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A General n-Port Network's Equivalent Current Sources Theorem

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Abstract: In this paper a general n-port network's equivalent current theorem has been derived out, for n = 1, 2…. the traditional Norton's Theorem is only a special case of it for n=1. When an n-port passive linear time-invariant network is connected to another n-port linear time-invariant network which contained sinusoidal sources with same frequency, this theorem provides a new way to calculate the port-current of the n-port passive network. But the short-port currents of the n-port network contained sinusoidal sources must be known at first. In sinusoidal networks, currents are vector quantity or complex quantity, including magnitude and phase angle. Ammeter can only be used to measure the magnitude of the current, not including its phase angle. So it is impossible to get the short-port currents by the short-port experiment. Moreover the short-port experiment may cause some dangerous events. So a special method to get the short-port currents is introduced in this paper, First to find out the open-port voltage vector (including magnitude and phase angle), by measuring the voltages magnitude between some two points of the open-port with a voltmeter and by drawing a series of voltage vector triangles that one side vector is the sum of other two side vectors, if the phase angle of one side vector in a triangle is known, the phase angles of the other side vectors in the same triangle can be decided. In the first triangle, the first open-port voltage vector is contained, its phase angle can be assigned to be zero, then the phase angles of the other two voltage vectors in the first triangle can be decided. In the second triangle, one of the two above voltage vectors is contained, then the phase angles of the other two voltage vectors in the second triangle can be decided. Thus go on step by step, all the open-port voltage vectors can be obtained. And the open-port voltage complex matrix has been obtained. The equation related the short-port current complex matrix and the open-port voltage complex matrix has been derived out in this paper. So the short-port current complex matrix can be obtained.

Keywords: Admittance Matrix, Equivalent Current Sources, Short-port Currents

1. Introduction

In the conventional circuits analysis there are four fundamental theorems: two-port reciprocity theorem (no math expression), one-port equivalent voltage source theorem (Thévenin's Theorem), one-port equivalent current source theorem (Norton's Theorem), and one-port maximum transfer power theorem. Since the multiport networks are met often, we should have corresponding theorems to analyze them.. A General n-Port Network's Reciprocity Theorem was derived out in 1985, not only establishing a math expression but also developing the meaming of reciprocity, it was published in a Chinese Journal of Wuhan Iron and Steel Institute [1], five years later it was published in the Journal of IEEE on educa- tion with Dr Waikai Cheng [2], A General n-Port Network's quivalent voltage source theorem and A General n-Port Network's Maxmum Transfer Power Theorem were derived out in detail long ago, both was published on Open Journal of Circuits and Systems in 2016, Hans [3, 4]. In this paper A General n-Port Network Equivalent Current Source Theorem has been presented, which for n = 1,2,⋯. the traditional Norton's equivalent current source theorem is only a special case of it for n=1.

Thus the four general n-port network's theorems have formed a complete systematical theory...

2. Derivation

Let $\mathrm{N_L}$ be a linear, time-invariant, passive, n-port network composed of resistors, inductors, coupled inductors and capacitors. The reference directions of the port voltages and the port currents are associated as $\mathrm{N_L}$ in Figure 1.

Let $\mathrm{N_s}$ be another linear, time-invariant, n-port network composed of resistors, inductors, coupled inductors and capacitors, and contained sinusoidal electrical sources with same frequency (constant dc source can be regarded as a special case of sinusoidal ac source while its frequency is zero.)

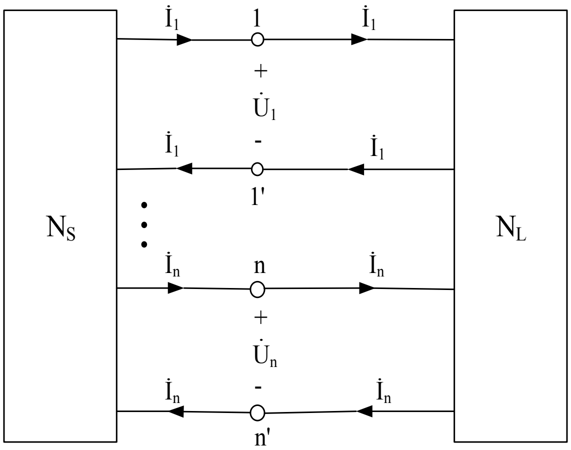

Figure 1. *N_L is connected to N_s.*

In steady state, the port currents are linear functions of the port voltages:

$$\dot{\mathrm{I}}_1 = \mathrm{Y_{L11}}\dot{\mathrm{U}}_1 + \cdots + \mathrm{Y_{L1n}}\dot{\mathrm{U}}_\mathrm{n}$$

....

$$\dot{\mathrm{I}}_\mathrm{n} = \mathrm{Y_{Ln1}}\dot{\mathrm{U}}_1 + \cdots + \mathrm{Y_{Lnn}}\dot{\mathrm{U}}_\mathrm{n}$$

In matrix form

$$\dot{\mathrm{I}} = \mathrm{Y_L}\dot{\mathrm{U}} \quad (1)$$

where

$$\dot{\mathrm{U}} = [\dot{\mathrm{U}}_1 \cdots \dot{\mathrm{U}}_\mathrm{n}]^\mathrm{T},\ \dot{\mathrm{I}} = [\dot{\mathrm{I}}_1 \cdots \dot{\mathrm{I}}_\mathrm{n}]^\mathrm{T}$$

"T" means transpose.

$\mathrm{Y_L} = \begin{bmatrix} \mathrm{Y_{L11}} & \cdots & \mathrm{Y_{L1n}} \\ \vdots & \ddots & \vdots \\ \mathrm{Y_{Ln1}} & \cdots & Y_{Lnn} \end{bmatrix}$, the short-port admittance matrix of network $\mathrm{N_L}$.

Its parameters can be obtained by short-port experiments of $\mathrm{N_L}$ step by step.

$\mathrm{Y_{Lhk}} = \frac{\dot{\mathrm{I}}_\mathrm{h}}{\dot{\mathrm{U}}_\mathrm{k}}|\dot{\mathrm{U}}_\mathrm{k} \neq 0$, all other port voltages=0

Next let the inner sources contained in network $\mathrm{N_s}$ do not work, but their devices are still connected in the network, only let the electromotive forces and current sources to be zero. Obviously, this new network is also a linear, time-invariant, passive, n-port network composed of resistors, inductors, coupled inductors and capacitors. Let us assign it as network $\mathrm{N_0}$.

Applying the same way as we found the short-port admittance matrix $\mathrm{Y_L}$ of network $\mathrm{N_L}$, we can also find the short-port admittance matrix $\mathrm{Y_0}$ of network $\mathrm{N_0}$

$$\mathrm{Y_0} = \begin{bmatrix} \mathrm{Y_{011}} & \cdots & \mathrm{Y_{01n}} \\ \vdots & \ddots & \vdots \\ \mathrm{Y_{0n1}} & \cdots & Y_{0nn} \end{bmatrix}$$

Its parameters can also be obtained by short-port experiments of $\mathrm{N_0}$ step by step.

$$\mathrm{Y_{0hk}} = \frac{\dot{\mathrm{I}}_\mathrm{h}}{\dot{\mathrm{U}}_\mathrm{k}}\ |\dot{\mathrm{U}}_\mathrm{k} \neq 0, \text{ all other port voltages=0}$$

Now suppose that the ports of the network $\mathrm{N_S}$ were short-ported as in Figure 2, and the short port currents $\dot{\mathrm{I}}_{\mathrm{S1}}, \cdots, \dot{\mathrm{I}}_{\mathrm{Sn}}$ could be obtained, then the port-voltage is zero, and the port-current of network $\mathrm{N_L}$ is zero too. Next each port is parallel in connection with another current source which is also equal to its short port current but opposite in direction as in Figure 3, thus the two current sources cancel out. The situation is the same as in Figure 1, the port-current of $\mathrm{N_S}$ and $\mathrm{N_L}$ is same.

To Figure 3, we apply superposition theorem,

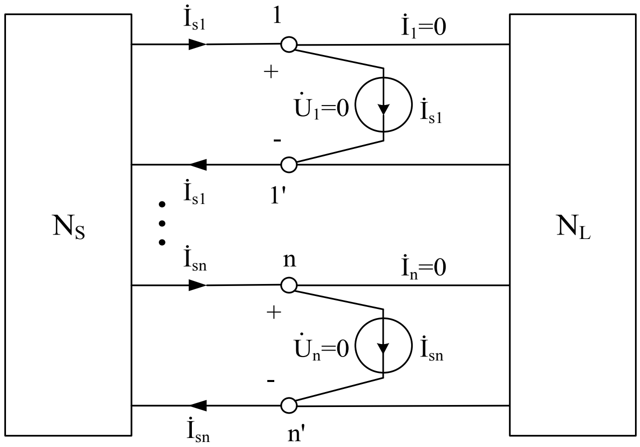

Figure 2. *The ports of N_S are shorted.*

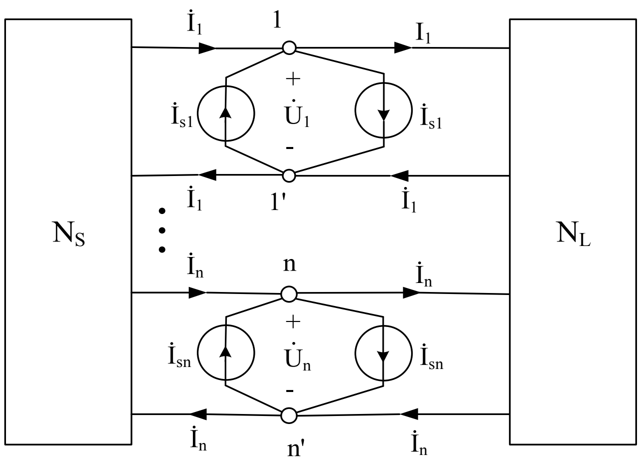

Figure 3. *Parallel connects equal but opposite current source.*

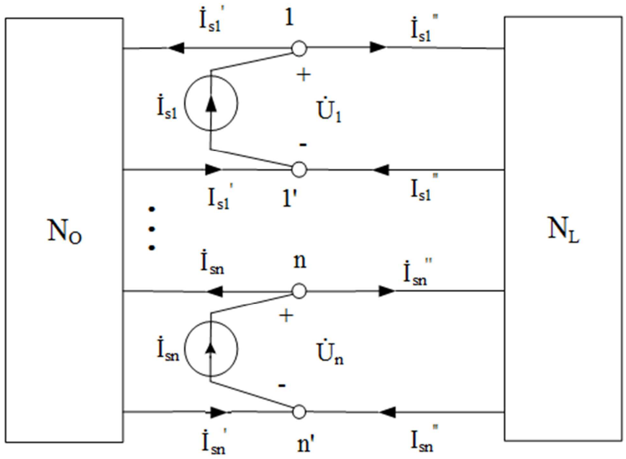

Figure 4. *The 2nd current source acts at N_L and N_0.*

Dividing all the sources into two groups, the first group is the inner sources of N_S and the first current source as in Figure 2; The second group is the second parallel connection current source acting on networks N_L and N_0 as in Figure 4.

Then finding the port-current and the port- voltage by Figure 2 and Figure 4 respectively, and superposing the results, we can get the port-current and the port-voltage of Figure 3, they are also the port-current and the port-voltage of Figure 1.

From Figure 2, the port-voltage of N_L is zero, and the port-current of N_L is zero too, the port-current of N_S is the short-port current $\dot{I}_S$.

From Figure 4, count them as follows:

$$\dot{I}_{s1}= \dot{I}'_{S1}+\dot{I}''_{S1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\dot{I}_{sn}=\dot{I}'_{Sn}+\dot{I}''_{Sn}$$

In matrix forms

$$\dot{I}_s= \dot{I}'_S + \dot{I}''_S \tag{2}$$

where $\dot{I}_S=[\dot{I}_{S1} \cdots \dot{I}_{Sn}]^T$, $\dot{I}'_S=[\dot{I}'_{S1} \cdots \dot{I}'_{Sn}]^T$,

$$\dot{I}''_S=[\dot{I}''_{S1} \cdots \dot{I}''_{Sn}]^T$$

As for N_0,

$$\dot{I}'_s=Y_0\dot{U} \tag{3}$$

As for N_L,

$$\dot{I}''_s=Y_L\dot{U} \tag{4}$$

Substituting equations (3) (4) to (2),
We get

$$\dot{I}_S=(Y_0 + Y_L)\dot{U}$$

$$\dot{U}=[Y_0 + Y_L]^{-1}\dot{I}_S \tag{5}$$

Where

Y_0 is the short-port admittance matrix of N_0,
Y_L is the short-port admittance matrix of N_L,
$\dot{U} = [\dot{U}_1 \cdots \dot{U}_n]^T$ is theport-voltage, The port-current of network N_L is

$$\dot{I} = 0 + \dot{I}''_s=Y_L\dot{U} = Y_L[Y_0 + Y_L]^{-1}\dot{I}_S. \tag{6}$$

Equation (6) is a general n-port network's equivalent current sources theorem.

It can be stated the theorem as follow:

An n-Port linear time-invariant network contained sinusoidal sources with same frequency can be expressed by an equivalent current source $\dot{I}_s$ which is equal to the short-port current but opposite in direction and a parallel passive network N_0 which is the original network N_S when its contained sources don't work, acting at another n-Port linear time-invariant passive network N_L as in Figure 4.

3. Discussion

How to obtain the short-port current $\dot{I}_s$ is the key question. By calculation, the structure of the contained sources network must be known and not too be complicated. By short-port experiments, the contained sources must be dc low voltage and the network must be composed of resistors only. Otherwise it may be dangerous to short ports of a network contained sinusoidal sources. So it is difficult to obtain the short-port current $\dot{I}_s$ by short-port experiments, An equation which expresses the relation between the short-port current complex matrix and the open-port voltage complex matrix is derived out as follows:

According to A General n-Port Network's Equivalent Voltage Theorem[3],, the port-current is:

$$\dot{I}=[Z_0 + Z_L]^{-1}\dot{U}_0$$

where Z_0 is the impendance matrix of network N_0, which is the network N_S when its inner sources don't work. Z_L is the impendance matrix of network N_L, $\dot{U}_0$ is the open-port voltage of network N_S.

According to A General n-Port Network's Equivalent Current Theorem, the port-current is:

$$\dot{I}=Y_L[Y_0 + Y_L]^{-1}\dot{I}_S.$$

Hence

$$Y_L[Y_0 + Y_L]^{-1}\dot{I}_S= [Z_0 + Z_L]^{-1}\dot{U}_0$$

$$\dot{I}_S= [Y_0 + Y_L]\, {Y_L}^{-1}\, [Z_0 + Z_L]^{-1}\dot{U}_0 \tag{7}$$

This is the equation wanted to calculate the short-port current $\dot{I}_S$ via the out-port voltage $\dot{U}_0$.

First we find $\dot{U}_0$, then applying equation (7), the short-port current $\dot{I}_S$ can be computed out.

Where

$$\dot{U}_0=[\dot{U}_{01}, \cdots, \dot{U}_{0n}]^T=[\dot{U}_{011'}, \cdots, \dot{U}_{0nn'}]^T,$$

The voltmeter can only measure the magnitude of voltage, but it can't measure the phase angle of voltage.

Here a special method is introduced to solve this problem:

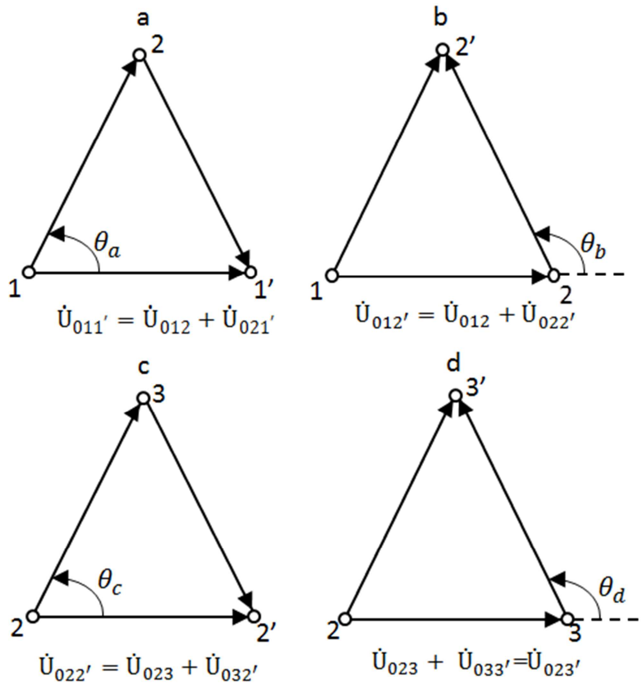

Figure 5. *Finding phase angle by drawing.*

With a voltmeter we can measure the voltage magnitude of any two points of network's ports. We chose such three points that their three voltage magnitudes can be formed a vector triangle which one side vector is the sum of the other two side vectors. If the phase angle of one vector in the triangle is known, the phase angles of the other two vectors in the same triangle can be decided. Let the first open-port voltage $\dot{U}_{01} = \dot{U}_{011'}$ be reference vector which phase angle is zero, forming the first voltage triangle as Figure 5 a, we have $\dot{U}_{011'} = \dot{U}_{012} + \dot{U}_{021'}$ since $\dot{U}_{011'}$ has been assigned, $\dot{U}_{012}$ can be decoded. Its phase angle is θ_a. Figure 5 b is the second triangle, $\dot{U}_{012'} = \dot{U}_{012} + \dot{U}_{022'}$ since $\dot{U}_{012}$ has been decided, $\dot{U}_{022'}$ can be decided. Its phase angle is $\theta_a + \theta_b$. Figure 5 c is the third triangle, $\dot{U}_{022'} = \dot{U}_{023} + \dot{U}_{032'}$, since $\dot{U}_{022'}$ has been decided, $\dot{U}_{023}$ can be decided, its phase angle is $\theta_a + \theta_b + \theta_c$. Figure 5 d is the fourth triangle, $\dot{U}_{023} + \dot{U}_{033'} = \dot{U}_{023'}$, since $\dot{U}_{023}$ has been decided, $\dot{U}_{033'}$ can be decided. its phase angle is $\theta_a + \theta_b + \theta_c + \theta_{d.}$ Thus go on step by step, we can find all the open port voltages $\dot{U}_{01}, \cdots, \dot{U}_{0n,}$and then the matrix$\dot{U}_0$.Finally the matrix $\dot{I}_S$can be obtained with equation (7).

Already got the short-port current $\dot{I}_S$, the port-current $\dot{I}$ of the passive network N_L can be obtained by A General n-Port Network's Equivalent current Source Theorem..

Another important problem must be pointed out: All the n ports of the networks $N_L$and N_0 in the general theorems must be interconnected. In other words, any port current should be linear functions of all port voltages and vice versa. Otherwise at least one column of determinant of the impedance or admittance matrix would all be zeros. The inverse matrix doesn't exist.

4. Special Case

In special case, when n=1, matrices Y_0 and Y_L are reduced to complex numbers. Equation (6) becomes

$$\dot{U} = \frac{\dot{I}_S}{Y_0 + Y_L}$$

$$\dot{I} = Y_L \dot{U} = \frac{Y_L}{Y_0 + Y_L} \dot{I}_S \tag{8}$$

Obviously this is Norton's theorem, it is only a special case for n=1 of the general theorem.

5. Conclusion

There are three achievements in this paper:

The first achievement is that A General n-Port Network's Equivalent Current Sources Theorem for n=1,2,⋯, has been derived out., Norton's Equivalent Current Source Theorem is only a special case of it for n=1, even be regarded as an important theorem in circuits theory. This general theorem should be regarded as one of the fundamental theorems in the circuits and systems theory. This theorem together with *A General n-Port Network's Reciprocity Theorem* for n=2,3,⋯,[1], *A General n-Port Network's Equivalent voltage Source Theorem* for n=1,2⋯,[3] and *A General n-Port Network's Maximum Transfer Power Theorem* for n=1,2. ⋯,[4], to form a complete systematical theory for dealing with n-port networks.

The second achievement is to obtain an equation (7) which gives a relation between the short-port current complex matrix and the open-port voltage complex martrix of an n-pot network contained sinusoidal sources, This equation is very useful to calculate the short -port current complex matrix via the open-port voltage complex matrix.

The third achievement is to find a method to obtain the open-port voltage phase angles to form complex matrix by drawing a series of voltage vector triangles with only a voltmeter to measure the voltage magnitudes between points of the open-ports of the network contained sinusoidal sources. This is a very useful method.

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