

A Note on (i,j) - $\pi g\beta$ Closed Sets in Intuitionistic Fuzzy Bitopological Spaces

S. Jothimani¹, T. Jenitha Premalatha²

¹Department of Mathematics, Government Arts and Science College, Coimbatore, India

²KPR Institute of Engineering and Technology, Coimbatore, India

Email address:

jothimanis09@gmail.com (S. Jothimani), joel.jensi@gmail.com (T. J. Premalatha)

To cite this article:

S. Jothimani, T. Jenitha Premalatha. A Note on (i,j) - $\pi g\beta$ Closed Sets in Intuitionistic Fuzzy Bitopological Spaces. *Mathematics and Computer Science*. Vol. 2, No. 2, 2017, pp. 14-19. doi: 10.11648/j.mcs.20170202.11

Received: April 22, 2017; Accepted: May 4, 2017; Published: June 27, 2017

Abstract: In this paper we introduce the concept of (i,j) - $\pi g\beta$ -closed set in intuitionistic fuzzy bitopological spaces as a generalization of $\pi g\beta$ -closed set in fuzzy bitopological space and study their related notions in bitopological spaces. Next, we introduce (i,j) - $\pi g\beta$ - open sets in intuitionistic fuzzy bitopological spaces, and investigate some of their basic properties. Using these concepts, the characterizations for the intuitionistic fuzzy pairwise (i,j) - $\pi g\beta$ continuous mappings are obtained. The relationships between intuitionistic fuzzy pairwise (i,j) - $\pi g\beta$ continuous mappings are discussed. Finally, we prove the irresoluteness in (i,j) - $\pi g\beta$ intuitionistic fuzzy bitopological spaces.

Keywords: IF Bitopological Spaces, IF (i, j) -Open Sets, IF (i, j) -Closed Sets, IF (I, j) $\pi g\beta$ -Open Sets, IF (i, j) $\pi g\beta$ -Closed Sets, IF (i, j) $\pi g\beta$ -Pairwise Continuous Function and IF (i, j) $\pi g\beta$ -Irresolute Function

1. Introduction

The notion of β -open set was introduced by Abd El-Monsef et al. [1] and Andrijevic [2]. Later on, as a generalization of the above mentioned set, $\pi g\beta$ sets have been introduced by Caldas and Jafari [4]. The concept of bitopological spaces (X, τ_i, τ_j) was introduced by Kelly J. C in 1963 [8] where X is a nonempty set and the bitopological spaces are equipped with two arbitrary topologies τ_i and τ_j , Where τ_i and τ_j are two topologies on X . After that several authors turned their attention towards generalizations of various concepts of topology for bitopological spaces. In 2005, the concept of (i,j) - β -open sets was defined and investigated by Raja Rajeswari and Lellis Thivagar [9]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [3]. Recently, Coker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. On the other hand, Kandil [7] introduced the concept of fuzzy bitopological spaces as a natural generalization of Chang's fuzzy topological spaces.

In 2012, the notion of bitopological space was introduced in intuitionistic fuzzy topology by - Jin Tae Kim, Seok Jong Lee [6]. In this paper, the concepts of $\pi g\beta$ -closed set have

been extended to the bitopological spaces in intuitionistic fuzzy topology and we introduce a new form of closed set called Intuitionistic fuzzy (IF) (i,j) - $\pi g\beta$ -closed set., The notion of IF (i,j) - $\pi g\beta$ -continuous function and irresolute function is introduced and studied.

2. Preliminaries

The interior and the closure of a subset A of an intuitionistic fuzzy bi topological space (IFBTS) (X, τ) are denoted by $\text{Int}(A)$ and $\text{Cl}(A)$, respectively.

In the following sections by X, Y and Z , we mean an intuitionistic fuzzy bi topological space (X, τ_i, τ_j) , (Y, σ_i, σ_j) and (Z, η_i, η_j) , respectively.

Throughout the paper the triplet (X, τ_i, τ_j) denotes an intuitionistic fuzzy bi topological space (IFBTS) and (X, τ_i, τ_j) be the intuitionistic fuzzy bi topological space, where $i, j \in \{1, 2\}$, and $i \neq j$.

For, a subset A of a bi topological space (X, τ_i, τ_j) , we denote the closure of A and the interior of A with respect to τ_i by $\tau_i \text{-Cl}(A)$ and $\tau_i \text{-Int}(A)$, respectively.

Definition 2.1. Let A be an intuitionistic fuzzy set in an IFBTS (X, τ_i, τ_j) , Then A is said to be an

(i) IF (i, j) -semi open [6] if there exists an IF, τ_i .open set U in X such that $U \subseteq A \subseteq \tau_j \text{-Cl}(U)$ and IF (i, j) -semi closed [6] if

there exists an IF, τ_i - closed set U in X such that j -Int(U) $\subseteq A \subseteq U$.

(ii) IF (i, j) -preopen [6], $A \subset \tau_i$. Int(τ_j - Cl(A)), and IF (i, j) -pre closed if

$$\tau_i \text{-Cl}((\tau_j - \text{Int}(A))).$$

(iii) IF (i, j) - β -open [7] if $A \subseteq \tau_j$ - Cl(τ_i - Int(τ_j - Cl(A))), where $i, j = 1, 2$ and $i \neq j$, and

$$(i, j)\text{-}\beta\text{-closed [7] if } (\tau_i \text{- Int } (\tau_j \text{- Cl}(\tau_i \text{- Int } (A)))) \subseteq A.$$

Definition 2.2. A subset A of an intuitionistic fuzzy bitopological space (X, τ_i, τ_j) is said to be IF (i, j) - g -closed if τ_j - Cl(A) $\subseteq U$ and $U \in \tau_i$ where $i, j = 1, 2$ and $i \neq j$.

Definition 2.3. A subset A of an intuitionistic fuzzy bitopological space (X, τ_i, τ_j) is said to be IF (i, j) - $g\beta$ -closed if (i, j) - β Cl(A) $\subseteq U$ whenever $A \subseteq U$ and $U \in \tau_i$.

Definition 2.4. [6]. Let A be an intuitionistic fuzzy subset of X , Then A is said to be IF (i, j) regular-open if

$A = (\tau_i \text{- Int}(\tau_j \text{- Cl}(A)))$. The union of all IF (i, j) regular - open set is known as IF (i, j) π -open.

The complement of IF (i, j) π -open set is IF (i, j) - π -closed.

3. Intuitionistic Fuzzy (i, j) - $\pi g\beta$ - Closed Set

Definition 3.1. A subset A of an intuitionistic fuzzy bitopological space (X, τ_i, τ_j) is said to be IF (i, j) - $\pi g\beta$ -closed [6] if IF (j, i) - β Cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is IF (i, j) - π -open.

Definition 3.2 A subset A of a bitopological space (X, τ_i, τ_j) is said to be

(i) IF (i, j) - $\pi g\beta$ -closure of A , is defined by the intersection of all IF (i, j) - $\pi g\beta$ -closed sets containing A . That is $\text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A) = \bigcap \{F \subseteq X: F \in (i, j)\text{-}\pi g\beta\text{-Cl}(X), A \subseteq F\}$ &

(ii) The IF (i, j) - $\pi g\beta$ -interior of A is defined by the union of all IF (i, j) - $\pi g\beta$ -open sets contained in A . Thus $\text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(A) = \bigcup \{F \subseteq X: F \in (i, j)\text{-}\pi g\beta\text{O}(X), F \subseteq A\}$

Theorem 3.1 Let (X, τ_1, τ_2) be an intuitionistic fuzzy bitopological space and A be a subset of X . Then $x \in \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A)$ if and only if for every IF (i, j) - $\pi g\beta$ -open set U containing x , such that $U \cap A \neq \emptyset$.

Proof:

Suppose that $x \in \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A)$, we shall show that $U \cap A \neq \emptyset$ for every $U \in \text{IF}(j, i)\text{-}\pi g\beta\text{O}(X, x)$.

Suppose that there exists $U \in \text{IF}(j, i)\text{-}\pi g\beta\text{O}(X, x)$ such that $U \cap A = \emptyset$.

Then $A \subseteq X \setminus U$ and $X \setminus U$ is IF (i, j) $\pi g\beta$ -closed.

Since $A \subseteq X \setminus U$, $\text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A) \subseteq \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(X \setminus U)$.

Since $x \in (j, i)\text{-}\pi g\beta\text{-Cl}(A)$, we have $x \in (j, i)\text{-}\pi g\beta\text{-Cl}(X \setminus U)$.

Since $X \setminus U$ is IF (i, j) $\pi g\beta$ -closed,

we have $x \in X \setminus U$; hence $x \notin U$, which is a contradiction that $x \in U$. Therefore, $U \cap A \neq \emptyset$.

Conversely, suppose that $U \cap A \neq \emptyset$ for every $U \in \text{IF}(j, i)\text{-}\pi g\beta\text{O}(X, x)$.

We can show that $x \in \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A)$. Suppose that $x \notin (j, i)\text{-}\pi g\beta\text{-Cl}(A)$.

Then there exists $U \in (j, i)\text{-}\pi g\beta\text{O}(X, x)$ such that $U \cap A = \emptyset$.

This is a contradiction to $U \cap A \neq \emptyset$, Hence $x \in \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A)$.

Cl(A).

Lemma 3.1 Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X . Then

$$(i) X \setminus \text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(A) = \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(X \setminus A).$$

$$(ii) X \setminus \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A) = \text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(X \setminus A).$$

Proof:

(i) Let $x \in \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A)$. There exists $V \in \text{IF}(i, j)\text{-}\pi g\beta\text{O}(X, x)$ such that $V \cap A \neq \emptyset$.

Hence we obtain $x \in (i, j)\text{-}\pi g\beta\text{-Int}(X \setminus A)$. This shows that $X \setminus \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A) \subseteq \text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(X \setminus A)$.

Let $x \in (i, j)\text{-}\pi g\beta\text{-Int}(X \setminus A)$. Since $\text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(X \setminus A) \cap A = \emptyset$, we obtain

$$x \notin \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A) \text{ hence } x \in X \setminus \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A).$$

Therefore, we obtain $\text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(X \setminus A) = X \setminus \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A)$.

(ii) Similar to (i)

Remark 3.1: If $A \subseteq B$, then $\text{IF}(i, j)\text{-}\beta\text{-Cl}(A) \subseteq \text{IF}(i, j)\text{-}\beta\text{-Cl}(B)$ & $\text{IF}(i, j)\text{-}\beta\text{-Int}(A) \subseteq \text{IF}(i, j)\text{-}\beta\text{-Int}(B)$ and hence $\text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(A) \subseteq \text{IF}(i, j)\text{-}\pi g\beta\text{-Cl}(B)$ & $\text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(A) \subseteq \text{IF}(i, j)\text{-}\pi g\beta\text{-Int}(B)$.

Definition 3.3

A space X is said to be IF (i, j) - $\pi g\beta T_1$ if for any two distinct points x, y of X , there exists IF

(i, j) - $\pi g\beta$ open sets U, V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.

Theorem 3.2 An intuitionistic fuzzy bitopological space X is IF (i, j) $\pi g\beta T_1$ if and only if $\{x\}$ is IF (i, j) - $\pi g\beta$ closed in X for every $x \in X$.

Proof. If $\{x\}$ is IF (i, j) $\pi g\beta$ closed in X for every $x \in X$, for $x \neq y$, $X \setminus \{x\}$, $X \setminus \{y\}$ are IF (i, j) - $\pi g\beta$ open sets such that $y \in X \setminus \{x\}$ and $x \in X \setminus \{y\}$. Therefore, X is IF (i, j) $\pi g\beta T_1$.

Conversely, if X is IF (i, j) - $\pi g\beta T_1$, if and if $y \in X \setminus \{x\}$ then $x \neq y$. Therefore, there exist IF (i, j) $\pi g\beta$ open sets $U_i, V_j \in X$ such that $x \in U_i$ but $y \notin U_i$ and $y \in V_j$ but $x \notin V_j$.

Let G be the union of all such V_j . Then G is an IF (i, j) $\pi g\beta$ - open set and $G \subset X \setminus \{x\} \subset X$.

Therefore, $X \setminus \{x\}$ is an IF (i, j) $\pi g\beta$ open set in X .

Theorem 3.3: If A is IF (i, j) $\pi g\beta$ closed and $A \subseteq B \subseteq (j, i)\text{-}\pi g\beta\text{-Cl}(A)$, then B is also IF (i, j) $\pi g\beta$ closed set.

Proof: Let U be an IF (i, j) π -open set in X such that $B \subseteq U$. Since A is IF (i, j) $\pi g\beta$ closed, $\text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A) \subseteq U$, $B \subseteq \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A)$, implies $\text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(B) \subseteq \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A)$, Hence $\text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(B) \subseteq U$. Hence B is also IF (i, j) $\pi g\beta$ closed.

Theorem 3.4 If A is IF (i, j) $\pi g\beta$ -closed set in X , then $A \cup (X \setminus \text{IF}(j, i)\text{-}\beta\text{Cl}(A))$ is also an

IF (i, j) $\pi g\beta$ closed set.

Proof: Let U be IF (i, j) π -open set in X such that $A \cup (X \setminus \text{IF}(j, i)\text{-}\beta\text{Cl}(A)) \subseteq U$, then

$(X \setminus U) \subseteq X \setminus \{A \cup (X \setminus \text{IF}(j, i)\text{-}\beta\text{Cl}(A))\} = X \cap \{A \cup (X \setminus \text{Int}(j\text{-Cl}(i\text{-Int}(A)))^c = X \cap \{A^c \cap [\text{IF}(j, i)\text{-}\beta\text{Cl}(A)] = \text{IF}(j, i)\text{-}\pi g\beta\text{-Cl}(A) \cap A^c = \text{IF}(j, i)\text{-}\beta\text{Cl}(A) \setminus A\}$, since $X \setminus U$ is a IF (i, j) π -closed set and A is a IF (i, j) $\pi g\beta$ -closed set, by theorem $X \setminus U = \emptyset$ and so $X = U$. Thus X is the only IF (i, j) $\pi g\beta$ -open set containing $A \cup (X \setminus \text{IF}(j, i)\text{-}\beta\text{Cl}(A))$.

Hence $A \cup (X \setminus \text{IF}(j, i)\text{-}\beta\text{Cl}(A))$ is a IF (i, j) - $\pi g\beta$ closed set.

Theorem 3.5: If A is IF (i, j) π -open and IF (i, j) $\pi g\beta$ -closed, then A is IF (i, j) β -closed.

Proof. Since A is $IF(i, j)$ π -open and $IF(i, j)$ $\pi g\beta$ -closed, $IF(j, i) \beta\text{-Cl}(A) \subseteq A$, but $A \subseteq IF(j, i) \beta\text{-Cl}(A)$, So $A = IF(j, i) \beta\text{-Cl}(A)$. Hence, A is $IF(i, j)$ β closed.

Theorem 3.6. Let A be an $IF(i, j)$ - $\pi g\beta$ -closed in intuitionistic fuzzy bitopological space X . Then $IF(j, i) \beta\text{-Cl}(A) \setminus A$ does not contain any nonempty $IF(i, j)$ π -closed set.

Proof. Let U be a nonempty $IF(i, j)$ π -closed subset of $IF(j, i) \beta\text{-Cl}(A) \setminus A$. Then $A \subseteq X \setminus U$, where A is $IF(i, j)$ - $\pi g\beta$ -closed and $X \setminus U$ is $IF(i, j)$ - π -open. Thus $IF(j, i) \beta\text{-Cl}(A) \subseteq X \setminus U$, or $U \subseteq X \setminus IF(j, i) \beta\text{-Cl}(A)$. Since by assumption $U \subseteq IF(j, i) \beta\text{-Cl}(A)$, we get a contradiction.

Corollary 3.2. Let A be $IF(i, j)$ - $\pi g\beta$ -closed in X . Then A is $IF(i, j)$ β -closed if and only if $IF(j, i) \beta\text{-Cl}(A) \setminus A$ is $IF(i, j)$ - π -closed.

Proof. Necessity: Let A be an $IF(i, j)$ $\pi g\beta$ -closed, By hypothesis $IF(j, i) \beta\text{-Cl}(A) = A$ and so $IF(j, i) \beta\text{-Cl}(A) \setminus A = \emptyset$ which is $IF(i, j)$ π -closed.

Sufficiency. Suppose $IF(j, i) \beta\text{-Cl}(A) \setminus A$ is $IF(i, j)$ π -closed Then by theorem 3.6,

$IF(j, i) \beta\text{-Cl}(A) \setminus A = \emptyset$, that is, $IF(j, i) \beta\text{-Cl}(A) = A$. Hence, A is $IF(i, j)$ β -closed.

Definition 3.4

In a bitopological space (X, τ_1, τ_2) , Let $B \subseteq A \subseteq X$. Then we say that B is $IF(i, j)$ $\pi g\beta$ -closed relative to A if $IF(j, i) \beta\text{Cl}_A(B) \subseteq U$ where $B \subseteq U$ and U is $IF(i, j)$ π -open in A .

Theorem 3.7 Let $B \subseteq A \subseteq X$ where A is a $IF(i, j)$ $\pi g\beta$ -closed and $IF(i, j)$ π -open set. Then B is $IF(i, j)$ $\pi g\beta$ -closed relative to A if and only if B is $IF(i, j)$ - $\pi g\beta$ -closed in X .

Proof. Here, $B \subseteq A$ and A is both a $IF(i, j)$ $\pi g\beta$ -closed and $IF(i, j)$ open set, then $IF(j, i) \beta\text{Cl}(A) \subseteq A$ and thus $IF(j, i) \beta\text{-Cl}(B) \subseteq IF(j, i) \beta\text{-Cl}(A) \subseteq A$. Now from, $A \setminus IF(j, i) \beta\text{-Cl}(B) = IF(j, i) \beta\text{Cl}_A(B)$, we have $IF(j, i) \beta\text{-Cl}(B) = IF(j, i) \beta\text{-Cl}_A(B) \subseteq A$. If B is $IF(i, j)$ - $\pi g\beta$ -closed relative to A and U is $IF(i, j)$ - π -open subset of X such that $B \subseteq U$, then $B = B \cap A \subseteq U \cap A$ where $U \cap A$ is $IF(i, j)$ π -open in A . Hence as B is $IF(i, j)$ - $\pi g\beta$ -closed relative to A , $IF(j, i) \beta\text{-Cl}(B) = IF(j, i) \beta\text{-Cl}_A(B) \subseteq U \cap A \subseteq U$. Therefore B is $IF(i, j)$ $\pi g\beta$ -closed in X .

Conversely if B is $IF(i, j)$ $\pi g\beta$ -closed in X and U is an Intuitionistic fuzzy bitopological

π -open subset of A such that $B \subseteq U$, then $U = V \cap A$ for some open subset V of X . As $B \subseteq V$ and B is $IF(i, j)$ $\pi g\beta$ -closed in X , $IF(j, i) \beta\text{-Cl}(B) \subseteq V$. Thus $IF(j, i) \beta\text{Cl}_A(B) = IF(j, i) \beta\text{-Cl}(B) \cap A \subseteq V \cap A = U$.

Therefore B is $IF(i, j)$ $\pi g\beta$ -closed relative to A .

Corollary 3.3. Let A be $IF(i, j)$ π -open $IF(i, j)$ $\pi g\beta$ -closed set. Then $A \cap U$ is $IF(i, j)$ $\pi g\beta$ closed whenever $U \in IF(j, i) \beta\text{-Cl}(X)$.

Proof. Since A is $IF(i, j)$ $\pi g\beta$ -closed and $IF(i, j)$ π -open, then $IF(j, i) \beta\text{-Cl}(A) \subseteq A$ and thus A is $IF(i, j)$ β -closed. Again, as $U \in IF(j, i) \beta\text{-Cl}(X)$ and $A \in IF(j, i) \beta\text{-Cl}(X)$, So $A \cap U \in IF(j, i) \beta\text{-Cl}(X)$.

Now, $A \cap U \subseteq A \Rightarrow IF(j, i) \beta\text{-Cl}(A \cap U) \subseteq A$ which means that $A \cap U$ is $IF(i, j)$ - $\pi g\beta$ -closed.

Theorem 3.8 Let A and B be any two Intuitionistic fuzzy subsets of (X, τ_1, τ_2) . Then the following properties hold:

$IF(i, j) \pi g\beta\text{-Int}(A \cap B) \subseteq IF(i, j) \pi g\beta\text{-Int}(A) \cap IF(i, j) \pi g\beta\text{-Int}(B)$.

$IF(i, j) \pi g\beta\text{-Int}(A) \cup (i, j) \pi g\beta\text{-Int}(B) \subseteq IF(i, j) \pi g\beta \text{Int}(A \cup B)$.

Proof. (i). Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, by Remark 3.1 we have $IF(i, j) \pi g\beta \text{Int}(A \cap B) \subseteq IF(i, j) \pi g\beta\text{-Int}(A)$ and $IF(i, j) \pi g\beta \text{Int}(A \cap B) \subseteq IF(i, j) \pi g\beta\text{-Int}(B)$.

Therefore, $IF(i, j) \pi g\beta \text{Int}(A \cap B) \subseteq IF(i, j) \pi g\beta\text{-Int}(A) \cap IF(i, j) \pi g\beta\text{-Int}(B)$.

(ii). We have $IF(i, j) \pi g\beta\text{-Int}(A) \subseteq IF(i, j) \pi g\beta \text{Int}(A \cup B)$ and

$IF(i, j) \pi g\beta\text{-Int}(B) \subseteq IF(i, j) \pi g\beta\text{-Int}(A \cup B)$.

Then $IF(i, j) \pi g\beta\text{-Int}(A) \cup IF(i, j) \pi g\beta\text{-Int}(B) \subseteq IF(i, j) \pi g\beta \text{Int}(A \cup B)$.

Theorem 3.9 A subset A of X is $IF(i, j)$ - $\pi g\beta$ -open if and only if $U \subseteq IF(j, i) \beta\text{-Int}(A)$ whenever U is $IF(i, j)$ π closed in X and $U \subseteq A$.

Proof. Necessity. Let A be $IF(i, j)$ - $\pi g\beta$ -open and $U \subseteq A$, where U is $IF(i, j)$ π -closed.

Then $(X \setminus A) \subseteq (X \setminus U)$ and $(X \setminus U)$ is $IF(i, j)$ π -open.

Therefore, $IF(j, i) \beta\text{-Cl}(X \setminus A) \subseteq (X \setminus U)$.

Hence $IF(j, i) \beta\text{-Cl}(X \setminus A) = X \setminus IF(j, i) \beta\text{-Int}(A) \subseteq (X \setminus U)$.

Thus we have $U \subseteq IF(j, i) \beta\text{-Int}(A)$.

Sufficiency. If U is $IF(i, j)$ π -closed and $U \subseteq IF(j, i) \beta\text{-Int}(A)$ whenever $U \subseteq A$, then $(X \setminus A) \subseteq (X \setminus U)$ and $(X \setminus U) \subseteq IF(j, i) \beta\text{-Int}(A) \subseteq (X \setminus U)$. That is, $IF(j, i) \pi g\beta\text{-Cl}((X \setminus A) \subseteq (X \setminus U))$.

Therefore, $(X \setminus A)$ is $IF(i, j)$ - $\pi g\beta$ -closed and hence A is $IF(i, j)$ $\pi g\beta$ -open.

Definition 3.5

The $IF(i, j)$ π -kernel of A is the intersection of all $IF(i, j)$ π -open sets containing A and the intersection of all the $IF(i, j)$ $\pi g\beta$ -open sets containing A which is by the usual notation, $IF(i, j) \pi g\beta\text{-ker}(A)$.

Lemma 3.2 Let X be an intuitionistic fuzzy bi topological space and $x \in X$. The following are equivalent.

(i) $x \in IF(i, j) \pi g\beta\text{ker}\{y\}$

(ii) $y \in IF(i, j) \pi g\beta\text{ker}\{x\}$

Proof. (i) \Rightarrow (ii): If $y \notin IF(i, j) \pi g\beta\text{ker}\{x\}$, then there exists an $IF(i, j)$ - $\pi g\beta$ open set U containing x such that $x \notin IF(i, j) \pi g\beta \text{ker}\{y\}$

(ii) \Rightarrow (i): *Proof* is similar.

Lemma 3.3 The following statements are equivalent for any two points x, y in an intuitionistic fuzzy bi topological space X .

(i) $IF(i, j) \pi g\beta\text{ker}\{x\} \neq IF(i, j) \pi g\beta\text{ker}\{y\}$

(ii) $IF(i, j) \pi g\beta\text{-Cl}\{x\} \neq IF(i, j) \pi g\beta\text{-Cl}\{y\}$

Proof. (i) \Rightarrow (ii): Let $IF(i, j) \pi g\beta\text{ker}\{x\} \neq IF(i, j) \pi g\beta\text{ker}\{y\}$

Then there exists a point z in X such that $z \in IF(i, j) \pi g\beta\text{ker}\{x\}$ & $z \notin IF(i, j) \pi g\beta\text{ker}\{y\}$ $z \in IF(i, j) \pi g\beta\text{ker}\{x\} \Rightarrow \{x\} \cap IF(i, j) \pi g\beta\text{ker}\{z\} \neq \emptyset$. Then we have $x \in IF(i, j) \pi g\beta \text{ker}\{z\}$. From

$z \notin IF(i, j) \pi g\beta\text{ker}\{y\}$ it follows that $\{y\} \cap IF(i, j) \pi g\beta\text{ker}\{z\} = \emptyset$. Since $x \in IF(i, j) \pi g\beta\text{ker}\{z\}$,

$IF(i, j) \pi g\beta\text{ker}\{x\} \subseteq IF(i, j) \pi g\beta\text{ker}\{z\}$ and $\{y\} \cap IF(i, j) \pi g\beta\text{ker}\{x\} = \emptyset$,

Hence $IF(i, j) \pi g\beta\text{ker}\{x\} \neq IF(i, j) \pi g\beta\text{ker}\{y\}$.

(ii) \Rightarrow (i): Let $IF(i, j) \pi g\beta\text{-Cl}\{x\} \neq IF(i, j) \pi g\beta\text{-Cl}\{y\}$, Then there exists a point

z in X such that $z \in IF(i, j) \pi g\beta\text{-Cl}\{x\}$ and $z \notin IF(i, j)$

$\pi\beta\text{-Cl}\{y\}$

Hence there exists an $\text{IF}(i, j) \pi\beta$ -open set containing z and x but not y . Therefore, $y \notin \text{IF}(i, j) \pi\beta\text{-ker}\{x\}$ and $\text{IF}(i, j) \pi\beta\text{-ker}\{x\} \neq \text{IF}(i, j) \pi\beta\text{-ker}\{y\}$

Definition 3.6 A space X is said to be $\text{IF}(i, j)$ - $\pi\beta R_0$ if every $\text{IF}(i, j)$ - $\pi\beta$ -open set contains the $\text{IF}(i, j)$ - $\pi\beta$ -closure of each of its singletons.

Lemma 3.4 A space X is $\text{IF}(i, j)$ - $\pi\beta R_0$ if and only if for any x and y in X and $\text{IF}(i, j)$ - $\pi\beta\text{-cl}\{x\} \neq \text{IF}(i, j)$ - $\pi\beta\text{-cl}\{y\}$ implies and $\text{IF}(i, j)$ - $\pi\beta\text{-cl}\{x\} \cap \text{IF}(i, j)$ - $\pi\beta\text{-cl}\{y\} = \emptyset$.

Proof. Necessity. If X is and (i, j) - $\pi\beta R_0$ and x, y in X such that and (i, j) - $\pi\beta\text{-cl}\{x\} \neq (i, j)$ - $\pi\beta\text{-cl}\{y\}$ then there exists $z \in$ and (i, j) - $\pi\beta\text{-cl}\{x\}$ such that $z \notin (i, j)$ - $\pi\beta\text{-cl}\{y\}$, Therefore, there exists $V \in$ and $(i, j) \pi\beta$ open set (X) such that $y \notin V$ and $z \in V$ and hence $x \in V$. Thus we get $x \notin (j, i) \pi\beta\text{-Cl}\{y\}$ and therefore, $x \in X \setminus (j, i) \pi\beta\text{-Cl}\{y\}$. This implies that $(i, j) \pi\beta\text{-Cl}\{x\} \subset X \setminus (j, i) \pi\beta\text{-Cl}\{y\}$, and therefore, $(j, i) \pi\beta\text{-Cl}\{x\} \cap (j, i) \pi\beta\text{-Cl}\{y\} = \emptyset$

Sufficiency. Let V be (i, j) - $\pi\beta$ open and $x \in V$. If $y \in X \setminus V$, then $x \neq y$ and $x \notin (j, i) \pi\beta\text{-Cl}\{y\}$. This shows that $(j, i) \pi\beta\text{-Cl}\{x\} \neq (j, i) \pi\beta\text{-Cl}\{y\}$ and hence by our assumption, $(j, i) \pi\beta\text{-Cl}\{x\} \cap (j, i) \pi\beta\text{-Cl}\{y\} = \emptyset$ Hence $y \notin (j, i) \pi\beta\text{-Cl}\{x\}$. Therefore, $(j, i) \pi\beta\text{-Cl}\{x\} \subset V$.

Theorem 3.10 A space X is (i, j) - $\pi\beta R_0$ if and only if for any x and y in X ,

$(i, j) \pi\beta\text{-ker}\{x\} \neq (i, j) \pi\beta\text{-ker}\{y\}$ implies $(j, i) \pi\beta\text{-Cl}\{x\} \cap (j, i) \pi\beta\text{-Cl}\{y\} = \emptyset$.

Proof. Suppose that X is $(i, j) \pi\beta R_0$ and if for any x and y in X , $(i, j) \pi\beta\text{-ker}\{x\} \neq (i, j) \pi\beta\text{-ker}\{y\}$ then by Lemma 3.3 $(j, i) \pi\beta\text{-Cl}\{x\} \neq (j, i) \pi\beta\text{-Cl}\{y\}$. If $z \in (i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{y\}$ then from $z \in (i, j) \pi\beta\text{-ker}\{x\}$ and by Lemma 3.2, it follows that $x \in (i, j) \pi\beta\text{-ker}\{z\}$. Since $x \in (i, j) \pi\beta\text{-ker}\{x\}$, by Lemma 3.4, $(i, j) \pi\beta\text{-ker}\{x\} = (i, j) \pi\beta\text{-ker}\{z\}$. Similarly, we have $(i, j) \pi\beta\text{-ker}\{y\} = (i, j) \pi\beta\text{-ker}\{z\}$ is a contradiction. Therefore, $(i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{y\} = \emptyset$

Conversely, let x, y be any two points in X such that $(i, j) \pi\beta\text{-ker}\{x\} \neq (i, j) \pi\beta\text{-ker}\{y\}$ implies

$(i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{y\} = \emptyset$; If $(i, j) \pi\beta\text{-ker}\{x\} \neq (i, j) \pi\beta\text{-ker}\{y\}$, then by Lemma 3.3, $(i, j) \pi\beta\text{-ker}\{x\} \neq (i, j) \pi\beta\text{-ker}\{y\}$, Hence $(i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{y\} = \emptyset$. $(i, j) \pi\beta\text{-Cl}\{x\} \cap (i, j) \pi\beta\text{-Cl}\{y\} = \emptyset$. For, if $z \in ((i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{z\}) \neq (i, j) \pi\beta\text{-ker}\{y\}$ and therefore, $(i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{y\} \neq \emptyset$. Therefore, by hypothesis, $((i, j) \pi\beta\text{-ker}\{x\} \cap (i, j) \pi\beta\text{-ker}\{y\}) \neq \emptyset$. Then $z \in ((i, j) \pi\beta\text{-Cl}\{x\} \cap (i, j) \pi\beta\text{-Cl}\{y\})$ implies that $(i, j) \pi\beta\text{-ker}\{x\} = (i, j) \pi\beta\text{-ker}\{z\} \neq (i, j) \pi\beta\text{-ker}\{y\}$ a contradiction. Therefore, by Lemma 3.4, X is $(i, j) \pi\beta R_0$.

Theorem 3.11. Let (X, τ_1, τ_2) be an intuitionistic fuzzy bitopological space. If A and B are two $\text{IF}(i, j)$ - $\pi\beta$ -open sets in X such that $\text{IF}(j, i) \pi\beta\text{-Cl}(A) \cap B = \emptyset$ and $A \cap \text{IF}(j, i) \pi\beta\text{-Cl}(B) = \emptyset$, then $A \cup B$ is $\text{IF}(i, j) \pi\beta$ -open.

Proof. Let A and B be two $\text{IF}(i, j) \pi\beta$ -open sets in X such that $\text{IF}(j, i) \pi\beta\text{-Cl}(A) \cap B = \emptyset$. and $A \cap \text{IF}(j, i) \pi\beta\text{-Cl}(B) = \emptyset$. Suppose V is an $\text{IF} \pi$ -closed and $V \subset A \cup B$. Clearly

$V \subset A$ and $V \subset B$. Then $V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A) \subset A \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A) = A$ and $V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(B) \subset B \cap \text{IF}(j, i) \pi\beta\text{-Cl}(B) = B$. By hypothesis we have $V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A) \subset$

$\text{IF}(j, i) \pi\beta\text{-Int}(A)$ and $(V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(B) \subset \text{IF}(j, i) \pi\beta\text{-Int}(B)$.

This implies $V \cap (j, i) \pi\beta\text{-Cl}(A) \subset \text{IF}(j, i) \pi\beta\text{-Int}(A)$ and $V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(B) \subset \text{IF}(j, i) \pi\beta\text{-Int}(B)$. Then $V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A) \subset \text{IF}(j, i) \pi\beta\text{-Int}(A) \cup (V \cap (j, i) \pi\beta\text{-Cl}(B) \subset \text{IF}(j, i) \pi\beta\text{-Int}(B)$, Which implies $(V \cap (j, i) \pi\beta\text{-Cl}(A) \cup (j, i) \pi\beta\text{-Cl}(B) \subset \text{IF}(j, i) \pi\beta\text{-Int}(A) \cup \text{IF}(j, i) \pi\beta\text{-Int}(B)$.

Thus $(V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A \cup B) \subset \text{IF}(j, i) \pi\beta\text{-Int}(A) \cup \text{IF}(j, i) \pi\beta\text{-Int}(B)$. Further,

$V = V \cap (A \cup B) \subset V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A \cup B)$, we have $V \subset (j, i) \pi\beta\text{-Int}(A \cup B) \subset (V \cap (j, i) \pi\beta\text{-Cl}(A \cup B))$

$\subset \text{IF}(j, i) \pi\beta\text{-Int}(A \cup B) \subset (V \cap \text{IF}(j, i) \pi\beta\text{-Cl}(A \cup B) \subset (j, i) \pi\beta\text{-Int}(A) \cup \text{IF}(j, i) \pi\beta\text{-Int}(B)$.

This shows that $V \subset \text{IF}(j, i) \pi\beta\text{-Int}(A \cup B)$. Hence $A \cup B$ is $(i, j) \pi\beta$ -open.

Theorem 3.12 Let (X, τ_1, τ_2) be an intuitionistic fuzzy bitopological space. If A is (i, j) - $\pi\beta$ -open set relative to B such that $A \subset B \subset X$ and B is (i, j) - $\pi\beta$ -open relative to X , then A is (i, j) - $\pi\beta$ -open relative to X .

Proof. Let $U \subset A$ and U be $\text{IF}(i, j) \pi$ -closed. Suppose A is $(i, j) \pi\beta$ -open relative to B . Then we have $U \subset \text{IF}(j, i) \beta\text{-Int}(A)$, Which implies that there exists an $\text{IF}(j, i) \beta$ -open set V_1 such that

$U \subset V_1 \cap B \subset A$. Let $U \subset B$ and U be $\text{IF} \pi$ -closed. Suppose B is $\text{IF}(i, j) \pi\beta$ -open relative to X . Then we have $U \subset \text{IF}(j, i) \pi\beta\text{-Int}(B)$ Which implies that there exists an $\text{IF}(j, i) \pi\beta$ -open set V_2 such that $U \subset V_2 \subset B$. Further $U \subset (V_1 \cap B) \cap V_2 \subset (V_1 \cap B) \cap B = V_1 \cap B \subset A$. This shows that $U \subset \text{IF}(j, i) \pi\beta\text{-Int}(A)$. Hence A is $\text{IF}(i, j) \pi\beta$ -open relative to X .

4. $\text{IF}(i, j)$ - $\pi\beta$ -Continuous Functions and Irresoluteness

Definition 4.1 Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a mapping from an IFBTS X to an IFBTS Y . Then f is said to be IF pairwise continuous if the inverse image of every σ_i -open set of Y is $\text{IF}(i, j)$ -open in X , where $i \neq j, i, j = 1, 2$.

Definition 4.2 Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a mapping from an IFBTS X to an IFBTS Y . Then f is said to be IF pairwise $\pi\beta$ continuous if $f^{-1}(A)$ is $\text{IF}(i, j) \pi\beta$ -open in X for each $\text{IF} \sigma_1$ -open set A in Y and $f^{-1}(B)$ is $\text{IF}(j, i) \pi\beta$ -open in X for each $\text{IF} \sigma_2$ -open set B in Y .

Theorem 4.1 For a function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

- (i) f is IF pairwise $\pi\beta$ -continuous.
- (ii) For each point x in X and each σ_i -open set F in Y such that $f(x) \in F$, there exists an $\text{IF}(i, j) \pi\beta$ -open set A in X such that $x \in A, f(A) \subset F$.
- (iii) The inverse image of each σ_i -closed set in Y is $\text{IF}(i, j) \pi\beta$ -closed in X .
- (iv) For each subset A of $X, f(\text{IF}(j, i) \pi\beta\text{-Cl}(A)) \subset \sigma_i\text{-Cl}(f(A))$.
- (v) For each subset B of $Y, \text{IF}(j, i) \pi\beta\text{-Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-Cl}(B))$.
- (vi) For each subset C of $Y, f^{-1}(\sigma_i \text{Int}(C)) \subset (j, i) \pi\beta\text{-Int}(f^{-1}(C))$.

Proof:(i) \Rightarrow (ii): Let $x \in X$ and U be a σ_i -open set of Y containing $f(x)$. By (i), $f^{-1}(U)$ is an $IF(i, j)$ - $\pi g\beta$ -open in X , Let $A = f^{-1}(U)$, Then $x \in A$ and $f(A) \subset U$.

(ii) \Rightarrow (i) Let U be σ_i -open in Y and let $x \in f^{-1}(U)$. Then $f(x) \in U$, By (ii) there is an $IF(i, j)$ - $\pi g\beta$ open set U_x in x such that $x \in U_x$ and $f(U_x) \subset U$, Then $x \in U_x \subset f^{-1}(U)$,

Hence $f^{-1}(U)$ is $IF(i, j)$ - $\pi g\beta$ -open in X .

(i) \Rightarrow (iii) For any subset B of Y , $f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$

(iii) \Rightarrow (iv) Let A be a subset of X . Since $A \subset f^{-1}(f(A))$, we have $A \subset f^{-1}(\sigma_i\text{-Cl}(f(A)))$, Now $\sigma_i\text{-Cl}(f(A))$ is σ_i -closed in Y . Hence $IF(i, j)$ - $\pi g\beta\text{-Cl}(A) \subset f^{-1}(\sigma_i\text{-Cl}(f(A)))$ for $IF(i, j)$ - $\pi g\beta\text{-Cl}(A)$ is the smallest $IF(i, j)$ - $\pi g\beta$ -closed set containing A .

Then $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(A)) \subset (\sigma_i\text{-Cl}(f(A)))$.

(iv) \Rightarrow (iii) Let U be any $IF(i, j)$ - $\pi g\beta$ -closed subset of Y .

Then $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(U))) \subset (\sigma_i\text{-Cl}(f(f^{-1}(U)))) = \sigma_i\text{-Cl}(U) = U$,

hence $IF(i, j)\text{-}\pi g\beta\text{-Cl}(A) \subset f^{-1}(U)$ which implies $f^{-1}(U)$ is $IF(i, j)$ - $\pi g\beta$ closed in X .

(iv) \Rightarrow (v) Let B be any subset of Y .

Now, $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B))) \subset \sigma_i\text{-}\pi g\beta\text{-Cl}(A(f^{-1}(B)))$

$\subset \sigma_i\text{-Cl}(B)$, then, $IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-Cl}(B))$.

(v) \Rightarrow (iv) Let $B = f(A)$, where A is a subset of X .

Then, $IF(i, j)\text{-}\pi g\beta\text{-Cl}(A) \subset IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B))$

$\subset f^{-1}(\sigma_i\text{-Cl}(B)) = f^{-1}(\sigma_i\text{-Cl}(f(A)))$,

Then $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(A)) \subset (\sigma_i\text{-Cl}(f(A)))$.

(i) \Rightarrow (vi) Let B be any σ_i -open set in Y , then $\sigma_i\text{-int}(B) = B$ and $f^{-1}(B) \setminus f^{-1}(\sigma_i\text{-int}(B)) \subset IF(i, j)\text{-}\pi g\beta\text{-int}(f^{-1}(B))$. Hence, $f^{-1}(B) \setminus IF(i, j)\text{-}\pi g\beta\text{-int}(f^{-1}(B)) = \emptyset$, Hence $f^{-1}(B)$ is $IF(i, j)$ - $\pi g\beta$ -open in X .

Theorem 4.2. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is ij - $\pi g\beta$ -continuous and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is ij - $\pi g\beta$ -irresolute function, then $g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is ij - $\pi g\beta$ -continuous.

Proof. Let $V \in ij$ - $\pi g\beta$ - $C(Z)$, since g is ij - $\pi g\beta$ -irresolute, then $g^{-1}(V) \in ij$ - $\pi g\beta$ - $C(Y)$. Since f is ij - $\pi g\beta$ -continuous, then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \in ij$ - $\pi g\beta$ - $C(X)$. Consequently, $g \circ f$ is ij - $\pi g\beta$ -continuous.

Theorem 4.3. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function. If $g: (X, \tau_1, \tau_2) \rightarrow (X \times Y, \sigma_1 \times \sigma_2)$ defined by $g(x) = (x, f(x))$ is an $IF(i, j)$ - $\pi g\beta$ -continuous function, then f is $IF(i, j)$ - $\pi g\beta$ -continuous.

Proof. Let V be a σ_i -open set of Y . Then $f^{-1}(V) = X \cap f^{-1}(V) = g^{-1}(X \times V)$. Since g is an $IF(i, j)$ - $\pi g\beta$ -continuous function and $X \times V$ is a $\sigma_i \times \tau_i$ -open set of $X \times Y$, $f^{-1}(V)$ is an $IF(i, j)$ - $\pi g\beta$ -open set of X . Hence f is an $IF(i, j)$ - $\pi g\beta$ -continuous.

5. $IF(i, j)$ - $\pi g\beta$ -Irresolute Functions

Definition 5.1 A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $IF(i, j)$ - $\pi g\beta$ -irresolute if for each $x \in X$ and each $V \in IF(i, j)\text{-}\pi g\beta O(Y, f(x))$, there exists $U \in IF(i, j)\text{-}\pi g\beta O(X, x)$ such that $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(U)) \subseteq (IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$.

Theorem 5.1 For a function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ the following are equivalent.

(i) f is $IF(i, j)$ - $\pi g\beta$ -irresolute.

(ii) $IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(B))$ for every

subset B of Y .

(iii) $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(A)) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(f(A))$ for every subset A of X .

Proof:

(i) \Rightarrow (ii): Let B be any subset of Y . Suppose that $x \notin f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(B))$. Then $f(x) \notin IF(i, j)\text{-}\pi g\beta\text{-Cl}(B)$ and there exists $V \in IF(i, j)\text{-}\pi g\beta O(X, f(x))$ such that $IF(i, j)\text{-}\pi g\beta\text{-Cl}(V) \cap B = \emptyset$; Since f is $IF(i, j)$ - $\pi g\beta$ -irresolute, there exists $U \in IF(i, j)\text{-}\pi g\beta O(X, x)$ such that $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(U)) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)$. Therefore, $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(U)) \cap B = \emptyset$; and $IF(i, j)\text{-}\pi g\beta\text{-Cl}(U) \cap f^{-1}(B) = \emptyset$.

Hence, $x \notin IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B))$, Therefore, $IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(B))$.

(ii) \Rightarrow (iii): Let A be any subset of X .

Then $IF(i, j)\text{-}\pi g\beta\text{-Cl}(A) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(f(A))) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(f(A)))$ and hence $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(A)) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(f(A))$.

(iii) \Rightarrow (ii): Let B be a subset of Y By (iii), $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B))) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(f(f^{-1}(B))) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(B)$ and $IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(B))$.

(ii) \Rightarrow (i): Let $x \in X$ and $V \in (IF(i, j)\text{-}\pi g\beta O(Y, f(x)))$. Then $IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)$ and

$Y \setminus (IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$ are disjoint and $f(x) \notin IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)$. Hence $x \notin f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(Y \setminus (IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))))$ and by (ii), $x \notin IF(i, j)\text{-}\pi g\beta\text{-Cl}(Y \setminus (IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)))$. Then there exists $U \in IF(i, j)\text{-}\pi g\beta O(X, x)$ such that $IF(i, j)\text{-}\pi g\beta\text{-Cl}(U) \cap f^{-1}(Y \setminus (IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))) = \emptyset$ and then $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(U)) \subseteq IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)$. Hence, f is $IF(i, j)$ - $\pi g\beta$ -irresolute.

Theorem 5.2. For a function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ the following properties are equivalent.

(i) f is $IF(i, j)$ - $\pi g\beta$ -irresolute.

(ii) $f^{-1}(V) \subseteq IF(i, j)\text{-}\pi g\beta\text{-int}(f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)))$ for every $V \in IF(i, j)\text{-}\pi g\beta O(Y)$.

(ii) $IF(i, j)\text{-}\pi g\beta\text{-}(f^{-1}(V)) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$ for every $V \in IF(i, j)\text{-}\pi g\beta O(Y)$.

Proof. (i) \Rightarrow (ii): Let $V \in IF(i, j)\text{-}\pi g\beta O(Y)$ and $x \in f^{-1}(V)$. Then $f(x) \in V$ and there exists $U \in (IF(i, j)\text{-}\pi g\beta O(X, x))$ such that $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(U)) \subseteq (IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$. Thus $x \in U \subseteq (IF(i, j)\text{-}\pi g\beta\text{-Cl}(U)) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$ and $x \in IF(i, j)\text{-}\pi g\beta\text{-int}(f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)))$. Hence $f^{-1}(V) \subseteq IF(i, j)\text{-}\pi g\beta\text{-int}(f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)))$.

(ii) \Rightarrow (iii): Let $V \in IF(i, j)\text{-}\pi g\beta O(Y)$ and $x \notin f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$.

Then $f(x) \notin IF(i, j)\text{-}\pi g\beta\text{-Cl}(V)$ and there exists $W \in IF(i, j)\text{-}\pi g\beta O(Y, f(x))$ such that $W \cap V = \emptyset$. and $IF(i, j)\text{-}\pi g\beta\text{-Cl}(W) \cap V = \emptyset$. Then $f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(W)) \cap f^{-1}(V) = \emptyset$.

Now $x \in f^{-1}(W)$ and by (ii), $x \in IF(i, j)\text{-}\pi g\beta\text{-int}(f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(W)))$.

There exists $U \in IF(i, j)\text{-}\pi g\beta O(X, x)$ such that $f(IF(i, j)\text{-}\pi g\beta\text{-Cl}(U))$

$\subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(W))$. Thus $IF(i, j)\text{-}\pi g\beta\text{-Cl}(U) \cap f^{-1}(V) = \emptyset$

and hence $x \notin IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(V))$.

Thus we get $IF(i, j)\text{-}\pi g\beta\text{-Cl}(f^{-1}(V)) \subseteq f^{-1}(IF(i, j)\text{-}\pi g\beta\text{-Cl}(V))$.

(iii) (i) Let $x \notin X$ and $V \notin IF(i, j)\text{-}\pi g\beta O(Y, f(x))$ Then

$$V \cap (Y \setminus \text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(V)) = \phi$$

$$\text{and } f(x) \notin \text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(Y \setminus \text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(V)).$$

Therefore, $x \notin f^{-1}(\text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(Y \setminus \text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(V)))$ and by (iii),

$$x \notin ((\text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(f^{-1}Y \setminus (\text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(V))))).$$

There exists $U \in (\text{IF}(i,j)\text{-}\pi g\beta\text{O}(X, x))$ such that

$$(\text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(U) \cap f^{-1}(Y \setminus (\text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(V)))) = \phi.$$

Hence $f(\text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(U)) \subseteq \text{IF}(i,j)\text{-}\pi g\beta\text{-Cl}(V)$ and hence f is $\text{IF}(i,j)\text{-}\pi g\beta$ -irresolute.

Theorem 5.3 Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be any two functions. Then

(i) $g \circ f$ is $\text{IF } \pi g\beta$ -continuous, if g is IF -continuous and f is $\text{IF-}\pi g\beta$ -continuous,

(ii) $g \circ f$ is $\text{IF-}\pi g\beta$ -continuous, if g is $\text{IF-}\pi g\beta$ -continuous and f is $\text{IF-}\pi g\beta$ -irresolute,

(iii) $g \circ f$ is $\text{IF-}\pi g\beta$ irresolute, if g is $\text{IF-}\pi g\beta$ -irresolute and f is $\text{IF-}\pi g\beta$ -irresolute.

Proof.(i) Let V be IF closed in Z . Then $g^{-1}(V)$ is closed in Y , since g is IF -continuous. $\text{IF}(i,j)$ $\pi g\beta$ -continuity of f implies that $f^{-1}(g^{-1}(V))$ is $\text{IF } \pi g\beta$ -closed in X . Hence $g \circ f$ is $\text{IF-}\pi g\beta$ -continuous.

(ii) Let V be closed in Z . Since g is $\text{IF-}\pi g\beta$ -continuous, $g^{-1}(V)$ is $\text{IF } \pi g\beta$ -closed in Y . As f is $\text{IF } \pi g\beta$ -irresolute, $f^{-1}(g^{-1}(V))$ is $\pi g\beta$ -closed in X . Hence $g \circ f$ is $\text{IF-}\pi g\beta$ -continuous.

(iii) Let V be $\text{IF}(i,j)$ $\pi g\beta$ -closed in Z . Then $g^{-1}(V)$ is $\text{IF } \pi g\beta$ -closed in Y , since g is $\text{IF}(i,j)$ $\pi g\beta$ -irresolute. Because f is $\text{IF } \pi g\beta$ -irresolute, $f^{-1}(g^{-1}(V))$ is $\text{IF } \pi g\beta$ -closed in X . Hence $g \circ f$ is $\text{IF } \pi g\beta$ -irresolute.

Definition 5.2: A function Let $f: X \rightarrow Y$ is called a $\text{IF}(i,j)\pi g\beta$ -homeomorphism if

(i) f is bijective.

(ii) f is $\text{IF}(i,j)\pi g\beta$ -irresolute.

(iii) f^{-1} is $\text{IF}(i,j)\pi g\beta$ -irresolute.

We denote the collection of all the $\text{IF}(i,j)\pi g\beta$ -homeomorphisms $f: X \rightarrow Y$ by $\text{IF}(i,j)\pi g\beta\text{h}(X)$.

Theorem 5.4: The collection $\text{IF}(i,j)\pi g\beta\text{h}(X)$ is a group.

Proof: Define a binary operation $*$: $\text{IF}(i,j)\pi g\beta\text{h}(X) \rightarrow \text{IF}(i,j)\pi g\beta\text{h}(X)$ by $*(f, g) = g \circ f$.

Then $*$ is well-defined and it is easily proved that under this binary operation $\text{IF}(i,j)\pi g\beta\text{h}(X)$ is a group.

6. Conclusion

In this paper, we introduce the concept of $\pi g\beta$ closed set in intuitionistic fuzzy bitopological spaces and study some of their properties. We also introduce the concept of $\pi g\beta$ continuous functions in bitopological spaces and some of their properties have been established. We hope that the findings in this paper are just the beginning of a new structure and not only will form the theoretical basis for further applications of topology on Intuitionistic fuzzy bitopological sets but also will lead to the development of information system and various fields in engineering.

References

- [1] Abd. El-Monsef. M. E, El Deeb. S. N. and Mahmoud. R. A, “open sets and-continuous mappings”, Bull. Fac. Sci. Assiut Univ., 12 (1983), 77-90.
- [2] Andrijevic. D, “Semi-preopen sets” Mat. Vesnik, 38 (1986), no. 1, 24-32.
- [3] K. Atanassova, S. Stoeva, Intuitionistic Fuzzy Sets, In Polish Symposium on Interval and Fuzzy Mathematics Poznan (1983) 23-26.
- [4] M. Caldas and S. Jafari - Weak and strong forms of β Irresoluteness - The Arabian Journal for Science and Engineering, Volume 31, Number 1A-31-39.
- [5] D. Coker, An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems 88(1997) 81-89.
- [6] Jin Tae Kim, Seok Jong Lee- Some Properties of Intuitionistic Fuzzy Bitopological Spaces- SCIS-ISIS 2012, Kobe, Japan, November 20-24, 2012.
- [7] A. Kandil, “Biproximities and fuzzy bitopological spaces,” Simon Stevin, vol. 63, pp. 45–66, 1989.
- [8] J. C. Kelly, Bitopological spaces, Proc. London Math. Soc., 13, 71-89 (1963).
- [9] Raja Rajeswari. R and Lellis Thivagar. M, “On Extension of Semi-pre open sets in Bitopological Spaces”, Proc. of the National Conference in Pure and Applied Mathematics, (2005), 28-32.