Journal of Electrical and Electronic Engineering
2017; 5(6): 215-227
http://www.sciencepublishinggroup.com/j/jeee
doi: 10.11648/j.jeee.20170506.12
ISSN: 2329-1613 (Print); ISSN: 2329-1605 (Online)

Design of Novel Fixed-time Disturbance Observer and Improved Sliding Mode Controller for PMSM

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To cite this article:
Hang Yang, Junkang Ni, Cheng Zhang. Design of Novel Fixed-time Disturbance Observer and Improved Sliding Mode Controller for PMSM. *Journal of Electrical and Electronic Engineering*. Vol. 5, No. 6, 2017, pp. 215-227. doi: 10.11648/j.jeee.20170506.12

Received: November 6, 2017; **Accepted**: December 13, 2017; **Published**: December 28, 2017

Abstract: This paper presents a novel fixed-time disturbance observer (FTDOB) and an improved sliding mode controller (IPSMC) with saturation function of adaptive variable fractional power times for the permanent magnet synchronous motor (PMSM) to solve the problems that PMSM are susceptible to parameter uncertainties and disturbances of external loads. The proposed FTDOB utilizes the uniform convergent part to drive the error trajectories into a compact set and then switches to the finite time convergent part to achieve exact convergence. It can achieve exact disturbance estimation within finite time independent of initial estimation error. In addition, the improved sliding mode control law for speed controller is developed through combining the rotate speed and q-axis stator current's second-order model. A saturation function of adaptive variable fractional power times is designed to realize the coordinated control between chattering and tracking accuracy. Numerical simulations are provided to demonstrate that IPSMC+FTDOB is robust to the parameter perturbation, model errors and external disturbances, and it can effectively weaken the chattering and ensure the steady speed precision of PMSM.

Keywords: Fixed-Time, Disturbance Observer, Sliding Mode Controller, Saturation Function, PMSM, Chattering

1. Introduction

PMSM has attracted more and more attention in the fields of robotics, aerospace and numerical control system because of its small size, simple structure, high torque to inertia ratio, good reliability and servo performance. However, PMSM is a complex nonlinear system. In the process of operation, the parameters will change and there are uncertainties and serious external disturbances which are the main factors that leading the descent of control system's performance. Disturbances widely exist in various systems and result in system instability, degrade their control performance. Thus, disturbance rejection plays a crucial and important role in design of control system. A feedforward compensation strategy can be used to eliminate the bad influence of the disturbance if the disturbance is measurable. However, for most cases, disturbance is unavoidable, unmeasurable or too expensive and complex to be measured. One feasible method to solve this problem is try to estimate the disturbance and design controller to compensate the disturbance [1].

Disturbance estimation motivates the development of various disturbance observers. Disturbance observer has been widely used in the motion control system such as servo motor control, robot control and pressure control because of its simple structure, less computation and independence on the accurate model. The nature of the disturbance observer proposed in [2] is equivalent to an acceleration negative feedback loop between the speed loop and the current loop, then disturbance can be rejected. [3] presented a generalized extended state observer to produce an estimate of the mismatched disturbances in nonintegral-chain systems by taking the external disturbance as an extended state. Uncertainty and disturbance estimator was designed in [4] to provide an estimate for disturbances and uncertainties under the assumption that the disturbances can be approximated using a filter with right bandwidth. Unknown input observer was utilized in [5] to simultaneously estimate disturbances. A finite time disturbance observer using high order sliding mode differentiator was proposed in [6] to estimate disturbance. However, disturbance cannot be eliminated within finite time because observers in [3], [4] and [5] are asymptotically convergent and they can only ensure the estimation errors converge to a bounded region, i.e., exact disturbance estimation cannot be guaranteed. What's more, the estimation time of finite time disturbance observer in [6] is dependent on initial estimation error. Fixed-time stability [7] is an extension of finite time stability. Fixed-time stability means that system stabilization can be achieved within a limited time independent of initial estimation error compared with finite time stability. Fixed-time stability has been applied to design differentiator [8]-[9], controller [10] and address network consensus problem [11]-[15] because of this attractive feature. It is worth saying that there are few literatures reporting the application of fixed-time stability into design of disturbance observer.

The sliding mode control has become a hot topic in the field of domestic and foreign scholars because of its strong robustness to parameter variation and external disturbance of PMSM system. According to the current state of system such as the error and its derivative, the sliding mode surface is constructed and the system is forced to make a small amplitude, high frequency sliding along the prescribed state trajectory. This control method is simple and can be applied to the control of PMSM effectively. In order to improve the performance of the system state reaching the sliding mode surface, the idea of reaching law was put forward by academician Weibing Gao. At present, the common reaching laws include constant reaching law, exponential reaching law and power reaching law. Variable rate reaching law has been proposed in some papers. An adaptive parameter tuning law in [16] was designed to adjust the gain of the switching function based on sliding mode surface and switching factor can be adjusted in real time. Thus, the dynamic performance of the system in the arrival process is optimized and the chattering is reduced.

Chattering problem has been one of the difficult points in sliding mode control. At present, the method of weakening chattering is Quasi sliding mode control [17], Neural sliding mode control [18], Fuzzy sliding mode control [19], Disturbance Observer [20], etc. A sliding mode variable structure observer was introduced by [21]-[22], the switching function is discontinuous, which can cause chattering. In order to reduce chattering, saturation function, Sigmoid function and hyperbolic tangent function were used in [23]-[24] to replace the switching function. However, it is difficult to meet the requirement of chattering and precision when the traditional fixed saturation function is used. At the same time, there are some problems in other methods, such as the difficulty of parameter matching, delay of response and difficulty of determining rules.

Motivated by aforementioned discussion, this paper presents a FTDOB and an IPSMC for PMSM. The main contributions of this paper are twofold. First, with rigorous mathematical derivation and precise logic discussion, the upper bound of the estimation time only depends on design parameters, which means that the proposed FTDOB can provide faster disturbance estimation within finite time independent of initial estimation error. Second, a novel saturation function of adaptive variable fractional power times is proposed to replace the sign function in the sliding mode control law. The high frequency chattering can be effectively reduced by this novel saturation function and dynamic response of system can be improved. The FTDOB and IPSMC can further improve the anti-disturbance performance of PMSM and the simulation results demonstrate the effectiveness of them.

The rest of this paper is organized as follows. Section II formulates the problem and presents some definitions and lemmas. Main results of this paper are presented in Section III and simulation results verifying the effectiveness of proposed FTDOB and IPSMC are given in Section IV. Finally, the conclusion is drawn in Section V.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation

Consider the following n-th order single input single output disturbed nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + d_1 \\ \quad \cdots \\ \dot{x}_{n-1} = x_n + d_{n-1} \\ \dot{x}_n = f(x) + d_n \\ y_O = x_1 \end{cases} \tag{1}$$

where $x_i\ (i = 1, 2, \ldots, n)$ are state variables; y_O is system output; $f(x)$ is known smooth nonlinear function. The system has disturbances $d_i\ (i = 1, 2, \ldots, n)$ in all channels and the disturbances are unknown and unmeasurable. System (1) is a disturbed Brunovsky system and the disturbances are supposed to satisfy the following assumption:

Assumption 1: The disturbances $d_i\ (i = 1, 2, \ldots, n)$ in system (1) satisfy $d_i^{(n)} = \sum_{j=0}^{n-1} b_{(n-j)i} d_i^{(j)}$, where n is a positive integer and $b_{(n-j)i}$ is a real number.

Remark 1: A wide variety of disturbances, such as periodic disturbances and exponential disturbances, satisfy Assumption 1. Many practical systems, such as joint manipulator, maglev suspension system and PMSM, have the same form as system (1) or can be transformed into system (1) by using feedback exact linearization.

2.2. The Second-Order Model of PMSM

Assumption 2: The magnetic field is not saturated, the hysteresis and eddy current losses are not considered, and the distribution of space magnetic field is sinusoidal.

The ideal mathematical model of the surface-mounted PMSM in d-q coordinate is obtained [25]:

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -R_s/L_d & P_n\omega & 0 \\ -P_n\omega & -R_s/L_q & -P_n\psi_f/L_q \\ 0 & P_n\psi_f/J & -B/J \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} + \begin{bmatrix} u_d/L_d \\ u_q/L_q \\ -T_L/J \end{bmatrix} \quad (2)$$

where R_s is the stator resistance. P_n is the number of pole-pairs. u_d, u_q are the stator voltage of the motor system in d-q coordinate. L_d, L_q are the stator equivalent inductance of the motor system in *d-q* coordinate. For surface-mounted PMSM, $L_d=L_q=L$. ω is the rotor angular velocity of motor. ψ_f is the rotor flux corresponding to permanent magnet. B is the vicious coefficient. T_L is the load torque. J is the sum of the moment of inertia of motor and load.

It can be seen from (2) that there is a coupling between i_d and i_q, so it is not easy to realize the linear control of torque. In practical engineering, the control strategy of $i_d^*=0$ is often adopted and two linear PI controllers are used for current controllers. Thus, the approximate decoupling of PMSM can be achieved. Block diagram of PMSM speed regulation system is shown in Figure 1.

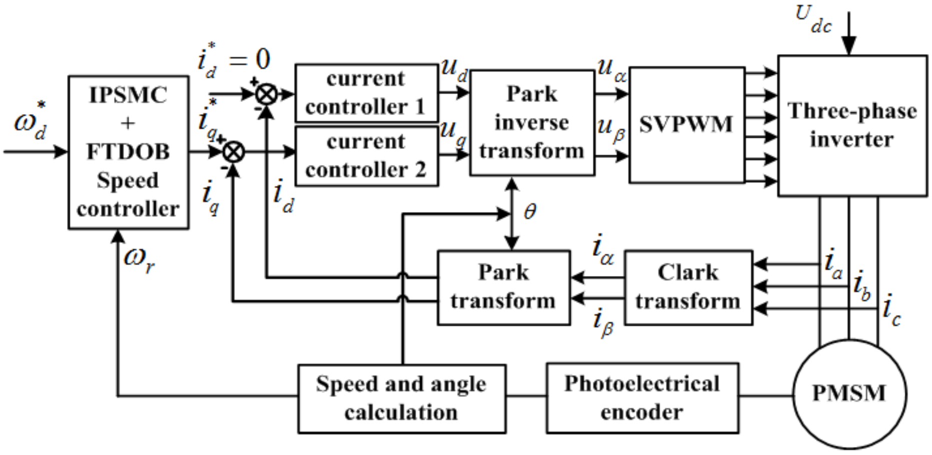

Figure 1. *Block diagram of PMSM speed regulation system based on IPSMC+FTDOB.*

The velocity equation of PMSM can be obtained by (2).

$$\dot{\omega} = \frac{(P_n\psi_f i_q - B\omega - T_L)}{J} = \frac{P_n\psi_f i_q}{J} + d(t) \quad (3)$$

where $d(t) = -(B\omega/J) - (T_L/J)$ represents parameter uncertainties and disturbances of external loads.

Equation (3) is a first-order differential model describing ω and i_q. The traditional sliding mode speed controller of PMSM speed regulation system uses the output of the speed controller i_q^* which is approximately equal to i_q as the input of the current controller 2. Thus, (3) is also the first-order model of ω and i_q^*. The current controller is controlled by the electrical signal, and the speed controller is affected by the mechanical parameters and inertia. Therefore, the response of current controller is faster than speed controller. When the difference between i_q^* and i_q is relatively large, the current controller can make a quick response. In this case, the error caused by $i_q^* \approx i_q$ can be ignored. However, in other cases, the motor speed and frequency are higher. Thus, the current controller and speed controller have shorter control period. At this moment, if the input error of the current controller is too large, it will take a long time to eliminate the error and the high dynamic response performance of closed-loop system will be reduced by $i_q^* \approx i_q$.

In order to obtain a more accurate mathematical model of ω and i_q^*, the second-order model of ω and i_q^* needs to be designed. (4) is transformed by applying Laplace transform.

$$s\omega(s) = \frac{P_n\psi_f}{J} i_q(s) + d(s) \quad (4)$$

where s is Laplace operator.

The transfer function can be obtained by current controller 2.

$$\frac{u_q(s)}{i_q^*(s) - i_q(s)} = k_p + \frac{k_i}{s} \quad (5)$$

where $u_q(s)$ is the output of current controller 2, k_p and k_i are the proportional gain and integral gain of current controller 2.

Therefore, (5) can be rewritten as

$$i_q(s) = i_q^*(s) - \frac{u_q(s)}{k_p + k_i/s} \quad (6)$$

Substituting (6) into (4), then

$$(s^2+\frac{k_i}{k_p}s)\omega(s)=\frac{P_n\psi_f}{J}(s+\frac{k_i}{k_p})i_q^*(s) -\frac{P_n\psi_f}{Jk_p}su_q(s)+(s+\frac{k_i}{k_p})d(s) \quad (7)$$

Equation (8) is defined as the output of IPSMC+FTDOB.

$$u(s)=\frac{P_n\psi_f}{J}(s+\frac{k_i}{k_p})i_q^*(s) \quad (8)$$

Thus, (6) can be rewritten as

$$i_q^*(s)=\frac{k_p}{(k_p s+k_i)}\frac{J}{P_n\psi_f}u(s) \quad (9)$$

Equation (7) can be rewritten as

$$(s^2+\frac{k_i}{k_p}s)\omega(s)=u(s)-\frac{P_n\psi_f}{Jk_p}su_q(s) +(s+\frac{k_i}{k_p})d(s) \quad (10)$$

The second-order model of PMSM can be obtained with inverse Laplace transform for (10) [25].

$$\ddot{\omega}=-\frac{Bk_p+Jk_i}{Jk_p}\dot{\omega}-\frac{Bk_i}{Jk_p}\omega-\frac{\dot{T}_L}{J}-\frac{k_iT_L}{Jk_p} -\frac{P_n\psi_f}{Jk_p}\dot{u}_q+u(t) \quad (11)$$

where $u(t)=\frac{P_n\psi_f}{J}\dot{i}_q^*+\frac{P_n\psi_f k_i}{Jk_p}i_q^*$.

Define state variables: $x_1=\omega_d^*-\omega_r=\Delta\omega$, $x_2=\dot{x}_1$ and set up $u(t)$ as control variable. Then, the state equation of the system is shown as below.

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-\frac{Bk_p+Jk_i}{Jk_p}x_2+\frac{Bk_i}{Jk_p}\omega+\frac{\dot{T}_L}{J} +\frac{k_iT_L}{Jk_p}+\frac{P_n\psi_f}{Jk_p}\dot{u}_q-u(t)\\ y_O=x_1\end{cases} \quad (12)$$

Consider the uncertainty of some parameters in (12).

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=(a+\Delta a)x_2+(b+\Delta b)\omega+d-u(t)\\ y_O=x_1\end{cases} \quad (13)$$

where Δa, Δb represent the uncertainties of the corresponding parameters and $a=-\frac{Bk_p+Jk_i}{Jk_p}$, $b=\frac{Bk_i}{Jk_p}$, $d=\frac{\dot{T}_L}{J}+\frac{k_iT_L}{Jk_p}+\frac{P_n\psi_f}{Jk_p}\dot{u}_q$.

*Re*mark 2: The initial condition of PMSM is assumed to be unknown and the problem is to design a FTDOB such that exact disturbance estimation can be guaranteed within finite time independent of initial condition.

2.3. Fixed-time Stability and Uniformly Finite-Time Stability

Consider the following differential equation system:

$$\dot{x}(t)=f(x(t)),x(0)=x_0 \quad (14)$$

where $x\in R$ and $f:R_+\times R^n\to R^n$ is a nonlinear function. Suppose that the origin is an equilibrium point of (14).

Definition 1: [12] The origin of system (14) is a finite time stable equilibrium if the origin is Lyapunov stable and there exists a function *T*: $R^n\to R^+$, called the settling time function, such that for every $x_0\in R^n$, the solution $x(t,x_0)$ of system (14) is defined on $[0,T(x_0))$, $x(t,x_0)\in R^n$, for all $t\in[0,T(x_0))$, and $\lim_{t\to T(x_0)}x(t,x_0)=0$.

Definition 2: [7] The origin of system (14) is said to be fixed-time stable equilibrium point if it is globally finite-time stable with bounded convergence time $T(x_0)$, i.e., there exists a bounded positive constant $T_{\max}$ such that $T(x_0)<T_{\max}$ satisfies.

Definition 3: [9] Consider the following system:

$$\dot{\tilde{x}}=f(\tilde{x}),\tilde{x}(0)=\tilde{x}_0 \quad (15)$$

where $\tilde{x}\in R$ and $f(\tilde{x})$ is a nonlinear function.

System (15) is said to be uniformly finite time convergent w.r.t initial value if there exist positive constants T and r such that for all $\tilde{x}_0\in R^n$, $\|\tilde{x}(t)\|\le r$ holds for all $t\ge T$. If the upper bound *T* can be explicitly indicated, the system (15) is called fixed-time convergent to the compact set $W=\{\tilde{x}(t):\|\tilde{x}(t)\|\le r,r>0\}$.

2.4. Homogeneity Property

Definition 4: [27] Let $r=(r_1,\ldots,r_n)$ be a generalized weight vector with $r_i>0$. The dilation associated to the weight vector r is: $\Lambda_r:(x_1,x_2,\ldots,x_n)\mapsto(\lambda^{r_1}x_1,\ldots,\lambda^{r_n}x_n)$ for $\lambda>0$. A vector field f is said to be a homogeneous function of degree m with respect to a generalized weight r iff for all $x\in R^n$ and $\lambda>0$, we have $\lambda^{-m}\Lambda_r^{-1}f(\Lambda_r,x)=f(x)$.

Homogeneity property can be used to obtain finite time stability property and uniform convergence property. Finite time convergence means that exact convergence can be achieved within finite time. The notion of homogeneity can be used to obtain finite time stability property as follows:

Lemma 1: [28] If $f: R^n \to R^n$ is a homogeneous vector field of degree $k<0$ and is locally attractive, then f is globally finite-time stable (FTS).

Uniform convergence property means that for any initial condition, the convergence time is uniformly bounded by a constant. Based on homogeneity property, uniform finite time convergence property can be obtained as:

Lemma 2: [9] System (15) is uniformly finite time convergent w.r.t. initial value $\tilde{x}_0$ if (i) its origin is globally asymptotically stable; (ii) f is a continuous homogeneous vector field of degree $m>0$.

2.5. Other Mathematical Lemmas

Lemma 3: [29] Consider the product space $X \times Y$, where Y is compact. If an open set N contains the slice $\{x_0\} \times Y$ of $X \times Y$, it also contains some tube $W \times Y$ about $\{x_0\} \times Y$ and W is a neighbor of x_0 in X.

Lemma 4: [30] If continuous real-value functions V_1 and V_2 are homogeneous with respect to v of degree $l_1>0$ and $l_2>0$, and V_1 is positive definite, the following inequality holds for every $x \in R^n$:

$$[\min_{\{z:V_1(z)=1\}} V_2(z)][V_1(x)]^{\frac{l_2}{l_1}} \le V_2(x) \le [\max_{\{z:V_1(z)=1\}} V_2(z)][V_1(x)]^{\frac{l_2}{l_1}} \quad (16)$$

3. Main Results

3.1. Design of Fixed-Time Disturbance Observer (FTDOB)

In this subsection, FTDOB will be presented to achieve exact estimation for the disturbances in system (1) within finite time independent of initial estimation error. The basic idea of disturbance observer is that actual object caused by external torque disturbance or the change of model parameters and output error of nominal model are equivalent to the input port of the speed controller. And then, the corresponding compensation is introduced in the control to achieve the complete rejection of the disturbance.

For system (1) with disturbances satisfying Assumption 1, define $v_{0i}=x_i$, $v_{1i}=d_i$, $v_{2i}=\dot{d}_i, \ldots, v_{ni}=d_i^{(n-1)}$ and we have $\dot{v}_{0i}=v_{1i}+h_i$, $\dot{v}_{1i}=v_{2i}, \ldots, \dot{v}_{ni}=\sum_{j=0}^{n-1} b_{(n-j)i} d_i^{(j)}=\sum_{j=0}^{n-1} b_{(n-j)i} v_{(j+1)i}$.

where $h_i=x_{i+1}$ for $i=1,2,\ldots,n-1$ and $h_n=f(x)$, b_{ij} is the coefficient of the dynamics that the disturbance d_i should satisfy in Assumption 1. Introduce intermediate variables $z_{0i}=v_{0i}$, $z_{1i}=v_{1i}-b_{1i}v_{0i}, \ldots, z_{ni}=v_{ni}-\sum_{j=0}^{n-1} b_{(n-j)i} v_{ji}$ and we have:

$$\begin{aligned} \dot{z}_{0i} &= z_{1i}+b_{1i}z_{0i}+h_i \\ \dot{z}_{ji} &= z_{(j+1)i}+b_{(j+1)i}z_{0i}-b_{ji}h_i \\ \dot{z}_{ni} &= -b_{ni}h_i \end{aligned} \quad (17)$$

The proposed FTDOB has the following form:

$$\begin{aligned} \dot{\hat{z}}_{0i} &= \hat{z}_{1i}+b_{1i}z_{0i}+h_i+k_{0i}\theta sig^{\alpha_0}(z_{0i}-\hat{z}_{0i}) \\ &\quad +k_{0i}(1-\theta)sig^{\beta_0}(z_{0i}-\hat{z}_{0i}) \\ \dot{\hat{z}}_{ji} &= \hat{z}_{(j+1)i}+b_{(j+1)i}z_{0i}-b_{ji}h_i+k_{ji}\theta sig^{\alpha_j}(z_{0i}-\hat{z}_{0i}) \\ &\quad +k_{ji}(1-\theta)sig^{\beta_j}(z_{0i}-\hat{z}_{0i}) \\ \dot{\hat{z}}_{ni} &= -b_{ni}h_i+k_{ni}\theta sig^{\alpha_n}(z_{0i}-\hat{z}_{0i}) \\ &\quad +k_{ni}(1-\theta)sig^{\beta_n}(z_{0i}-\hat{z}_{0i}) \end{aligned} \quad (18)$$

where $i=1,2,\ldots,n$, $j=1,2,\ldots,n-1$, $\hat{z}_{0i},\hat{z}_{1i},\cdots,\hat{z}_{ni}$ are estimation for $z_{0i},z_{1i},\cdots,z_{ni}$, $sig^{\alpha}(\cdot)=|\cdot|^{\alpha} sign(\cdot)$, α_i, β_i, k_{ji} and θ are observer parameters and they can be selected to satisfy the following conditions:

1) The exponents $\alpha_i=(i+1)\alpha-i$, $\beta_i=(i+1)\beta-i$, where $\alpha \in (1-\varepsilon_1,1)$ and $\beta \in (1,1+\varepsilon_2)$ with ε_1 and ε_2 being small positive real numbers.

2) The observer coefficients $k_{ji}(j=0,1,\ldots,n)$ are assigned such that the following matrix is Hurwitz:

$$A=\begin{bmatrix} -k_{0i} & 1 & 0 & \cdots & 0 \\ -k_{1i} & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -k_{ni} & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (19)$$

The function $\theta:[0,\infty) \to \{0,1\}$ is selected as:

$$\theta(t)=\begin{cases} 0 & \text{if } t \le T_u \\ 1 & \text{if } t > T_u \end{cases}$$

where T_u is switching time, a design parameter that is typically selected through trial and error.

Using the observer (18), the disturbance can be estimated as:

$$\begin{aligned} \hat{d}_i &= \hat{z}_{1i}+\Delta_{1i}\hat{z}_{0i} \\ \dot{\hat{d}}_i &= \hat{z}_{2i}+\Delta_{1i}\hat{z}_{1i}+\Delta_{2i}\hat{z}_{0i} \\ \ddot{\hat{d}}_i &= \hat{z}_{3i}+\Delta_{1i}\hat{z}_{2i}+\Delta_{2i}\hat{z}_{1i}+\Delta_{3i}\hat{z}_{0i} \\ &\cdots \\ \hat{d}_i^{(n-1)} &= \hat{z}_{ni}+\Delta_{1i}\hat{z}_{(n\text{-}1)i}+\Delta_{2i}\hat{z}_{(n\text{-}2)i}+\ldots+\Delta_{ni}\hat{z}_{0i} \end{aligned} \quad (20)$$

where $\Delta_{1i}=b_{1i}, \Delta_{2i}=b_{1i}\Delta_{1i}+b_{2i}, \ldots, \Delta_{ni}=b_{1i}\Delta_{(n-1),i}+$ $=b_{1i}\Delta_{(n-1),i}+b_{2i}\Delta_{(n-2),i}+\cdots+b_{ni}$, $\hat{d}_i, \dot{\hat{d}}_i, \ldots, \hat{d}_i^{(n-1)}$ are estimation values for disturbance d_i and its derivatives.

Define the estimation error $\sigma_{ji} = z_{ji} - \hat{z}_{ji}$ and the dynamics of the estimation error can be expressed as:

$$\begin{aligned}\dot{\sigma}_{0i} &= \sigma_{1i} - k_{0i}\theta sig^{\alpha_0}(\sigma_{0i}) - k_{0i}(1-\theta)sig^{\beta_0}(\sigma_{0i}) \\ \dot{\sigma}_{ji} &= \sigma_{(j+1)i} - k_{ji}\theta sig^{\alpha_j}(\sigma_{0i}) - k_{ji}(1-\theta)sig^{\beta_j}(\sigma_{0i}) \\ \dot{\sigma}_{ni} &= -k_{ni}\theta sig^{\alpha_n}(\sigma_{0i}) - k_{ni}(1-\theta)sig^{\beta_n}(\sigma_{0i})\end{aligned} \tag{21}$$

For $t \le T_u$, $\theta = 0$, system (21) becomes:

$$\begin{aligned}\dot{\sigma}_{0i} &= \sigma_{1i} - k_{0i}sig^{\beta_0}(\sigma_{0i}) \\ \dot{\sigma}_{ji} &= \sigma_{(j+1)i} - k_{ji}sig^{\beta_j}(\sigma_{0i}) \\ \dot{\sigma}_{ni} &= -k_{ni}sig^{\beta_n}(\sigma_{0i})\end{aligned} \tag{22}$$

Select a Lyapunov function $V(\beta,\sigma_i) = \xi_1^T(\sigma_i)P\xi_1(\sigma_i)$ where $\xi_1(\sigma_i) = [\sigma_{0i}, \sigma_{1i}^{\frac{1}{\beta_0}}, \cdots, \sigma_{ni}^{\frac{1}{\beta_{n-1}}}]^T$ and P is symmetric positive definite matrix satisfying $A^T P + PA = -Q$ with Q a positive definite matrix. If $\beta = 1$, error system (22) becomes $\dot{\sigma}_i = A\sigma_i$ where $\sigma_i = [\sigma_{0i}, \sigma_{1i}, \cdots, \sigma_{ni}]^T$. Since the matrix A is Hurwitz, error state σ_i is asymptotically stable. Since V is proper, $S = \{\sigma_i \in R^{n+1} : V(1,\sigma_i) = 1\}$ is a compact set. Since $M = \{(\beta,\sigma_i)\}$ is an open set containing the slice $\{1\}\times S$ and S is a compact set, it follows from *Lemma 3* that M contains some tube $(1-\varepsilon_1, 1+\varepsilon_2)\times S$ around $\{1\}\times S$. Since $\dot{V}(\beta,\sigma_i)$ is a continuous in β and σ_i, $\dot{V}(\beta,\sigma_i)$ is uniformly continuous in the set $W = \{(\beta,\sigma_i) \in R\times R^{n+1} \mid \beta = 1, \sigma_i \in S\}$. The time derivative of $V(1,\sigma_i)$ satisfies:

$$\dot{V}(1,\sigma_i) = \sigma_i^T(A^T P + PA)\sigma_i = -\sigma_i^T Q\sigma_i < 0 \tag{23}$$

$\dot{V}(1,\sigma_{\rm i})=0$ if $\sigma_{\rm i} = 0$. When $\sigma_{\rm i} = 0$, we have $\dot{\sigma}_{\rm i} = 0$ for system (22). This means that there exists a small constant ε_2 such that for all $\beta \in (1, 1+\varepsilon_2)$, $\dot{V}(1,\sigma_i) < 0$ also implies $\dot{V}(\beta,\sigma_i) < 0$. Therefore, $\dot{V}(\beta,\sigma_i)$ is a Lyapunov function of system (23) and the error system (22) is asymptotically stable. Further, the right hand side of the system (22) is homogeneous of degree $m = \beta - 1 > 0$ with respect to weights $r_i = i\beta - (i-1)$, which implies the system (22) is uniformly finite time convergent.

It is easy to prove that the Lyapunov function $V(\beta,\sigma_i)$ is homogeneous of degree $l_1 = 2 > \max(-m, 0)$ and its derivative $\dot{V}(\beta,\sigma_i)$ is homogeneous of degree $l_2 = l_1 + m = 1 + \beta$ with respect to the same weights $r_i = i\beta - (i-1)$. Therefore, according to *Lemma 4*, the following inequality holds:

$$\dot{V}(\beta,\sigma_i) \le -\kappa_1(\beta,\sigma_i)(V(\beta,\sigma_i))^{\frac{\beta+1}{2}} \tag{24}$$

Since $\lim_{\beta\to 1}\kappa_1(\beta,\sigma_i) = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$ and the right hand side of system (22) is continuous with respect to β, there exists a small constant $\varepsilon_2 > 0$ such that for $\beta \in (1, 1+\varepsilon_2)$, $\kappa_1(\beta,\sigma_i) \ge \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}$ holds and we have:

$$\dot{V}(\beta,\sigma_i) \le -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}(V(\beta,\sigma_i))^{\frac{\beta+1}{2}} \tag{25}$$

When $t = T_u$, the Lyapunov function $V(\beta,\sigma_i)$ satisfies:

$$\begin{aligned}V(T_u) &\le \left(\frac{\beta-1}{4}\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}T_u + V(0)^{\frac{1-\beta}{2}}\right)^{\frac{2}{1-\beta}} \\ &< \left(\frac{\beta-1}{4}\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}T_u\right)^{\frac{2}{1-\beta}}\end{aligned} \tag{26}$$

For $t > T_u$, $\theta = 1$, system (21) becomes:

$$\begin{aligned}\dot{\sigma}_{0i} &= \sigma_{1i} - k_{0i}sig^{\alpha_0}(\sigma_{0i}) \\ \dot{\sigma}_{ji} &= \sigma_{(j+1)i} - k_{ji}sig^{\alpha_j}(\sigma_{0i}) \\ \dot{\sigma}_{ni} &= -k_{ni}sig^{\alpha_n}(\sigma_{0i})\end{aligned} \tag{27}$$

A Lyapunov function is selected $V(\alpha,\sigma_i) = \xi_2^T(\sigma_i)P\xi_2(\sigma_i)$, where $\xi_2(\sigma_i) = [\sigma_{0i}, \sigma_{1i}^{\frac{1}{\alpha_0}}, \cdots, \sigma_{ni}^{\frac{1}{\alpha_{n-1}}}]^T$. Similarly, the system (27) is finite time convergent can be proved and the time derivative of Lyapunov function $V(\alpha,\sigma_i)$ satisfies:

$$\dot{V}(\alpha,\sigma_i) \le -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}(V(\alpha,\sigma_i))^{\frac{\alpha+1}{2}} \tag{28}$$

Since $\alpha < 1$, the Lyapunov function (28) with the initial condition $V(T_u)$ will converge to zero within finite time. After the convergence of error system (21), the observer (18) can give exact disturbance estimations and the disturbances and its derivative can be expressed as:

$$\begin{aligned}d_i &= z_{1i} + \Delta_{1i}z_{0i} \\ \dot{d}_i &= z_{2i} + \Delta_{1i}z_{1i} + \Delta_{2i}z_{0i} \\ \ddot{d}_i &= z_{3i} + \Delta_{1i}z_{2i} + \Delta_{2i}z_{1i} + \Delta_{3i}z_{0i} \\ &\cdots \\ d_i^{(n-1)} &= z_{ni} + \Delta_{1i}z_{(n-1)i} + \Delta_{2i}z_{(n-2)i} + \ldots + \Delta_{ni}z_{0i}\end{aligned} \tag{29}$$

From (20) and (29), it can be seen that if the intermediate variable z_{ij} can be estimated precisely, the disturbance d_i and its derivative can be estimated accurately.

Remark 3: Different from the existing disturbance observers, even if the initial estimation error tends to infinity, the proposed FTDOB can achieve exact disturbance estimation within finite time independent of initial estimation error.

3.2. Design of Improved Sliding Mode Controller (IPSMC)

Rewrite (13) and consider the following second-order model of PMSM which satisfied *Assumption 1.*

$$\begin{cases} \dot{x}_1 = x_2 + D_1(t) \\ \dot{x}_2 = ax_2 + b(\omega_d^* - x_1) - u(t) + D_2(t) \\ y_O = x_1 \end{cases} \quad (30)$$

where the disturbances are supposed to be $D_1(t)$ and $D_2(t)$ which represent parameter uncertainties and model errors.

The control strategy of IPSMC+FTDOB speed controller is: the inputs of the speed controller are the reference speed ω_d^* and the actual rotor speed signal ω_r from the speed sensor. In the starting process of the motor system, only the IPSMC part is used as the speed controller. By a careful calculation and trial and error, e_c is selected as 0.2. When the speed error is reduced to $|15e_c|(e_c > 0)$, the proposed FTDOB is used to observe various kinds of disturbances which are unavoidable, unmeasurable or too expensive and complex to be measured caused by parameter perturbation, model errors and load. The disturbance estimation of the FTDOB is considered as compensation to eliminate the disturbance. Finally, through the calculation of sliding mode surface function and reaching law, i_q^* is calculated as the output of the speed controller.

According to IPSMC, the control law is usually composed of the equivalent control part $u_{eq}(t)$ and the switching control part $u_{sw}(t)$. The equivalent control part keeps the system state on the sliding mode surface. In the switching control part, the state of the system is forced to move on the sliding mode surface by high frequency switching and ensured to slide toward the stable point along the sliding line. Therefore, robust control for parameter uncertainties and disturbances of external loads is achieved. The sliding mode surface function can be selected as follows.

$$S = cx_1 + x_2 \quad (31)$$

where c is a constant. $x_1 = \omega_d^* - \omega_r = \Delta\omega$ and $x_2 = \dot{x}_1$.

The goal of the sliding mode control law is that the state of system is reached and maintained on the sliding mode surface in finite time. Variable structure control method using switching function has the following form:

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (32)$$

where $u_{eq}(t)$ is the equivalent control part of IPSMC when $\dot{S} = 0$ and $D_1(t) = D_2(t) = 0$. $u_{sw}(t)$ is the switching control part of IPSMC. According to the condition of sliding mode equivalent control, i.e., (33):

$$\dot{S} = c\dot{x}_1 + \dot{x}_2 = 0 \quad (33)$$

Sliding mode equivalent control part is designed as:

$$u_{eq}(t) = (a + c)x_2 + b(\omega_d^* - x_1) \quad (34)$$

Usually, there are many factors such as inertia and time delay in the actual system. Due to the discontinuity of the switching function, when the moving point reaches the sliding mode surface, the chattering appears near the sliding mode surface. Chattering not only affects the control accuracy of the controller, increases the loss, but also stimulates the high frequency unmodeled dynamics. Thus, it is necessary to eliminate the chattering. However, chattering must exist. The elimination of chattering means that the anti-disturbance ability of sliding mode control is lost.

Saturation function $sat(S)$ is used to replace the switching function $\operatorname{sgn}(S)$ to weaken the chattering, that is, the continuous feedback control is used in the boundary layer, and the normal switching control is adopted outside the boundary layer. The switching control part of IPSMC is designed as follow.

$$u_{sw}(t) = K \cdot sat(S) \quad (35)$$

where K represents switching control gain. $sat(S)$ represents the novel saturation function of adaptive variable fractional power times and it is a continuous linear function in the boundary layer, which can weaken the chattering by adjusting the size of the boundary layer δ. Its form is designed as follow.

$$\operatorname{sat}(S) = \begin{cases} \left(\dfrac{S}{\delta}\right)^{\frac{1}{n}} & |S| \le \delta \\ \operatorname{sgn}(S) & |S| > \delta \end{cases} \quad (36)$$

where $1/n$ is the adaptive variable fractional power series and n is a positive odd number. $\operatorname{sgn}(\cdot)$ is the sign function. $\delta > 0$ is the thickness of boundary layer. The value of saturation function is solved in the range of real number. δ is usually set in a range, i.e., $0.2 \le \delta \le 0.5$ through trial and error.

The switching function $\operatorname{sgn}(\cdot)$ is a discontinuous nonlinear function and the amount of switching is large in the vicinity of zero. Its form can be expressed as:

$$\operatorname{sgn}(S) = \begin{cases} 1 & S > 0 \\ 0 & S = 0 \\ -1 & S < 0 \end{cases} \quad (37)$$

When switching gain K is large, the control gain is changed greatly, and then the chattering is serious. For the saturation function in (36), when the sliding mode surface is constant and δ is small, the saturation function in the linear region has a large control gain, the response speed is fast and the control precision is higher, but the chattering is enhanced. When δ is large, the chattering is reduced, but the control gain is reduced, too. The response speed is slow and the overshoot and the steady-state error are large.

Remark 4: Similarly, when n is smaller, the chattering is reduced, but the control gain is reduced, too. The response speed is slow and the overshoot and the steady-state error are large. However, when n is larger, the saturation function in the linear region has a large control gain, the response is faster and the control precision is higher, but due to the same δ, the enhancement of chattering can be ignored.

In order to overcome the shortcoming that the saturation function of fixed boundary layer can not weaken the chattering and ensure the control precision at the same time, a novel saturation function of adaptive variable fractional power times is designed to replace the sign function. n is small at the beginning and increase the value of n when the error is reduced to a certain value $e_c\ (e_c > 0)$. By a careful calculation and trial and error, e_c is selected as 0.2. In this way, it not only weakens the chattering effectively, but also improves the control performance of the system. The variable fractional power series can be expressed as:

$$n = 2\gamma + 1 \tag{38}$$

where $\gamma = 0,1,2,3$ and it satisfies:

$$\gamma = \begin{cases} 0 & |e| > 3e_c \\ 1 & 2e_c < |e| \le 3e_c \\ 2 & e_c < |e| \le 2e_c \\ 3 & |e| \le e_c \end{cases} \tag{39}$$

The novel saturation function of adaptive variable fractional power times is shown in Figure 2.

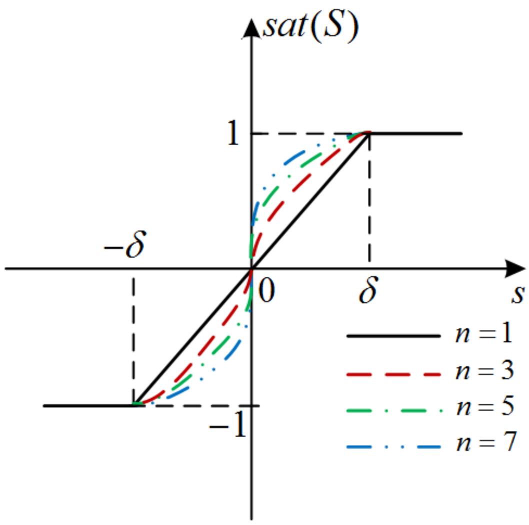

Figure 2. *Novel saturation function of adaptive variable fractional power times.*

In conclusion, the output of IPSMC is:

$$\begin{aligned} u(t) &= u_{eq}(t) + u_{sw}(t) \\ &= (a+c)\cdot x_2 + b\cdot(\omega_d^* - x_1) + K\cdot sat(S) \end{aligned} \tag{40}$$

In order to ensure the stability of the system, the Lyapunov function is chosen as

$$V = \frac{1}{2}S^2 \tag{41}$$

Proof: Assume $K \ge \varepsilon > |D(t)|$ and take the derivative of $V = \frac{1}{2}S^T S$, we have:

$$\begin{aligned} \dot{V} &= S\cdot\dot{S} \\ &= S\cdot(c\cdot\dot{x}_1 + \dot{x}_2) \\ &= S\cdot[(a+c)\cdot x_2 + b\cdot(\omega_d^* - x_1) - u(t) + c\cdot D_1(t) + D_2(t)] \\ &= S\cdot[-K\cdot sat(S) + D(t)] \\ &\le -K\cdot|S| + D(t)\cdot|S| \\ &= -[K - D(t)]\cdot|S| \\ &\le 0 \end{aligned} \tag{42}$$

where $D(t) = cD_1(t) + D_2(t)$.

The proof is completed. Thus, the existence and stability of IPSMC are guaranteed, i.e., when $S \ne 0$, $S\cdot\dot{S} < 0$. According to the above design process, the output of the IPSMC has both the continuous part and the switching part. The part of equivalent control keeps the state of the system on the sliding mode surface. The part of switching control compensates the estimation error of the equivalent control and force the system state to slide on the sliding mode surface.

Although this paper is aimed at the PMSM speed regulation system of multi closed loop structure, all the current loop, including the current controller and PMSM is considered as a generalized controlled object when design IPSMC. The generalized object can be considered as a second-order model.

Refer to the relationship between the output of IPSMC+FTDOB $u(t) = \frac{P_n\psi_f}{J}\dot{i}_q^* + \frac{P_n\psi_f k_i}{Jk_p} i_q^*$ and the output of the speed controller i_q^*, the control structure diagram of the second-order model of PMSM based on IPSMC+FTDOB can be simplified as shown in Figure 3.

Remark 5: As can be seen from Figure 3, the output of the speed controller, i.e., the input of the q-axis current i_q^* is obtained by the output of the IPSMC+FTDOB through a first order low-pass filter. The chattering has been filtered to some extent by the low-pass filter.

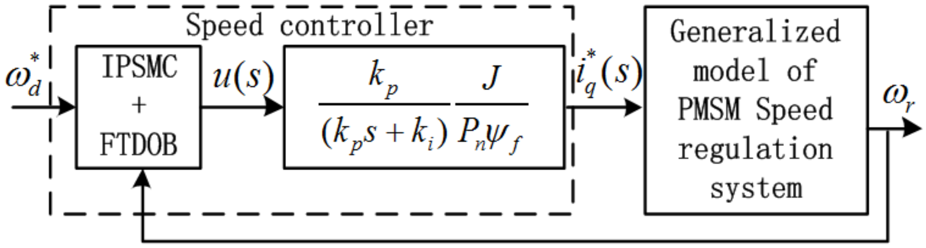

Figure 3. *Simplified control structure diagram of the second-order model of PMSM speed regulation system based on IPSMC+FTDOB.*

4. Simulation Results

A simulation model based on MATLAB is established and the vector control strategy of $i_d^*=0$ is adopted and two linear PI controllers are used for current controllers to realize the decoupling control of PMSM. The speed controller of the system is designed as IPSMC+FTDOB to realize the fast speed response. Put parameters in Appendix A plug into system (30) and initial condition is chosen as $(x_1(0),\ x_2(0))=(500,0)$. By a careful calculation and trial and error, the parameters for IPSMC+FTDOB speed controller are selected as $c=5$, $\delta=0.5$, $K=3.5$, $e_c=0.2$, $\varepsilon=3$, $n(0)=7$, $k_{01}=k_{02}=100$, $k_{11}=k_{12}=100$, $k_{21}=k_{22}=100$, $\alpha=0.75$, $\beta=1.5$, $T_u=0.1$. The proposed FTDOB is applied to estimate the disturbances in system (30). The initial value for the observer is selected as ($\hat{z}_{01}(0)$, $\hat{z}_{11}(0)$, $\hat{z}_{21}(0)$, $\hat{z}_{02}(0)$, $\hat{z}_{12}(0)$) = $(2,-0.8,-0.8,-2,-0.8)$.

Disturbances $D_1(t)=0.5\cos(t)$ and $D_2(t)=0.3\sin(t)$ are imposed to PMSM (30), which represent parameter uncertainties and model errors. Figure 4 presents the actual disturbances and their estimations. It is clear that the observer can give exact disturbances estimation within 2s.

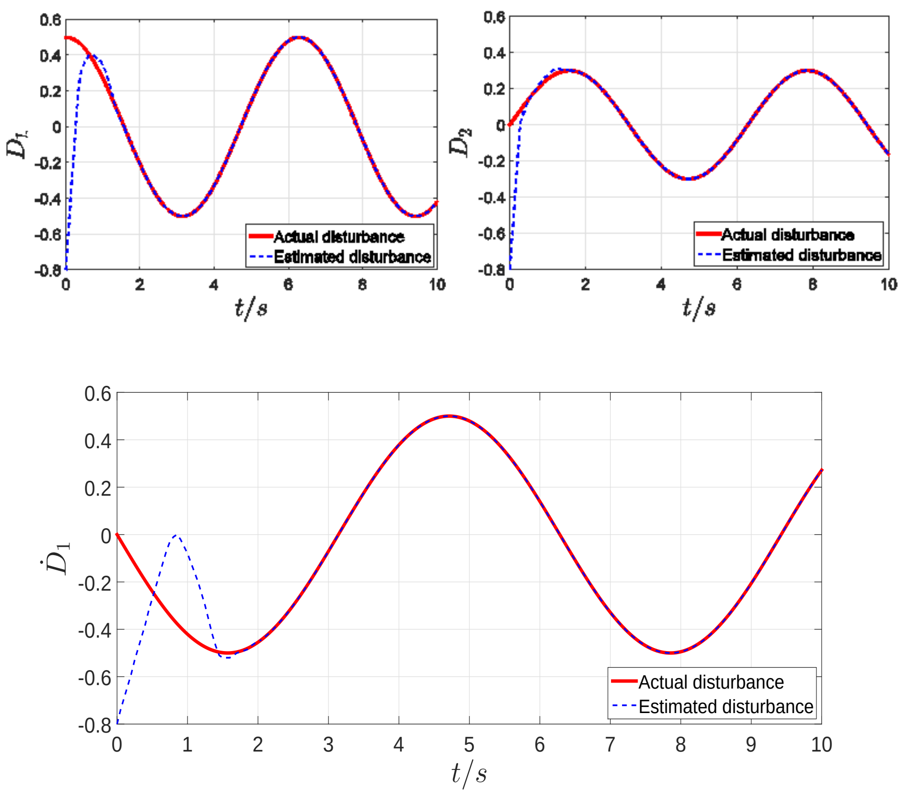

Figure 4. *Disturbance D_1, D_2, $\dot{D}_1$ and their estimated values.*

In order to objectively verify and compare the performance of the proposed algorithm, IPSMC+FTDOB, IPSMC and PID controller are used for the speed controller of PMSM speed regulation system. Parameters of these three controllers are selected based on the following principles. Firstly, the same part of IPSMC+FTDOB and IPSMC sets the same parameters. Secondly, the outputs of these speed controllers are in the same order of magnitude. The specific parameters of PMSM are described in Appendix.

Reference speed $\omega_d^* = 500\text{r/min}$ are set as the step signal. When the actual speed tends to reference speed and the fractional power times n=7, a sudden load torque $T_L = 2N \cdot m$ is increased to the PMSM at $0.1s$. When the fractional power times n=1, a sudden load torque $T_L = 2N \cdot m$ is increased to the PMSM at $0.14s$. When the fractional power times is variable, a sudden load torque $T_L = 2N \cdot m$ is increased to the PMSM at $0.1s$. Figure 5, Figure 6 and Figure 7 are the current chattering curves corresponding to different n of fractional power times.

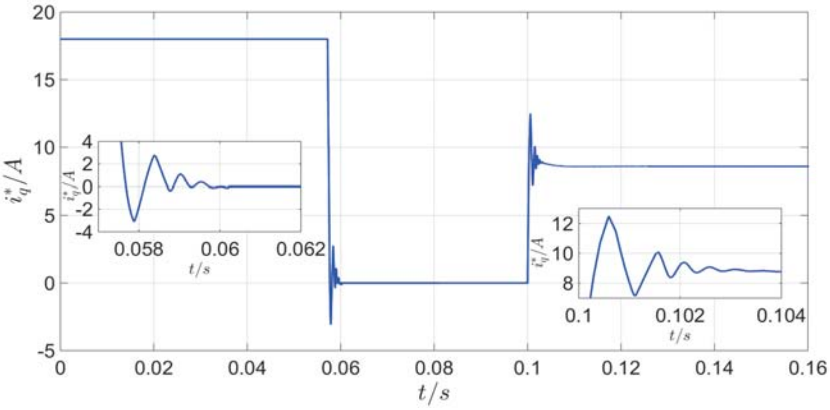

Figure 5. *Chattering curve of* i_q^* *under saturation function of fractional power times n=7.*

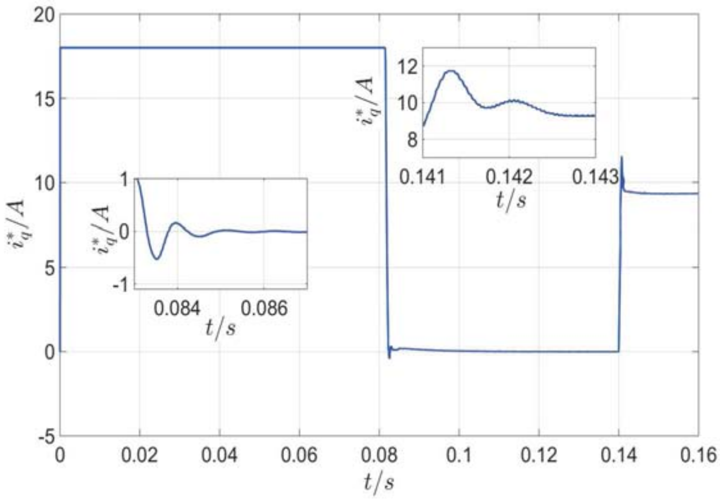

Figure 6. *Chattering curve of* i_q^* *under saturation function of fractional power times n=1.*

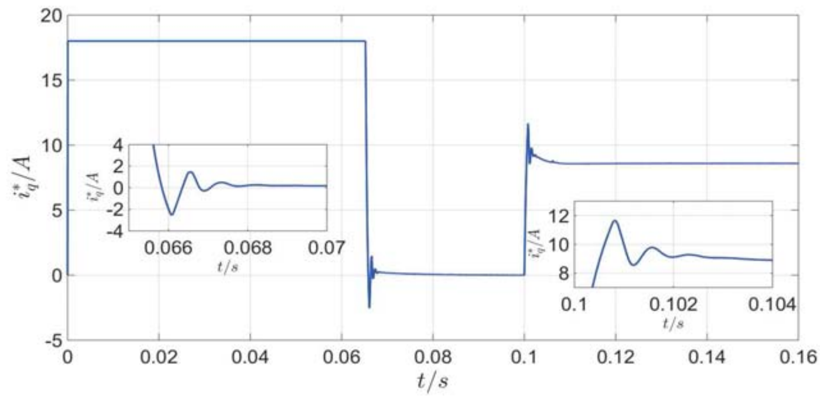

Figure 7. *Chattering curve of* i_q^* *under saturation function of adaptive variable fractional power times.*

As can be seen from Figure 5, when the number of power times of the saturation function is large (n=7), the chattering of current is large and i_q^* is stable after a violent oscillation. However, When the system is disturbed by external load, the actual speed can accurately track the reference speed, and the steady state error is small. As can be seen from Figure 6, when the number of power times of the saturation function is small (n=1), the chattering of current is small. When the system is disturbed by external load, the speed tracking error is large and can not be restored to the reference value. In Figure 7, the saturation function of adaptive variable fractional power times can not only weaken the chattering, but also realize stability and fast speed response.

When the actual speed tends to reference speed at 0.1s, a sudden load torque $T_L = 2\text{N}\cdot\text{m}$ is increased to the PMSM. At the same time, disturbances $D_1(t) = 0.5\cos(t)$ and $D_2(t) = 0.3\sin(t)$ are imposed to PMSM (30). Figure 8 presents the speed response curve of IPSMC+FTDOB and IPSMC. Figure 9 presents the speed response curve of IPSMC+FTDOB and PID controller.

As can be seen from Figure 8, under the control of IPSMC+FTDOB, the motor speed enters steady state at 0.057s in the start-up period. Under the control of IPSMC, it costs nearly 0.060s. Descent value of speed is $\Delta n_1 = 1.8\text{r/min}$ by using IPSMC+FTDOB and the recovery time of reference speed is less than $10ms$. Meanwhile, the value of speed decreases by $\Delta n_2 = 2.2\text{r/min}$ when using IPSMC and the recovery time of reference speed is about 10ms.

Remark 6: IPSMC+FTDOB and IPSMC both have the characteristics of small overshoot and small descent value of speed. They both can realize the fast dynamic response, accurate tracking, and almost no oscillation of speed in transient process. What's more, the performance of IPSMC+FTDOB is better than IPSMC. It is clear that the proposed FTDOB can observe the disturbance from external and model errors accurately.

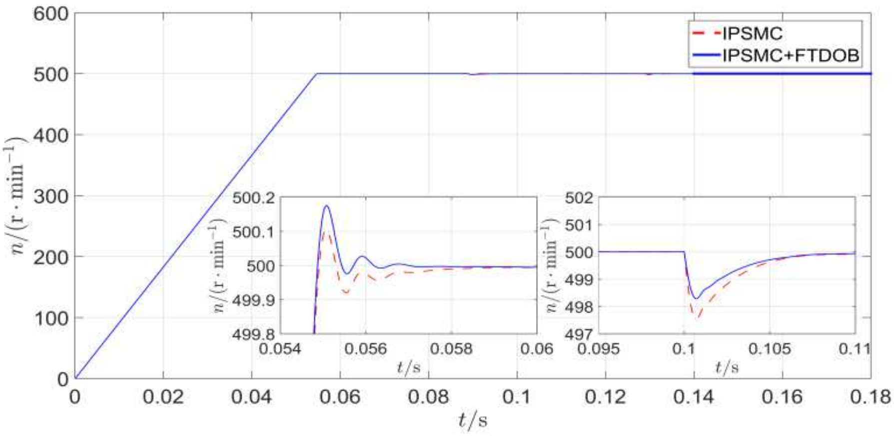

Figure 8. *Speed response curve of IPSMC+FTDOB and IPSMC.*

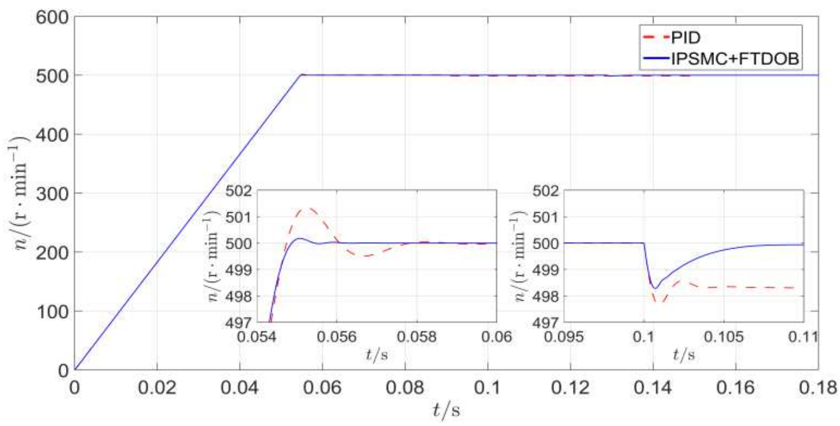

Figure 9. *Speed response curve of IPSMC+FTDOB and PID controller.*

It can be observed from Figure 9 that the overshoot and fluctuation of the speed is large, and the adjustment time is nearly 0.060s with PID controller. On the contrary, the simulation results prove that the IPSMC+FTDOB controller can make PMSM speed regulation system almost no overshoot, short adjustment time, fast response, the strong robust performance. Descent value of speed is $\Delta n_1 = 1.8\text{r/min}$ by using IPSMC+FTDOB controller and the recovery time of reference speed is less than 10ms. Meanwhile, the value of speed decreases by about $\Delta n_3 = 2.4\text{r/min}$ when using PID controller. Although the speed of PMSM controlled by PID controller will be stable to a certain value (498.2r/min), it is affected by parameter uncertainties and disturbances of external loads. Thus, its steady state error is large and can not recover to reference speed.

Remark 7: Numerical simulation results and comparison between IPSMC+FTDOB controller and PID controller show that the proposed IPSMC+FTDOB achieves smaller overshoot and faster response, and that the steady-state error is near to zero. Moreover, the IPSMC+FTDOB has strong robustness in parameter perturbation and disturbances of external loads, and ensures higher steady-state precision of speed with lower chattering.

5. Conclusion

A novel FTDOB and an IPSMC for PMSM are proposed in this paper to solve the problem of parameter uncertainties, model errors and disturbances of external loads. The stability proofs are given respectively. The main contributions of this paper are twofold. First, the proposed FTDOB combines uniform convergent disturbance observer and finite time convergent disturbance observer to achieve exact disturbance estimation within finite time independent of initial estimation error. Second, a novel saturation function of adaptive variable fractional power times is proposed to replace the sign function in sliding mode control law. The high frequency chattering can be effectively weakened by this proposed saturation function. To summarize, IPSMC+FTDOB is robust to the parameter perturbation, model errors and external disturbances, and it can effectively weaken the chattering and ensure the steady speed precision of PMSM speed regulation system.

Appendix

The parameters of permanent magnet synchronous motor is: rated power $P_N = 3\text{kW}$, rated speed $n_N = 1500\text{r/min}$, rated torque $T_N = 2.387\text{N}\cdot\text{m}$, rated line current $I_N = 18\text{A}$, rated line voltage $U_N = 220\text{V}$, resistance of stator coil $R_s = 1.75\Omega$, inductance of armature coil $L_q = L_d = L = 4\text{mH}$, Inertia $J = 1.78\times10^{-4}\text{kg}\cdot\text{m}^2$, damping coefficient $B = 7.403\times10^{-5}\text{N}\cdot\text{ms/rad}$, number of pole-pairs $P_n = 5$, PM flux linkage $\psi_f = 0.1267\text{Wb}$. The output limit of speed controller is $\pm18\text{A}$.

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