Performance of Tuned Mass Dampers in Vibration Response Control of Base-Excited Structures

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To cite this article:

Received: March 7, 2017; Accepted: March 29, 2017; Published: May 3, 2017

Abstract: The behavior of structures in recent years indicates that moderate and severe earthquakes lead to substantial damages, extensively higher than what is expected. One solution in order to reduce the seismic response of the structures, especially in relative story displacements is usage of tuned mass dampers (TMD). In this study, comparisons between uncontrolled and controlled cases have been evaluated. Results for an 8-story shear building under specified records show that active tuned mass dampers (ATMD) have more appropriate efficiency in structural displacement response reduction compared to passive tuned mass dampers (PTMD) in spite of high cost of installation. In addition, implementation of PTMD would lead to more desirable results in comparison with uncontrolled case regarding acceleration and displacement time history responses, especially when the natural structures’ frequency is different from the dominant frequency of the records. In addition, usage of ATMD results in significant reduction in the story shear response, whereas PTMD equipped system decreases story shear with a limited margin.

Keywords: Tuned Mass Damper (TMD), Structural Control, Time-History Analysis, Shear Building

1. Introduction

Natural hazards such as earthquakes especially in countries with severe seismicity call for the use of new and innovative approaches to limit structural responses [1, 2]. Control of structures has been introduced as an efficient way to decrease attributed damages and provide enough safety against extreme excitations [3-7]. For many structures, damping often becomes an important issue. The steady-state vibrations at resonance are limited by the damping of the structure. Therefore increased damping could be valuable solution to address such problems.

Deep interest in seismic response reduction of structures leads to design and development of more creative dissipation devices [8, 9]. In this regard, active and passive controlling approaches could be implemented to limit the movements of buildings’ higher stories within acceptable range, especially in tall buildings [10, 11]. There is increasing number of innovative approaches toward implementation of the passive control concepts to design more efficient control systems [12-14]. The history of vibration control dates back to year 1909, when for the first time, Frahm invented a control device named dynamic vibration absorber [15]. The early applications of tuned mass dampers have been directed toward mitigation of wind-induced excitations [16]. From then on, a number of comprehensive studies and projects have been conducted on the behavior of equipped structures with tuned mass dampers. After 1971, tuned mass dampers (TMDs) as an effective device was implemented in a number of new and existing buildings. The 244m high John Hancock Tower in Boston and 280m high Citicorp Centre Office building in New York City [17] are among a number of buildings equipped with tuned mass dampers. TMDs could be used in a wide range of structures in various civil engineering applications as well [18]. These commonly-used devices are found to be reliable and inexpensive means of seismic response reduction for a long period of time. Tuned mass dampers are generally set based on the natural period of structure. In this regard, passive tuned mass dampers (PTMDs) is tuned with the first mode of the structure. Accordingly, when specified frequency is excited, the damper will resonate out the corresponding phase which causes influential reduction in the first-mode response as the
most influential mode in structures’ response [17]. Nevertheless, this device could only be tuned with a single structural frequency, and extensively reduces the specified mode response of the structure. However, it is probable that the higher mode responses, especially in high-rise buildings would increase [16].

The solution to reduce the response of structures under earthquakes has been evolving from passive control systems to active ones. Although passive systems are regarded as suitable means of energy dissipation, they are unable to adapt to changes in the response and seismic nature of excitations. On the other hand, active control systems have significant adaptability with happened changes in response. In this regard, the usage of TMD with active capability in infrastructures would result in significant reduction in structures’ response [16].

There are a number of distinctive issues associated with the design of controlling systems: performance objective, control methodology, uncertainties and non-linearities to only name a few. For the above-mentioned factors, the feedback gained from the response of structures is used in active tuned mass dampers to provide more effective response decrease, Nanjing Communication Tower with 310m height in China, and Riverside Sumida Central Tower with 134m in Japan are among a number of practical implementations of active tuned mass dampers ATMDs in high-rise buildings [18]. Despite substantial costs of installation and maintenance, usage of ATMD in high-rise buildings results in significant decrease in story drifts, leading to less non-structural damages.

2. Passive Tuned Dampers

Tuned mass damper as an energy dissipative device is viscous spring-mass equipment used to reduce the modal responses of the structures [19]. This device oscillates with shifted-phase frequency compared to that of the original structure to provide efficient response reduction [20]. Energy would be dissipated through relative movement between PTMD and structure. Hence, generally TMDs are implemented in the top floors of the buildings to have more effectiveness in seismic response of the structures. Figure 1 shows a simple configuration of a structure equipped with PTMD.

The equations of motion for this device are as follows:

\[ m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = c_2 \dot{y} + k_2 y + f \]  
\[ m_2 \ddot{y} + c_2 \dot{y} + k_2 x_1 = -m_2 \ddot{x}_1 \]  
\[ y = y(t) = x_2(t) - x_1(t) \]

where \( m_1, k_1, c_1 \) are the mass, stiffness and damping of the main system, respectively. \( m_2, k_2, c_2 \) are PTMD characteristics and \( y \) is the relative displacement between PTMD and the structure, and \( f \) is the external force applied to the mass. It has been observed that by increasing the PTMD mass ratio, building’s damping coefficient would be increased linearly [21]. The tuned mass dampers are effective provided that the dominant frequency of the record would be near to the first mode frequency of the structure.

This study would investigate the general concepts associated with passive and active control procedures in details. The comparison of the applications of PTMD and ATMD in a typical eight-story building seismic behavior is determined. Subsequently, the recommendations to improve the passive control system’s general performance to approach ATMD performance are made.

3. Active Tuned Mass Dampers

Implementation of passive tuned mass dampers could not be highly effective in order to decrease the response of the structure whose dominant frequency is not compatible with the natural frequency of the device; PTMD could lead to undesirable results if higher modes of the structure would be resonated under excitation. Therefore, in order to overcome the mentioned shortcomings of PTMD equipped systems, active tuned mass dampers were introduced. Figure 2 shows a simple configuration of structure equipped with ATMD, which imposes external forces by usage of actuators.

Active tuned mass damper works with feedback control of the response. This could be done by real-time monitoring of the structures’ response, and generating a force known as corrective control force by ATMD. The corresponding dynamic equations of motion for multi-degree of freedom
(MDOF) systems equipped with ATMD could be solved based on state-space approach [21]. In what follows, dynamic equations of active tuned mass dampers would be investigated [22-24].

\[ m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = c_2 \dot{y} + k_2 y + f - u \]  
\[ m_2 \ddot{y} + c_2 \dot{y} + k_2 y = u - m_2 \ddot{x}_1 \]

The above equations could be indicated in state-space format:

\[ \dot{z}(t) = Az(t) + Bu(t) + Hf(t) \]

in which:

\[ z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \]

\[ A = \begin{bmatrix} 0 & I \\ -K \times M^{-1} & -C \times M^{-1} \end{bmatrix} \]

\[ B = \begin{bmatrix} 0 \\ D \times M^{-1} \end{bmatrix} \]

\[ H = \begin{bmatrix} I \\ E \times M^{-1} \end{bmatrix} \]

In this regard, \( A \) is the system matrix due to comprising of mass, stiffness and damping coefficients. \( B \) and \( H \) have dimensions of \( 2n \times m \) and \( 2n \times r \) introduced to determine the controllers’ locations and external excitations, respectively. \( n \) and \( m \) are number of DOFs and number of controllers. The \( n \times m \) matrix \( D \) and \( n \times r \) matrix \( E \) are location matrices in general; they define the locations of control force and excitation, respectively.

For implementation of effective dissipating systems, different structural safety levels, minimum costs and structures’ importance should be considered. While the structural safety for human comfort, and non-structural damages could be achieved by imposing a maximum allowable deflection in pre-set point in the structure, the cost of utilizing ATMD seems to be a significant constraint as well due to the fact that these system are expensive to install and maintain. Subsequently, to consider the conflicting requirements, a performance index \( J \) was introduced as an appropriate weighting function to balance between involving parameters. In what follows, the performance index \( J \) suggested by Soong [21] is shown:

\[ J = \frac{1}{2} \int_{t_0}^{t_f} [z^T(t)Qz(t) + u^T(t)Ru(t)]dt \]  

In which \( t_f \) is the time duration over which the control force operates. This performance index for structural control is quadratic in \( z(t) \) and \( u(t) \). Performance index is an integral evaluated over the control interval; moreover, in eqn (11), \( J \) is a scalar which could be minimized with respect to \( u(t) \). \( Q \) is a \( 2n \times 2n \) positive semi-definite matrix and \( R \) is an \( m \times m \) positive definite matrix. The weighting matrices \( R \) and \( Q \) are set regarding the desirable response reduction and the effects of corrective control force. The assignment of higher values to \( Q \), or lesser ones to \( R \) would lead to significant reduction in the responses, in spite of increase in control forces. Matrix \( Q \) could be estimated by solving the eigenvalues of controlled structure and checking the stability conditions [21]. Additionally, if the value of \( t_f \) approaches infinity, we could obtain the optimal feedback control law for corrective control force:

\[ u(t) = Gz(t) \]

\[ G = -\frac{1}{2} R^{-1}B^TP \]

where \( G \) is the gain matrix. If the feedback gain matrix is positive definite, the closed-loop system is asymptotically stable. The symmetric matrix \( P \) known as Ricatti matrix could be evaluated from the following formula:

\[ \dot{P} + PA - \frac{1}{2} PBR^{-1}B^TP + A^TP + 2Q = 0 \]  

The Ricatti matrix is extensively dependent on the structural characteristics and weighting matrices. It has been recommended that Ricatti matrix \( P \) remains constant over the earthquake [25].

Control system remains unaffected provided that the Ricatti matrix is supposed as a constant matrix; hence, the first term of eqn (14), \( P \) could be assigned zero on the condition that \( t_f \) would be longer than earthquake duration [25]. This would lead to simplification of the matrix algebraic equations and less computing time.

### 4. Results

The inability of PTMDs to limit the response in the excitations with different dominant frequencies from that of the original building resulted in designing active systems. In this regard, Northridge and El-Centro records have been evaluated for proper indication of ATMD and PTMD efficiency. Furthermore, in order to determine the effects of active systems, an eight-story shear building controlled passively and actively has been investigated and compared with uncontrolled case. Table 1 indicates the geometrical and mechanical characteristics of the mentioned structure and tuned mass damper.

<table>
<thead>
<tr>
<th>Characteristic of the structure</th>
<th>Mass (ton)</th>
<th>Stiffness ((\times 10^5 \text{ N/m}))</th>
<th>Damping ((\times 10^5 \text{ N.s/m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure (each story)</td>
<td>320</td>
<td>930</td>
<td>15.69</td>
</tr>
<tr>
<td>Active/ Passive-TMD</td>
<td>44</td>
<td>3.7</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The weighting matrix \( Q \) is considered with two different configurations as indicated in Table 2, and \( R \) as a scalar is considered \( 10^{-3} \) and \( 10^{-4} \) to evaluate the effects of different weighting matrices. The Ricatti equation was solved
regardless of the real-time behavior of the response in ATMD-equipped structures.

Table 2. ATMD parameters and results.

<table>
<thead>
<tr>
<th>ATMD parameters</th>
<th>Excitation</th>
<th>Northridge</th>
<th>El-Centro</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 10^{-3}, Q = \text{diag}[1 \ 0 \ 0]$</td>
<td>Maximum displacement (m)</td>
<td>0.1408</td>
<td>0.1371</td>
</tr>
<tr>
<td></td>
<td>Maximum Acceleration (g)</td>
<td>0.210</td>
<td>0.255</td>
</tr>
<tr>
<td></td>
<td>Required force for ATMD (ton)</td>
<td>100.84</td>
<td>86.5</td>
</tr>
<tr>
<td>$R = 10^{-4}, Q = \text{diag}[1 \ 1 \ 0]$</td>
<td>Maximum displacement (m)</td>
<td>0.0846</td>
<td>0.0974</td>
</tr>
<tr>
<td></td>
<td>Maximum Acceleration (g)</td>
<td>0.400</td>
<td>0.330</td>
</tr>
<tr>
<td></td>
<td>Required force for ATMD (ton)</td>
<td>185.18</td>
<td>202.98</td>
</tr>
<tr>
<td>$R = 10^{-3}, Q = \text{diag}[1 \ 0 \ 0]$</td>
<td>Maximum displacement (m)</td>
<td>0.3514</td>
<td>0.2239</td>
</tr>
<tr>
<td></td>
<td>Maximum Acceleration (g)</td>
<td>0.681</td>
<td>0.591</td>
</tr>
<tr>
<td></td>
<td>Required force for ATMD (ton)</td>
<td>212.3</td>
<td>149.87</td>
</tr>
<tr>
<td>$R = 10^{-4}, Q = \text{diag}[1 \ 0 \ 0]$</td>
<td>Maximum displacement (m)</td>
<td>0.3365</td>
<td>0.2128</td>
</tr>
<tr>
<td></td>
<td>Maximum Acceleration (g)</td>
<td>0.680</td>
<td>0.682</td>
</tr>
<tr>
<td></td>
<td>Required force for ATMD (ton)</td>
<td>218.6</td>
<td>173.57</td>
</tr>
</tbody>
</table>

Figure 3 and Figure 4 show the displacement time history of each floor under Northridge and El-Centro records, respectively. In addition, the PTMD-equipped shear building has an insignificant reduction in response due to incompatibility between the dominant frequency of structure and records, leading to substantial increase in higher modes of the structure response.

As it is shown, usage of PTMD leads to 22.1% and 10% reduction in top-story maximum displacement; however, ATMD decreases the maximum displacement response, 79.7% and 72.1% under Northridge and El-Centro records, respectively. It is concluded that by making any changes in the values of the weighting matrices, significant changes occur in the results. Displacement time history responses of the eight-story shear building under Northridge and El-Centro records indicate that each story of the shear building follows similar time history trend response. The values of story shear and drift have been indicated in Figure 5 under Northridge and El-Centro excitations. It is noted that implementation of ATMD decreases the story displacement response considerably; however, due to lack of compatibility between structures’ dominant frequency under El-Centro record, the effect of PTMD on structure responses is significantly limited. It is observed that by increasing the value of $R$, the control forces imposed by ATMD decrease and stories’ displacements increase. Hence, for implementation of ATMD a trade-off between control forces and appropriate response reduction should be considered. It is significant to note that PTMD displacement is considerably higher than that of the ATMD in order to limit story displacement (Figure 6).
Figure 3. Time-history story responses under Northridge ground motion.
Figure 4. Time-history story responses under El-Centro ground motion.

Figure 5. Story shear and story drifts under a) Northridge b) El-Centro excitations.

Figure 6. ATMD and PTMD displacement time history under a) Northridge b) El-Centro excitations.
It can be seen that the eight-story shear building equipped with ATMD shows highly limited story drift and story shear regarding the studied strong ground motions; however, implementation of PTMD leads to slight reduction in story drift and shear. In addition, it has been shown that tuned mass dampers whether passive or active, could have substantial effect on the relative displacements compared with absolute accelerations. The absolute acceleration is obtained based on the summation of relative acceleration and ground acceleration.

It is significant to note that structure equipped with PTMD follows a similar trend to that of the uncontrolled one with respect to displacement time history; however, ATMD equipped structure shows an approximately different trend compared to uncontrolled case. In addition, the ratio of the maximum movement amplitude of ATMD to that of PTMD are approximately 10% and 17% under Northridge and El-Centro excitations indicating less needed space to move for ATMD device. Table 3 shows the maximum amplitude of the tuned mass dampers’ movement and the percentage of the maximum movement amplitude of ATMD to that of the PTMD.

<table>
<thead>
<tr>
<th>Amp. (cm)</th>
<th>TMD</th>
<th>ATMD</th>
<th>Movement Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northridge</td>
<td>89.2</td>
<td>9.7</td>
<td>10%</td>
</tr>
<tr>
<td>El-Centro</td>
<td>61.9</td>
<td>10.5</td>
<td>17%</td>
</tr>
</tbody>
</table>

### 5. Conclusions

In this study an eight-story shear building controlled actively and passively has been investigated and compared with uncontrolled case. Active tuned mass damper, despite the costs, shows a significant effect on reduction of the responses, especially in story displacement. In addition, PTMD could be substantially efficient provided that dominant frequency of the excitation would be approximately close to fundamental frequency of the building. Therefore, the human comfort modes would not be considered, since it is highly influenced by higher modes of the PTMD equipped structure. It is concluded that utilizing ATMD leads to appropriate response reduction, regardless of the compatibility of records dominant frequency and structures natural frequency. Although ATMD is commonly used for the purpose of reducing story displacement, this device could have a noticeable impact on story shear response as well. Furthermore, it is observed that by reducing the weighting matrix of R, the story displacement time history response has been increased, and the configuration of Q=diag [1 1 0 0] would lead to more efficient displacement response control.

The future applications of active control methods is in need of significant reduction in cost while passive control methods could be optimized for improving the efficiency [26, 27, 28].

### Acknowledgements

This material is based upon the work initially developed by the Sharif University of Technology, Tehran, Iran.

### References


