in I} v^{ji} \cdot \beta^{ji} + \sum_{j \in I} v^j \cdot \beta^j + \sum_{j \in I} v^{n+j} \cdot \beta^{n+j} \leq 0.$$

In our case necessary and sufficient conditions have a form

$$\begin{aligned} \sum_{j \in I} \mu^j \cdot (v^{ji} + (v^j - v^{n+j}) \cdot c^{ij}) &= 0 \text{ for } \forall i \in I, \\ \sum_{j \in I} (\sum_{i \in I} v^{ji} \cdot \ell^i \cdot v^{ji} + (v^j - v^{n+j}) \cdot (\ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij})) &\leq 0. \end{aligned} \quad (13)$$

Let $u^j = v^j - v^{n+j}, \forall i \in I$.

Then it follows from (13) that u^j are such that

$$\sum_{j \in I} \mu^j \cdot u^j \cdot c^{ij} \leq 0 \text{ for } \forall i \in I.$$

Substituting u^j ($j \in I$) into (13) we get (8), (9).

Lemma is proved.

Consider the matrix $C = (c^{ij})_{i,j=1}^n$. Let $|C|$ be its determinant. It is valid Lemma 3. Let the numbers $v^{ji} \geq 0, c^{ij} > 0$ ($i, j \in I$) be given and $|C| \neq 0$. The following conditions are equivalent

1. numbers $v^{ji} \geq 0, u^j$ ($i, j \in I$) are such that (8) and (9) are fulfilled;
2. numbers ℓ^i, μ^j ($i, j \in I$) are such that for $\forall i, j \in I$ is valid

$$\ell^i \cdot v^{ji} + \frac{1}{|C|} \cdot \sum_{i \in I} (-1)^{i+k+1} \cdot \frac{\mu^j}{\mu^k} (\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk}) \cdot |C_i^k| \leq 0, \quad (14)$$

where C_i^k is $(n-1) \times (n-1)$ matrix obtained from the matrix C by removing k -th column and i -th row.

Proof. The system of equalities (8) we rewrite as follows

$$\sum_{j \in I} \mu^j \cdot u^j \cdot c^{ij} = -\sum_{j \in I} \mu^j \cdot v^{ji}, \forall i \in I. \quad (15)$$

Consider the equation

$$A \cdot u = -b, \quad (16)$$

where

$$A = \begin{pmatrix} \mu^1 \cdot c^{11} & \mu^2 \cdot c^{12} & \dots & \mu^n \cdot c^{1n} \\ \mu^1 \cdot c^{21} & \mu^2 \cdot c^{22} & \dots & \mu^n \cdot c^{2n} \\ \dots & \dots & \dots & \dots \\ \mu^1 \cdot c^{n1} & \mu^2 \cdot c^{n2} & \dots & \mu^n \cdot c^{nn} \end{pmatrix}, b = \begin{pmatrix} b^1 \\ b^2 \\ \dots \\ b^n \end{pmatrix},$$

$$b^m = \sum_{k \in I} \mu^k \cdot v^{km} \quad (m \in I).$$

Note that

$$|A| = \prod_{j \in I} \mu^j \cdot |C|,$$

where $|C|$ is a determinant of the matrix.

As is known [8] the equation (16) has a solution (note that $|C| \neq 0$):

$$u^j = -\frac{|A^j|}{|A|} \quad (j \in I),$$

where A^j is $n \times n$ matrix obtained from the matrix A by replacing j -th column by the row b .

Expanding the determinant of the matrix A^j over the element of the j -th column (it refers the column b), we get

$$|A^j| = \frac{1}{\mu^j} \cdot \prod_{k \in I} \mu^k \cdot |C^j|$$

$$= \frac{1}{\mu^j} \cdot \prod_{k \in I} \mu^k \cdot \sum_{m \in I} (-1)^{m+j} \cdot b^m \cdot |C_m^j| \quad (j \in I),$$

here C_m^j is $(n-1) \times (n-1)$ matrix obtained from the matrix C by removing j -th column and m -th row.

Substituting the values of $|A|, |A^j|$ into the solution of the equation (16), we obtain

$$u^j = \frac{1}{\mu^j \cdot |C|} \cdot \sum_{m \in I} (-1)^{m+j+1} \cdot \left(\sum_{k \in I} \mu^k \cdot v^{km} \right) \cdot |C_m^j| \quad (j \in I). \quad (17)$$

Note that (16) is indeed matrix form of the system of equations (15) relatively u .

Then

$$\sum_{j \in I} \left(\sum_{i \in I} v^{ji} \cdot \ell^i \cdot v^{ji} + u^j \cdot \left(\ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij} \right) \right) =$$

$$= \sum_{j \in I} \left[\sum_{i \in I} v^{ji} \cdot \ell^i \cdot v^{ji} + \frac{1}{\mu^j \cdot |C|} \cdot \left(\ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij} \right) \times \right. \quad (18)$$

$$\left. \times \sum_{m \in I} (-1)^{m+j+1} \cdot \left(\sum_{k \in I} \mu^k \cdot v^{km} \right) \cdot |C_m^j| \right],$$

where $|C| \neq 0$.

The expression (17) after some transformations may be reduced to the form

$$\sum_{j \in I} \sum_{i \in I} d^{ji} \cdot v^{ji} = [d, v], \quad (19)$$

where $d = (d^{11}, d^{12}, \dots, d^{nn})$, $v = (v^{11}, v^{12}, \dots, v^{nn})$, and d^{ji} are corresponding coefficients at v^{ji} , depending on $\ell^i, v^{ji}, c^{ij}, \mu^j$:

$$d^{ji} = d^{ji}(\ell^i, v^{ji}, c^{ij}, \mu^j) \quad (i, j \in I).$$

To reduce the expression (19) to the form (19), it is necessary in (18) pass from v^{km} to v^{ji} . To do this we accept reindexing " k " $\leftrightarrow$ " j ", " m " $\leftrightarrow$ " i ":

$$\sum_{j \in I} \frac{1}{\mu^j \cdot |C|} \cdot \left(\ell^j + \sum_{i \in I} \ell^i \cdot v^{ji} \cdot c^{ij} \right) \cdot \sum_{m \in I} (-1)^{m+j+1} \cdot \left(\sum_{k \in I} \mu^k \cdot v^{km} \right) \cdot |C_m^j| =$$

$$= \sum_{k \in I} \frac{1}{\mu^j \cdot |C|} \cdot \left(\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk} \right) \cdot \sum_{m \in I} (-1)^{i+k+1} \cdot \left(\sum_{j \in I} \mu^j \cdot v^{ji} \right) \times$$

$$\times |C_i^k| = \sum_{j \in I} \frac{1}{|C|} \cdot \sum_{k \in I} \frac{\mu^j}{\mu^k} \left(\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk} \right) \sum_{i \in I} (-1)^{i+k+1} \cdot |C_i^k| \cdot v^{ji} =$$

$$= \sum_{j \in I} \sum_{i \in I} \frac{1}{|C|} \cdot \sum_{k \in I} (-1)^{i+k+1} \cdot \frac{\mu^j}{\mu^k} \cdot \left(\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk} \right) \cdot |C_i^k| \cdot v^{ji}.$$

Then (18) turns to

$$\sum_{j \in I} \sum_{i \in I} \left(\ell^i \cdot v^{ji} + \frac{1}{|C|} \cdot \sum_{k \in I} (-1)^{i+k+1} \times \right.$$

$$\left. \times \frac{\mu^j}{\mu^k} \cdot \left(\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk} \right) \cdot |C_i^k| \cdot v^{ji} \right).$$

Thus comparing the last one with (19), we arrive to

$$d^{ji} = \ell^i \cdot v^{ji} + \frac{1}{|C|} \cdot \sum_{k \in I} (-1)^{i+k+1} \cdot \frac{\mu^j}{\mu^k} \cdot (\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk}) \cdot |C_i^k|. \quad (20)$$

Then the condition (9) is equivalent to

$$[d, v] \leq 0,$$

where scalar product $[d, v]$ is defined by the formula (19). It does not exist $v = (v^{11}, v^{12}, \dots, v^{1n})$ such that $[d, v] > 0$ then and only then when тогда и all $d^{ji} \leq 0$ for $\forall i, j \in I$, where d^{ji} is defined by (20).

Lemma is proved.

It takes place

Theorem 2. Let the numbers $v^{ji} \geq 0$ и $c^{ij} > 0$ ($i, j \in I$) are such that $\max_{j \in I} v^{ji} > 0$ и $|C| \neq 0$. The equilibrium prices without loss by given v^{ji}, c^{ji} and some $\ell^i > 0$ ($i \in I$) and $\mu^j > 0$ ($j \in I$) exist only and only when (14) is satisfied; the coefficients μ^k ($k \in I$) and equilibrium prices P are related by the formula

$$\mu^k \cdot [P, c^k] = \ell^k + [\ell_v^k, c^k] \quad (k \in I), \quad (21)$$

where $c^k = (c^{1k}, \dots, c^{nk})$, $\ell_v^k = (\ell^1 \cdot v^{k1}, \dots, \ell^n \cdot v^{kn})$.

The proof immediately follows from the lemmas 2 and 3.

Note 1. Note that the condition (14), that is necessary and sufficient condition of existence equilibrium prices without loss does not depend on the vector of distributed resources y .

Note 2. By given $v^{ji} \geq 0, c^{ij} > 0$ ($i, j \in I$) the parameters ℓ^i, μ^j ($i, j \in I$) is a solution of the system of inequalities (14).

Example. Rewrite the conditions (14) for the cases $n = 2, n = 3$ и $v^{ji} = 0, \mu^i = \mu^j$ ($i \neq j$) ($i, j \in I$).

a) by $n = 2$ we have

$$\begin{aligned} & \frac{1}{|C|} \cdot [-(c^{22} + v^{11} \cdot c^{21} \cdot c^{12}) \cdot \ell^1 + (1 + v^{22} \cdot c^{22}) \cdot c^{21} \cdot \ell^2] \leq 0, \\ & \frac{1}{|C|} \cdot [-(1 + v^{11} \cdot c^{11}) \cdot c^{22} \cdot \ell^1 + (1 + v^{22} \cdot c^{22}) \cdot c^{21} \cdot \ell^2] \leq 0, \\ & \frac{1}{|C|} \cdot [(1 + v^{11} \cdot c^{11}) \cdot c^{12} \cdot \ell^1 - (1 + v^{22} \cdot c^{22}) \cdot c^{11} \cdot \ell^2] \leq 0, \\ & \frac{1}{|C|} \cdot [(1 + v^{11} \cdot c^{11}) \cdot c^{12} \cdot \ell^1 - (c^{11} + v^{22} \cdot c^{12} \cdot c^{21}) \cdot \ell^2] \leq 0, \end{aligned}$$

where $|C|$ is a determinant of the 2×2 matrix C ;

b) by $n = 3$ conditions (14) turn to:

$$\left(v^{11} - \frac{|C_1^1|}{|C|} \cdot (1 + v^{11} \cdot c^{11}) \right) \cdot \ell^1 + \frac{|C_1^1|}{|C|} \cdot (1 + v^{22} \cdot c^{22}) \cdot \ell^2 -$$

$$- \frac{|C_1^3|}{|C|} \cdot (1 + v^{33} \cdot c^{33}) \cdot \ell^3 \leq 0,$$

$$\frac{1}{|C|} \cdot [-(1 + v^{11} \cdot c^{11}) \cdot |C_1^1| \cdot \ell^1 + (1 + v^{22} \cdot c^{22}) \cdot |C_1^2| \cdot \ell^2 -$$

$$-(1 + v^{33} \cdot c^{33}) \cdot |C_1^3| \cdot \ell^3] \leq 0,$$

$$\frac{1}{|C|} \cdot [(1 + v^{11} \cdot c^{11}) \cdot |C_2^1| \cdot \ell^1 - (1 + v^{22} \cdot c^{22}) \cdot |C_2^2| \cdot \ell^2 -$$

$$-(1 + v^{33} \cdot c^{33}) \cdot |C_2^3| \cdot \ell^3] \leq 0,$$

$$\frac{|C_2^1|}{|C|} \cdot (1 + v^{11} \cdot c^{11}) \cdot \ell^1 + \left(v^{22} - \frac{|C_2^2|}{|C|} \cdot (1 + v^{22} \cdot c^{22}) \right) \cdot \ell^2 +$$

$$+ \frac{|C_2^3|}{|C|} \cdot (1 + v^{33} \cdot c^{33}) \cdot \ell^3 \leq 0,$$

$$\frac{1}{|C|} \cdot [-(1 + v^{11} \cdot c^{11}) \cdot |C_3^1| \cdot \ell^1 + (1 + v^{22} \cdot c^{22}) \cdot |C_3^2| \cdot \ell^2 -$$

$$-(1 + v^{33} \cdot c^{33}) \cdot |C_3^3| \cdot \ell^3] \leq 0,$$

$$- \frac{|C_3^1|}{|C|} \cdot (1 + v^{11} \cdot c^{11}) \cdot \ell^1 + \frac{|C_3^2|}{|C|} \cdot (1 + v^{22} \cdot c^{22}) \cdot \ell^2 +$$

$$+ \left(v^{33} - \frac{|C_3^3|}{|C|} \cdot (1 + v^{33} \cdot c^{33}) \right) \cdot \ell^3 \leq 0,$$

where $|C|$ is a determinant of the 3×3 matrix C , C_j^i is 2×2 matrix obtained from C by removing i -th column and j -th row.

Introduce the numbers

$$d_i^{kj} = \begin{cases} \ell^i \cdot v^{ji} + (-1)^{i+j+1} \cdot \frac{|C_i^j|}{|C|} \cdot \left(\ell^j + \sum_{m \in I} \ell^m \cdot v^{jm} \cdot c^{mj} \right), & \text{if } k = j, \\ (-1)^{i+k+1} \cdot \frac{|C_i^k|}{|C|} \cdot \left(\ell^k + \sum_{m \in I} \ell^m \cdot v^{km} \cdot c^{mk} \right), & \text{if } k \neq j, (i, j, k \in I) \end{cases}$$

and vector $\tilde{d}^{kj}$:

$$\tilde{d}^{kj} = \begin{pmatrix} -d_1^{kj} \\ \vdots \\ -d_n^{kj} \end{pmatrix} \quad (k, j \in I). \quad (22)$$

Proposition 2. The numbers $\mu^j > 0$ ($j \in I$), satisfying (14), exist if and only if when there exists the index $k_o \in I$ and α^k such that

$$\alpha^{k_o} > 0, \sum_{k=1}^{n^2} \alpha^k \cdot \tilde{d}^{kj} \geq 0 \quad (j \in I), \quad (23)$$

where $\tilde{d}^{kj}$ is defined by (22).

Proof. Necessity. Let there exist the numbers $\mu^j >$

Thus by this way the system of n^2 inequalities (24) is reduced to the system of n superlinear inequalities.

3. Results

In the paper the following results are obtained:

- The necessary and sufficient conditions are derived for the optimality of the branch trajectories;
- The maximal growth rate is defined for the branches in the without loss case;
- The necessary and sufficient condition is derived for the existence of the equilibrium prices without loss;
- The form of the superdifferential is given for the utility function of the consumer;
- The conditions are defined for the reducing the system of n^2 linear inequalities to the system of n superlinear inequalities of the same variables.

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