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Pricing of the Quanto Game Option with Asian Feature

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Abstract: The game option, which is also known as Israel option, is a new type of American option to give the option writer the right to cancel the contract before the maturity. This article studies the pricing behaviors of the quanto game option with Asian features based on partial differential equation and the stochastic analysis. The Asian feature in an option model refers to the payoff of the option depends on both the average asset price over the life of the option. The quanto options (currency-translated foreign equity options) are contingent claims where the payoff depends on exchange rate level at the option exercise time. The Asian quanto game options can be regarded as double-barrier European options for the features that both the holder and the writer can exercises the options contract at any time over the life of the option. We derive the pricing equation and provide the integral expression of pricing formula for the option. The option price is decomposed into the corresponding European option price and the penalty paid by the option writer for an early callable and the penalty paid by the option holder for early exercise of the option. In addition, we discuss optimal exercise strategies and continuation regions of the option.

Keywords: American Option, Quanto Game Option, Asian Feature, Callable Strategy

1. Introduction

The game option is an innovative American option, in which the contract seller can exercise the contract at any time over the life of the option. If the option holder exercises the contract at time t before the maturity T, he gets the payoff X_t. On the other hand, if the contract is cancelled by the option writer, the option writer gives the option holder payoff Y_t. If the option holder exercises the contract at the time when the writer cancels the contract, the option holder can only get the payoff $X_t = Y_t$. In the call game option case, $X_t = (S-K)^+$ and $Y_t = (S-K)^+ + \delta$, where δ denotes the penalty. There is a strong interest in this area of the American style derivatives in the recent years which is confirmed by the large amount of publications – see for example Park and Jeon (2017), Le and Dang (2017), Balajewicz and Toivanen (2017), Gong and Zhuang (2017), Kang et al. (2017), Zhao and Yang (2018), Chen et al. (2018), Madi et al. (2018), Soleymani et al. (2018), Chen et al. (2019), Zaevski (2019) and Gao et al. (2020).

Following the arguments by Kifer (2000) and Kyprianou (2004) we know that the optimal strategy for the writer is to stop only when $S_t = K$. Similarly to the case of the American option, the goal of the game option holder is to maximize his payoff. The game option writer, however, has to hedge his short position and at the same time, when he cancels the contract, minimizes the payoff obtained by the option holder. In some sense, the game option can be viewed as an American option that the writer has the right to cancel the contract before the maturity. Since the writer could cancel the contract before the maturity, the price of the game option should not be higher than the price of the corresponding American option. The relevant conclusions of the game option pricing can be found in Kyprianou (2004), Baurdoux (2004), Ekstrom (2006), Kuhn, C., and a. e. Kyprianou. (2007), Guo Peidong (2014), Yam et al.(2014), Tsvetelin et al.(2020).

The aim of this article is to study the pricing behavior of the quanto game option with the Asian feature whose payoffs depend on the mean of the underlying asset price during the life of the option. We analyze the optimal exercise strategies and derive the pricing formula of the quanto game option with the Asian feature. The paper is organized as follows. In Sect. 2 we derive the pricing equation of the quanto game option with the Asian feature. We also study the optimal exercise strategies and the characteristics of continuation regions of the option. In Sect. 3 we derive the pricing formula of the quanto game option with the Asian feature with floating strike in domestic currency. The numerical simulation analysis and the conclusion are in Sect. 4.

2. The Pricing Equation

Under the risk neutral condition, the asset price process follows a lognormal diffusion process

$$dS^* = \mu_{S^*}S^*dt + \sigma_{S^*}S^*dW_1 \quad (1)$$

$$dS = \mu_S Sdt + \sigma_S SdW_2 \quad (2)$$

$$dF = \mu_F Fdt + \sigma_F FdW_3 \quad (3)$$

where W_i (i= 1,2,3) denotes the standard Wiener process, and σ_Δ, μ_Δ (Δ= S^*, S, F) represent the drift rate and the volatility of the asset return respectively. Let ρ_{SF} be correlation coefficient of the standard Wiener process W_2 and W_3.

Further let T denote the expiration time of the option. Denote the divided yield, the domestic and foreign risk-free interest rate by $r > 0, r_f > 0, q > 0$, respectively. According to $S^* = FS$, we have

$$\mu_{S^*} = \mu_S + \mu_F + \rho_{SF}\sigma_S\sigma_F \quad (4)$$

$$\sigma_{S^*}^2 = \sigma_S^2 + \sigma_F^2 + \rho_{SF}\sigma_S\sigma_F SF. \quad (5)$$

Now we consider the quanto Asian game (QAG) option where the strike price is denominated in domestic currency and the strike price follows the geometric average distribution, namely

$$G_{S^*} = \exp\left\{\frac{1}{t}\int_0^1 \ln S^*(\tau)d\tau\right\}, 0 \le t \le T. \quad (6)$$

According to the principle of the no-arbitrage pricing and Itô lemma, the QAG option satisfies the following equation

$$\begin{aligned} &V_t + \tfrac{1}{2}\sigma_S^2 S^2 V_{SS} + \rho_{SF}\sigma_S\sigma_F SFV_{SF} + \tfrac{1}{2}\sigma_F^2 F^2 V_{FF} \\ &+\delta_S SV_S + \delta_F FV_F + \left(\frac{G_{S^*}}{t}\ln\frac{S^*}{G_{S^*}}\right)V_{G_{S^*}} - rV = 0, \end{aligned} \quad (7)$$

and $\delta_S = r_f - q - \rho_{SF}\sigma_S\sigma_F, \delta_F = r - r_f$.

Noting that

$$X_t = (FS - \alpha G_{S^*})^+,\ Y_t = (FS - \alpha G_{S^*})^+ + \delta S^*.$$

3. Optimal Exercise Strategy

Define $\emptyset = \alpha G_{S^*}/FS$. Noting the QAG option price $\tilde{V}(\emptyset, t)$, we have $\tilde{V} = V/SF$, and $\tilde{X} = (1-\emptyset)^+, \tilde{Y} = (1-\emptyset)^+ + \delta$. Hence the region of the QAG option price is

$$(1-\emptyset)^+ \le \tilde{V}(\emptyset,t) \le \min\{(1-\emptyset)^+ + \delta, \tilde{V}^A(\emptyset,t)\}, \quad (8)$$

where $\tilde{V}^A(\emptyset,t)$ represent corresponding American option.

Lemma 1 *Suppose* $\delta^* = \tilde{V}^A(1,0)$. *When* $\delta \ge \delta^*$ *the writer should not exercise the option early. Consequently, the option is worth the same as the corresponding American option.*

Proof: By the nature of American options, we know that the corresponding American option price $\tilde{V}^A(\emptyset,t)$ is monotonically decreasing on t and $\emptyset$, further $\emptyset_0 \le 1$ due to the non-negative payoff of the option holder at the optimal exercise boundary $\emptyset_0$.

$FS < \alpha G_{S^*}$, namely $\emptyset > 1$.

We know that the price of an American option varies depending on the underline asset price. For two American options, the difference between their option values is bounded by the maximal deviation between the two exercise processes (Shreve 2004). In addition, because the game option gives the writer the early callable right, its price should be less than the corresponding American option price. That is,

$$\tilde{V}(\emptyset,t) \le \tilde{V}^A(\emptyset,t) \le \tilde{V}^A(\emptyset,0) < \tilde{V}^A(1,0) + (1-\emptyset)^+ \le \delta + (1-\emptyset)^+, \quad (9)$$

therefore

$$\tilde{V}(\emptyset,t) \le \delta + (1-\emptyset)^+. \quad (10)$$

$FS > \alpha G_{S^*}$ and $FS \in (\alpha G_{S^*}, S_0^*)$, namely $0 < \emptyset < 1$, where S_0 denote the corresponding option optimal exercise boundary.

Due to the option price $\tilde{V}(\emptyset,t)$ is convexity and monotonically decreasing on t, and further consider the Bonding conditions

$$\left(\frac{\partial\tilde{V}^A(\emptyset,t)}{\partial\emptyset}\right)_{\emptyset=\emptyset_0} = -1 \text{ at } \emptyset = \emptyset_0. \quad (11)$$

Then we know that $\tilde{V}^A(\emptyset,t) - (1-\emptyset)^+$ is monotonically increasing on $\emptyset$ when $\emptyset \in (\emptyset_0, 1)$. That is

$$\tilde{V}(\emptyset,t) - (1-\emptyset)^+ \le \tilde{V}^A(\emptyset,t) - (1-\emptyset)^+ < \tilde{V}^A(1,t) < \tilde{V}^A(1,0) \le \delta. \quad (12)$$

Based on the conclusion in (1) and (2), we know that the corresponding American option price $\tilde{V}^A(\emptyset,t)$ satisfies the following condition

$$\tilde{V}^A(\emptyset,t) \le \delta + (1-\emptyset)^+, \quad (13)$$

implying that the writer should not cancel the contract. Hence, the QAG option is worth the same as the corresponding American option. #

Lemma 2 *Let* $\delta = \tilde{V}^A(1,t^*)$, *and* $\tilde{V}^A(1,T) < \delta < \tilde{V}^A(1,0)$. *Then, when* $t \ge t^*$ *the writer should not exercise the option. Namely, only* $t \in [0,t^*)$ *the writer can exercise the option early.*

Proof: Following the arguments in lemma 1, we know $\delta < \tilde{V}^A(1,0)$. Considering the corresponding American option price $\tilde{V}(\emptyset,t)$ is monotonically decreasing on t and

$\widetilde{V}(\emptyset, T) = (1-\emptyset)^+$, then we have $\delta < 0$ if $\delta < \widetilde{V}^A(1, T)$. In this case, the QAG option is invalid in the market transactions.

When the option is near expiration, there have

$$\widetilde{V}(\emptyset, t) - (1-\emptyset)^+ \le \widetilde{V}^A(\emptyset, t) - (1-\emptyset)^+ < \widetilde{V}^A(1, t) < \widetilde{V}^A(1, t^*) = \delta. \quad (14)$$

Hence in the case the writer should not exercise the option early. #

Lemma 3 *As soon as asset price process $\emptyset_t$ first hits the strike price 1 (that is $FS = \alpha G_{S^*}$), the writer cancels the contract with optimal stopping strategy that*

$$\hat{t} = inf\{t \ge 0, \emptyset_t = 1\}$$

Proof: In the case, there have the optimal recall time for the QAG option writer. The optimal recall time for the writer correspond to the time that option holders obtains the smallest payoff. The proof proceeds along the similar lines as the proof of lemma 1.

$FS < \alpha G_{S^*}$, namely $\emptyset > 1$.

In the case, the option is out of the money. The writer will lose the time value of the penalty δS^* if the writer chose to recall the option early. Hence it would not make sense for the writer to exercise early at all.

$FS > \alpha G_{S^*}$ and $FS \in (\alpha G_{S^*}, S_0^*)$, namely $0 < \emptyset < 1$.

In contrast to (1), in the case the option is in the money. The writer will pay an additional $(1-\emptyset)$ if the writer chose to recall the option early in this time. Based on minimizing the total payment, the writer should not recall the option at this time.

To sum up, the QAG option writer have the optimal stopping strategy $\hat{t} = \inf\{t \ge 0, \emptyset_t = 1\}$, that is $FS = \alpha G_{S^*}$. #

4. Pricing Formula

When $t \in [0, t^*)$, the QAG option satisfies the pricing equation (4). Considering the optimal exercise boundary

$$S_0^*(T) = \max\left(\alpha G_{S^*}, \frac{\alpha G_{S^*} r}{q}\right) \quad (15)$$

we discuss the QAG option price under different condition according to the size of $S_0^*(T)$.

$$S_0^*(T) = \alpha G_{S^*}.$$

In the case, the writer and the holder exercise the option at the same time. From the model assumption, we put this case as the holder exercise the option early. Hence, the QAG option price is equivalent to the corresponding American Asian option.

$S_0^*(T) = \frac{\alpha G_{S^*} r}{q}$, namely $S_0^*(T) > \alpha G_{S^*}$.

We discuss the option price in the interval $(0, \infty)$ according to delayed compensation theory.

(i) $S^* \in (0, \alpha G_{S^*}) \cup (\alpha G_{S^*}, S_0^*)$.

The QAG option price satisfies following equation

$$LV(S^*, G_{S^*}, t) = V_t + \frac{G_{S^*}}{t}\ln\left(\frac{S^*}{G_{S^*}}\right)V_{G_{S^*}} + (r-q)S^*V_{S^*} + \frac{1}{2}\sigma_{S^*}^2 S^{*2} V_{S^*S^*} - rV = 0. \quad (16)$$

(ii) $S^* = \alpha G_{S^*}$.

In the case, the writer should cancel the option contract, and the option price is equal to δS^*. The price equation can be written by

$$LV(S^*, G_{S^*}, t) = -q\delta S^*. \quad (17)$$

(iii) $S^* \in (S_0^*, +\infty)$.

In the case, according to the feature of American option the holder should exercise the option contract, namely $V(S^*, G_{S^*}, t) = (S^* - \alpha G_{S^*})^+$. Noting that

$$LV(S^*, G_{S^*}, t) = -qS^* - \alpha\frac{\partial G_{S^*}}{\partial t} + r\alpha G_{S^*}. \quad (18)$$

To sum up, the QAG option price satisfies following equation:

$$LV(S^*, G_{S^*}, t) = \begin{cases} 0, & S^* \in (0, \alpha G_{S^*}) \cup (\alpha G_{S^*}, S_0^*), \\ -q\delta S^*, & S^* = \alpha G_{S^*}, \\ -qS^* - \alpha\frac{\partial G_{S^*}}{\partial t} + r\alpha G_{S^*}, & S^* \in (S_0^*, +\infty). \end{cases} \quad (19)$$

By solving the above pricing model, we will get the QAG option pricing formula.

Theorem *the quanto Asian game option where the strike price is denominated in domestic currency price is given by*

$$\delta > \tilde{V}^A(1,0)$$

$$V(S, F, G_{S^*}, t) = V^A(S^*, G_{S^*}, t), 0 \le t \le T.$$

$$\delta = \tilde{V}^A(\emptyset, t^*), S^*(T) = \alpha G_{S^*}$$

$$V(S,F,G_{S^*},t)=V^A(S^*,G_{S^*},t), 0 \le t \le T.$$

$$\delta = \tilde{V}^A(\emptyset,t^*), S^*(T) = \frac{\alpha G_{S^*} r}{q}$$

$$V(S,F,G_{S^*},t)=\begin{cases} V^E(S^*,G_{S^*},t) + V^e(S^*,G_{S^*},t) - V^D(S^*,G_{S^*},t), & 0 \le t < t^*, \\ V^A(S^*,G_{S^*},t), & t^* \le t \le T. \end{cases} \quad (20)$$

Where

$$V^A(S^*,G_{S^*},t) = V^E(S^*,G_{S^*},t) + V^e(S^*,G_{S^*},t),$$

$$V^E(S^*,G_{S^*},t) = S^* e^{(\delta_{S^*}-r)(T-t)}\left[N(d_1) - \alpha\left(\frac{G_{S^*}}{S^*}\right)^{\frac{t}{T}} e^{-Q} N(d_2)\right],$$

$$V^D(S^*,G_{S^*},t) = \delta S^*\left[\left(\frac{\alpha G_{S^*}}{S^*}\right)^{\frac{\left(\mu_{S^*}-\sqrt{2\sigma_{S^*}^2 r+\mu_{S^*}^2}\right)y}{\sigma_{S^*}^2}} N(f_2) + \left(\frac{\alpha G_{S^*}}{S^*}\right)^{\frac{\left(\mu_{S^*}+\sqrt{2\sigma_{S^*}^2 r+\mu_{S^*}^2}\right)y}{\sigma_{S^*}^2}} N(h_2)\right],$$

$$V^e(S^*,G_{S^*},t) = S^*\int_t^T e^{(\delta_{S^*}-r)(u-t)}\left\{(\delta_{S^*}-r)N(\widehat{d_1}) - \alpha\left(\frac{G_{S^*}}{S^*}\right)^{\frac{t}{u}} e^{-\hat{Q}}\left[\left(r - \frac{ln\alpha}{u} + \widehat{d_3}\right)N(\widehat{d_2}) + \frac{\widehat{\sigma_{S^*}}}{u^2} n(\widehat{d_2})\right]du\right., f_2 = \frac{y-\sqrt{2\sigma_{S^*}^2 r+\mu_{S^*}^2}(t^*-t)}{\sigma_{S^*}\sqrt{t^*-t}}, h_2 = \frac{y+\sqrt{2\sigma_{S^*}^2 r+\mu_{S^*}^2}(t^*-t)}{\sigma_{S^*}\sqrt{t^*-t}},$$

$$\delta_{S^*} = r - q, \mu_{S^*} = \delta_{S^*} + \frac{1}{2}\sigma_{S^*}^2, y = tln\frac{\alpha G_{S^*}}{S^*},$$

$$d_1 = \left[tln\left(\frac{S^*}{\alpha G_{S^*}}\right) - (T-t)ln\alpha + \frac{1}{2}\mu_{S^*}(T^2-t^2)\right]/\left(\sigma_{S^*}\sqrt{(T^3-t^3)/3}\right),$$

$$d_2 = d_1 - \frac{\sigma_{S^*}}{T}, Q = \frac{\mu_{S^*}}{2}\left(\frac{T^2-t^2}{T}\right) - \frac{\sigma_{S^*}^2}{6}\left(\frac{T^3-t^3}{T^2}\right),$$

$$\widehat{d_1} = \frac{uln\frac{\alpha G_{S^*}}{S_0^*} - tln\frac{\alpha G_{S^*}}{S^*} + \frac{\mu_{S^*}}{2}(u^2-t^2)}{\hat{\sigma}}, \widehat{d_2} = \widehat{d_1} - \frac{\hat{\sigma}}{u}, \hat{\sigma}^2 = \frac{\sigma_{S^*}^2(u^3-t^3)}{3},$$

$$\widehat{d_3} = \frac{tln\frac{G_{S^*}}{S^*} + uln\alpha - \frac{\mu_{S^*}}{2}(u^2-t^2) + \frac{\hat{\sigma}^2}{u}}{u^2}, \hat{Q} = \frac{\hat{\sigma}}{2u^2} - \frac{\mu_{S^*}}{2u}(u^3-t^3).$$

N(·) and n(·) denote cumulative normal distribution function and the standard normal distribution function respectively.

5. Conclusion

In this paper, we studied the pricing behaviors of the quanto Asian game option where the strike price is denominated in domestic currency and the strike price follows the geometric average distribution and obtaining the integral expression of pricing formula under the finite horizon case. Furthermore, we discussed optimal exercise strategies and continuation regions of options. As a consequence, the quanto Asian game option can be analyzed as a mixture of two exotic options, i.e., American and European quanto Asian barrier options. The game options with callable features are more flexible than American options. After the issuance of an option, the writer is no longer a passive player; he may terminate the contract to safeguard his own interest before the expiration of the option. The game options with callable features are also cheaper than the American-style options and thus are more conducive to the writer.

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