An Elementary Proof of Fermat's Last Theorem

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Abstract: This paper presents an elementary proof of Fermat's Last Theorem using mathematics that was available to Fermat. It shows that equation $a^n = b^n + c^n$ has no solution in natural numbers if a is prime or a power of a prime then uses that result to present a general proof.

Keywords: Fermat's Last Theorem, Pierre de Fermat

1. Introduction

Fermat's Last Theorem was the conjecture that equation

$$a^n = b^n + c^n \quad (1)$$

has no solution in natural numbers a , b , c , and n if $n > 2$. I will first present a partial proof showing Equation 1 to have no solution in natural numbers if a is prime or a power of a prime and then use that result to present a general proof.

2. Partial Proof

Fermat himself proved the case of $n = 4$ and thereby proved all cases in which n is a multiple of 4. Since all other integers greater than 2 are divisible by an odd prime, it is sufficient to assume that n is an odd prime. It can also be assumed that a , b , and c are relatively prime.

The relationships among a , b , and c are: $a > b$, $a > c$, and $(b + c)/2 < a < b + c < 2a$. The identity

$$\begin{aligned} a^n &= b^n + c^n \\ &= b^n - (-c)^n \\ &= (b + c)[b^{n-1} + b^{n-2}(-c) + \dots + b(-c)^{n-2} + (-c)^{n-1}] \end{aligned}$$

shows that $b + c$ divides a^n . Since $b + c > a$, there exists a natural number y such that $b + c = a + y$. If a is either prime or a power of a prime, there exist natural numbers p and m , where p is prime, such that $p^m = a$, $p^{mn} = b^n + c^n$, and $p^m + y$ divides p^{mn} . Since p is the only prime divisor of p^{mn} , any divisor of the form $p^m + y$ must equal p^k , where the natural

number $m < k \leq mn$. Thus $p^m + y = p^k$, $y = p^k - p^m = p^m(p^{k-m} - 1)$, and a divides y . But that is a contradiction because $a > (a + y)/2$ and $a > y$.

3. General Proof

Since $p^{mn} \neq b^n + c^n$, to prove Fermat's Last Theorem it remains to show that p^{mn} is not equal to the sum of the n th powers of two rational numbers. For if $a^n = b^n + c^n$, there must exist a natural number $w > 1$ such that $p^mw = a$, $p^{mn}w^n = b^n + c^n$, and $p^{mn} = b^n/w^n + c^n/w^n$.

Since $p^{mn} \neq b^n + c^n$, there must exist a natural number $t > 1$ such that $p^{mn} = b^n + t$ and $t^{1/n}$ is irrational. If there is a prime number $p' < p$ such that $p'^{mn} = b^n/w^n + t/w^n$, then p'^{mn} is not equal to the sum of the n th powers of two rational numbers because $(t/w^n)^{1/n}$ is irrational.

Consequently Equation 1 has no solution in natural numbers if $n > 2$.

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References

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